

Real-Time Evolution:

- Time evolution of a state described by the *time-dependent Schrödinger* equation:

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle$$

- Formal solution can be given as:

$$\hat{U}(t, t_0) = \mathcal{T} \left\{ \exp \left(-\frac{i}{\hbar} \int_{t_0}^t \hat{H}(t') dt' \right) \right\} \quad |\psi(t)\rangle = \hat{U}(t, t_0) |\psi(t_0)\rangle$$

- In case of time independent Hamiltonian H :

$$\hat{U}(t, t_0) = e^{-i\hat{H}(t-t_0)/\hbar} \quad |\psi(t)\rangle = e^{-i\hat{H}(t-t_0)/\hbar} |\psi(t_0)\rangle$$

Nineteen Dubious Ways to Compute the Exponential of a Matrix, Twenty-Five Years Later*

Cleve Moler[†]

Charles Van Loan[‡]

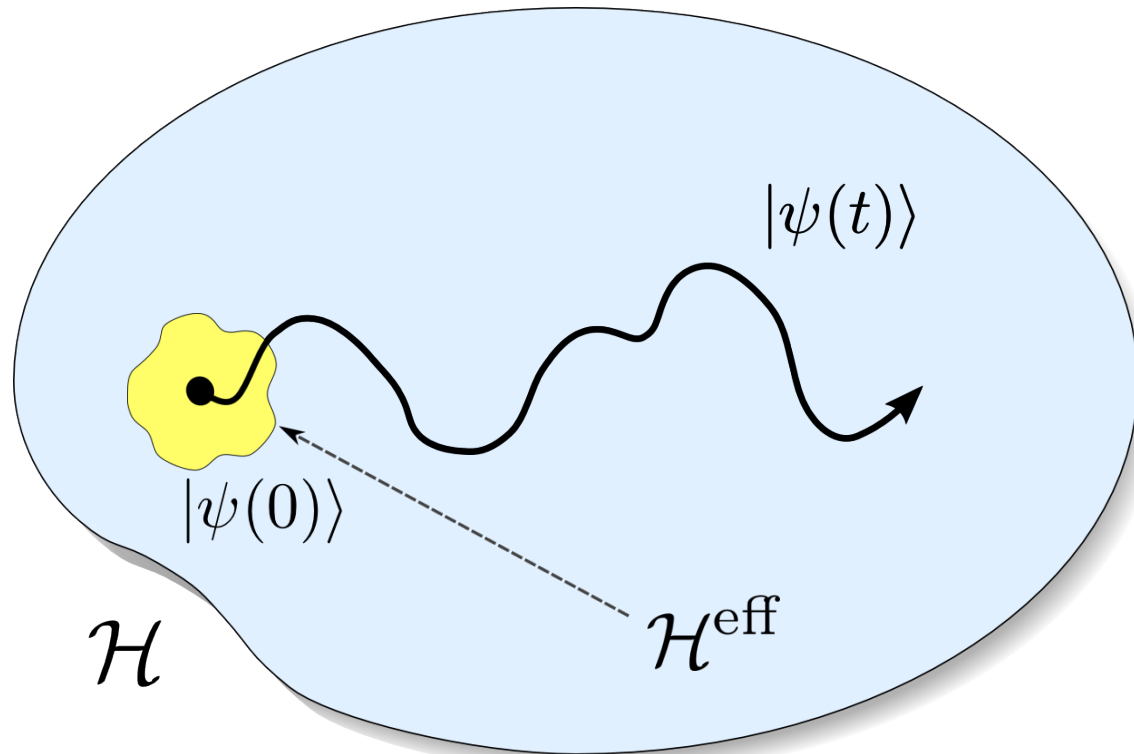
Abstract. In principle, the exponential of a matrix could be computed in many ways. Methods involving approximation theory, differential equations, the matrix eigenvalues, and the matrix characteristic polynomial have been proposed. In practice, consideration of computational stability and efficiency indicates that some of the methods are preferable to others, but that none are completely satisfactory.

Most of this paper was originally published in 1978. An update, with a separate bibliography, describes a few recent developments.

