How spin currents defy our high-school intuition

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Outline

• Spin currents in addition to electric currents;
• Interplay between spin and electric currents;
• Resistor vs. “spin resistor”. What’s the difference?
• Spin-orbit interaction changes the game.
Spin currents in metallic conductors

Electric and spin currents are carried by itinerant electrons

- Normal metals (Cu, etc.)
- Ferromagnets (Fe, Ni, etc.)
- Metals with spin-orbit interaction (Pt, etc.)
Electric and spin currents

Normal metal: pure electric current

\[ j_i^e = e \langle v_i \rangle \quad \text{electric current} \]

\[ s_\alpha = \langle \sigma_\alpha \rangle \quad \text{average spin} \]

\[ j_{\alpha, i}^s = \langle \sigma_\alpha v_i \rangle \quad \text{spin current} \]

Ferromagnetic metal: electric and spin currents

OR (or both)

Pure spin current
Driving currents with E field

Normal metal: electric current, no spin current

Ferromagnet: both electric and spin current

"Current spin polarization" 

$$ p = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow} $$

How can one ever produce a pure spin current? Or at least some spin current in a normal metal?
Diffusive model

Campbell, Fert, and Pomeroy, Philos. Mag. 15, 977 (1967)

Assumptions:
  a) Short mean free path of electrons \(\Rightarrow\) diffusive regime
  b) Spin flip time is much longer than momentum relaxation time

\(\uparrow\) and \(\downarrow\) electrons form separate Fermi gases with slow equilibration between the two

Metallic regime: \(k_B T \ll \mu\)

\[ n_\uparrow + n_\downarrow = n = \text{const} \]
Diffusive regime

Without spin accumulation

\[ \tau_p \ll \tau_s \]

With spin accumulation

\[ l_p \ll \lambda_s \sim L \]

\[ \mu_{\uparrow} + \mu_{\downarrow} \quad \Rightarrow \quad \mu \]

non-equilibrium spin accumulation

\[ \mu_s = \mu_{\uparrow} - \mu_{\downarrow} \]
Spin diffusion length
Coupled diffusion equations

\[ \begin{align*}
  j &= -\frac{\sigma}{\varepsilon^2} \left[ \nabla \mu + p \left( \frac{\nabla \mu_s}{2} \right) \right] \\
  j_s &= -\frac{\sigma}{\varepsilon^2} \left[ \left( \frac{\nabla \mu_s}{2} \right) + p \nabla \mu \right]
\end{align*} \]

\[ p = \frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}} \]

Time-dependent flow

\[ \begin{align*}
  \dot{n} + \text{div} j &= 0 \\
  \dot{s} + \text{div} j_s &= -\frac{s}{\tau_s}
\end{align*} \]
Coupled diffusion equations

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Spin relaxation
Coupled diffusion equations

\[
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\]

Stationary flow

\[
\begin{align*}
\text{div} j &= 0 \\
\text{div} j_s &= -\frac{s}{\tau_s}
\end{align*}
\]

Spin relaxation
Coupled diffusion equations

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\end{align*}
\]

\[
p = \frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}}
\]

\[
\begin{align*}
    \Delta \mu &= 0 \\
    \Delta \mu_s &= \frac{\mu_s}{\chi_s^2}
\end{align*}
\]

Time-dependent flow

\[
\begin{align*}
    \dot{n} + \text{div} j &= 0 \\
    \dot{s} + \text{div} j_s &= -\frac{s}{\tau_s}
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Stationary flow

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\]

Spin relaxation
Injection from a ferromagnet

Electric and spin currents in diffusion regime
Point injection produces pure spin current

Electric current completes the circuit; no I in dangling part
Spin current diffuses symmetrically in both directions
Non-local spin injection

“Non-local resistance”

- Diffusive regime
- Slow spin relaxation (Valet-Fert)
- Collinear magnetization

\[ \mu_s(t_F) \to 0 \]

\[ V = \frac{p\mu_s(0)}{2e} \]

Johnson-Silsbee formula
Johnson-Silsbee formula from diffusive equations

\[
\begin{align*}
    j &= -\frac{\sigma}{e^2} \left[ \nabla \mu + p \left( \frac{\nabla \mu_s}{2} \right) \right] \\
    j_s &= -\frac{\sigma}{e^2} \left[ \left( \frac{\nabla \mu_s}{2} \right) + p \nabla \mu \right]
\end{align*}
\]

Effective EMF developed in a ferromagnet

\[
V = \frac{\mu(t_F) - \mu