Real-Time Evolution Using *DMRG*

time-dependent DMRG

- Quantum system with L sites with *local Hilbert* spaces \mathcal{H}_j ($j = 1, \dots, L$)
 - The *Hilbert* space of the whole system: $\mathcal{H} = \bigotimes_{j=1}^L \mathcal{H}_j$
 - the dimension of the whole Hilbert space : $\dim(\mathcal{H}) = \prod_j^L d_j$



M. A. Cazalilla and J. B. Marston,
PRL **88**, 256403 (2002),
PRL **91**, 049702 (2003)

Real-Time Evolution:

■ Suzuki-Trotter decomposition:

$$e^{-i\hat{H}\delta t} = e^{-i\hat{H}_{\text{odd}}\delta t/2} e^{-i\hat{H}_{\text{even}}\delta t} e^{-i\hat{H}_{\text{odd}}\delta t/2} + O((\delta t)^3)$$

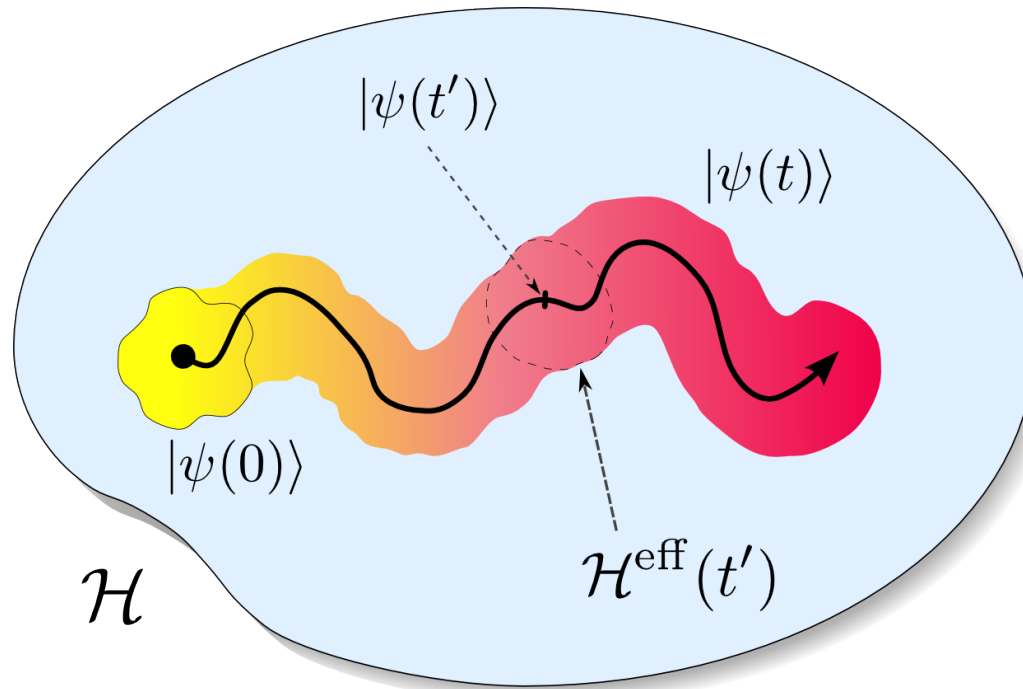
G. Vidal, *PRL* **91**, 147902 (2003)
PRL **93**, 040502 (2004)

A.J Daley, C. Kollath, U. Schollwöck, G. Vidal
J. Stat. Mech.: Theor. Exp. (2004)

S.R. White and A. E. Feiguin,
PRL **93**, 076401 (2004)

Real-Time Evolution Using *DMRG*

Adaptive t-DMRG (Suzuki-Trotter)



G. Vidal, *PRL* **91**, 147902 (2003)
PRL **93**, 040502 (2004)

A.J Daley, C. Kollath, U. Schollwöck, G. Vidal
J. Stat. Mech.: Theor. Exp. (2004)

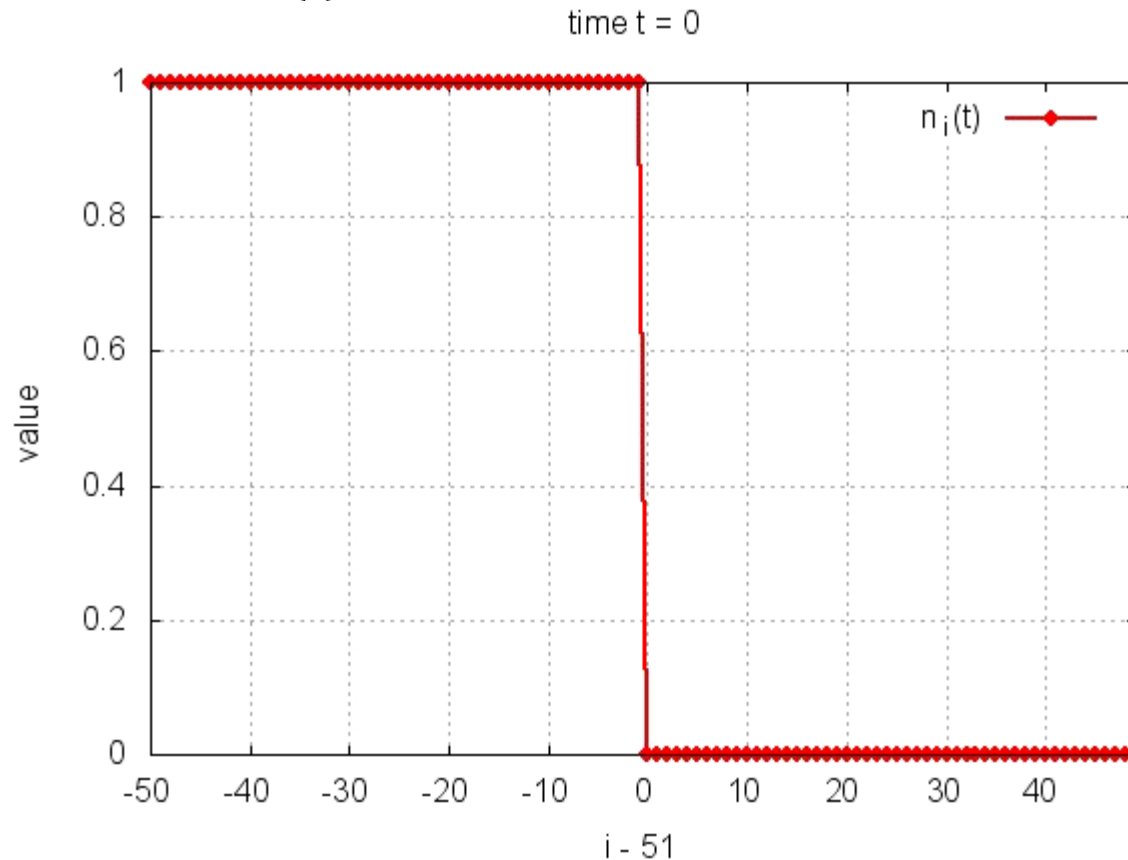
S.R. White and A. E. Feiguin, *PRL* **93**, 076401 (2004)

Accuracy of Adaptive Time-Dependent DMRG

1. Domain wall dynamics

$$c_j(t) = \frac{2}{L+1} \sum_k \sum_l \sin(kj) \sin(kl) e^{i\epsilon(k)t} c_l(0) \quad |\psi\rangle = \left(\prod_{j \leq N_e} c_j^\dagger \right) |0\rangle$$

$$\langle \psi | n_i(t) | \psi \rangle = \frac{4}{(L+1)^2} \sum_{k,k'} \sum_{\{j\}} e^{-it(\epsilon(k) - \epsilon(k'))} \sin(ki) \sin(kj) \sin(k'i) \sin(k'j)$$

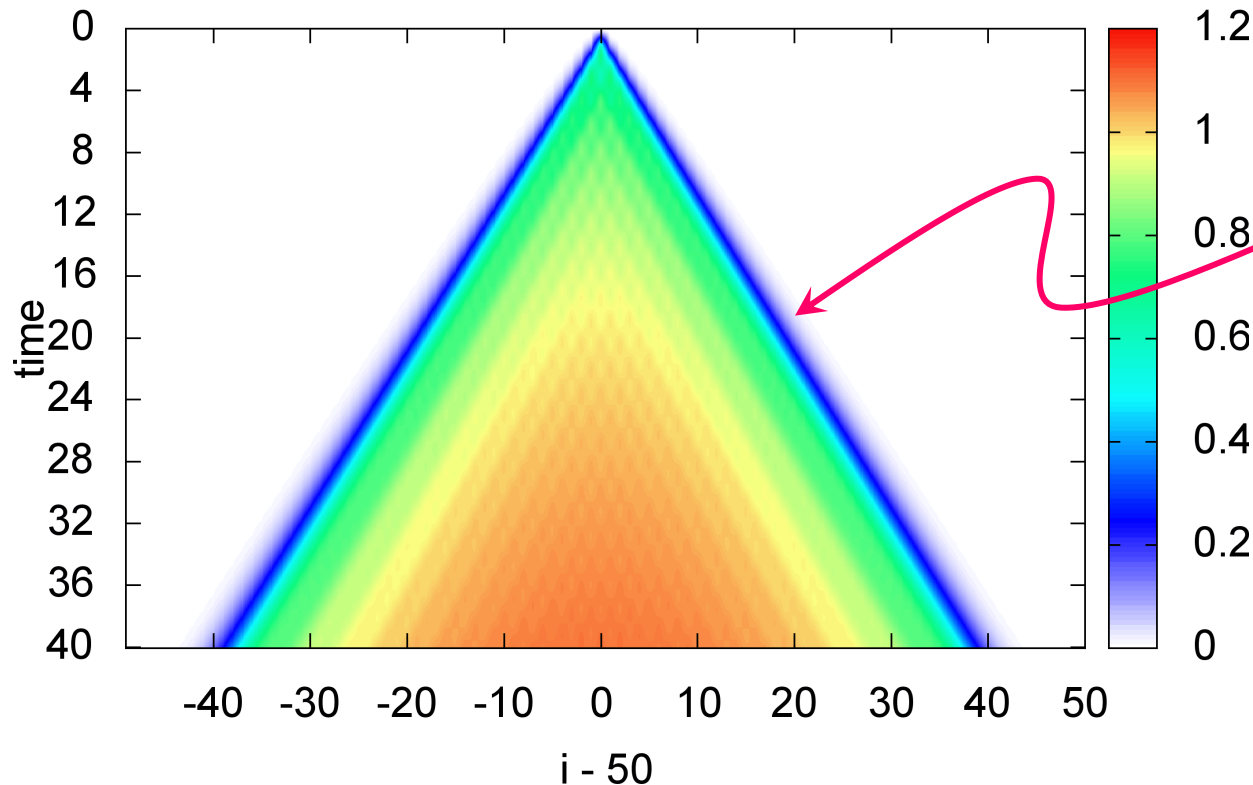


1. Domain wall dynamics:

von Neumann entropy of the bipartition:

$$S_{\text{vN}}(\hat{\rho}^S) \equiv -\text{Tr}(\hat{\rho}^S \log_2(\hat{\rho}^S)) = -\sum_{\mu=1}^{n_{\text{Sch}}} \lambda_{\mu}^2 \log_2(\lambda_{\mu}^2)$$

$S_i - S_i^{\text{GS}}$



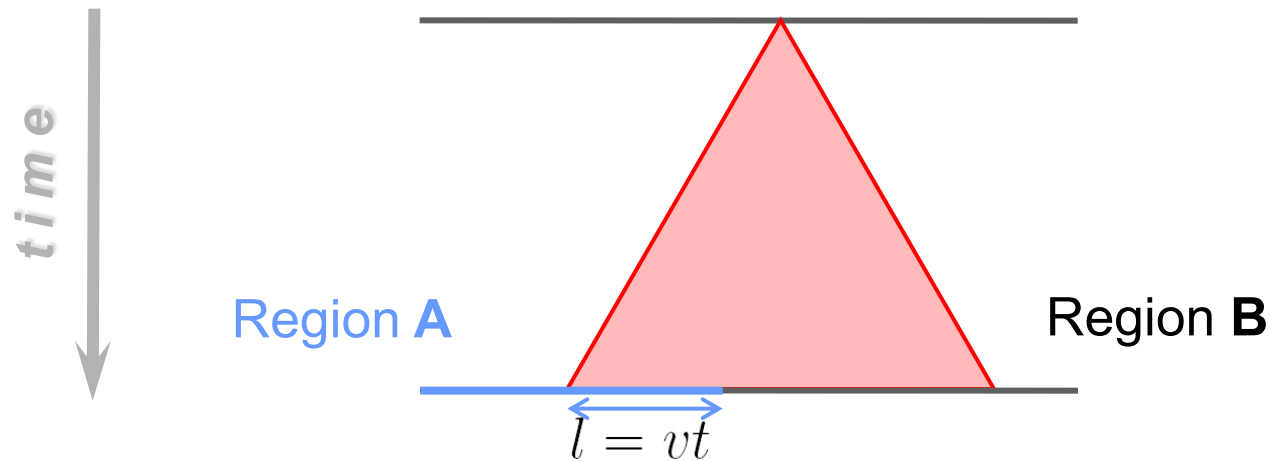
“Light cone”

Information can propagate in the system only with a finite group velocity

E.H. Lieb D.W. Robinsob
*Commun.Math.Phys.***28** 251-157
(1972)

Local Quench

Local quench:



Von Neumann entropy of the bipartition: $S = S_0 + c \log(vt)$

P. Calabrese, J. Cardy, *J.Stat.Mech.* P04010 (2005)

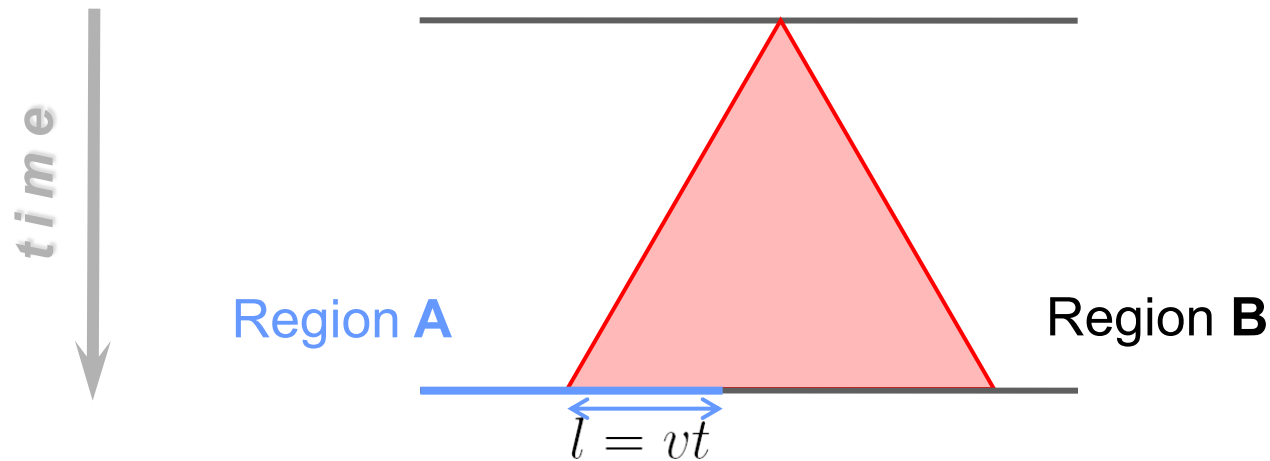
P. Calabrese, J. Cardy, *PRL* **96**, 136801 (2006)

P. Calabrese, J. Cardy, *J.Stat.Mech.* P10004 (2007)

V. Eisler, I. Peschel, *J.Stat.Mech.* P06005 (2007)

Local Quench

Local quench:



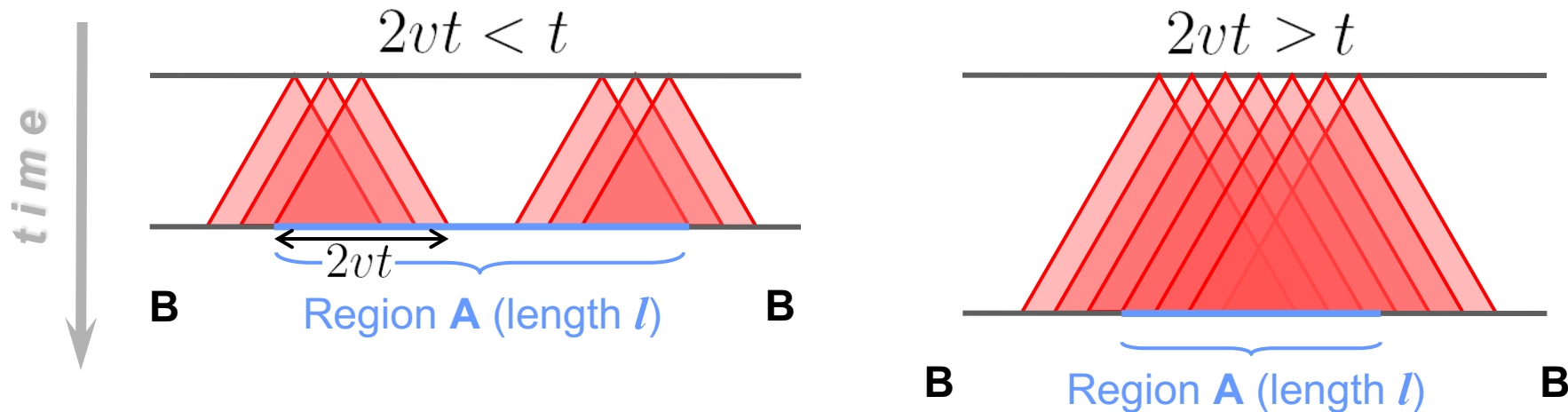
Von Neumann entropy of the bipartition: $S = S_0 + c \log(vt)$

■ Implications for t -DMRG:

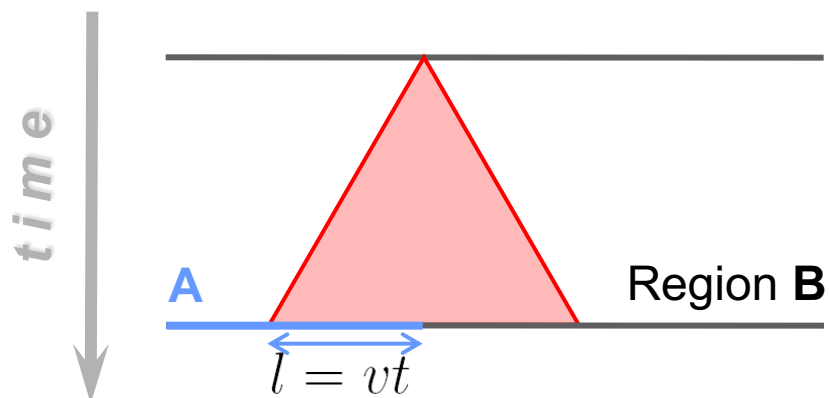
Number of kept states should be increased $\sim t^\alpha$

Global vs. Local Quenches

Global quench: $S = S_0 + ct$



Local quench: $S = S_0 + c \log(vt)$



- P. Calabrese, J. Cardy, *J.Stat.Mech.* P04010 (2005)
- P. Calabrese, J. Cardy, *PRL* **96**, 136801 (2006)
- P. Calabrese, J. Cardy, *J.Stat.Mech.* P10004 (2007)
- V. Eisler, I. Peschel, *J.Stat.Mech.* P06005 (2007)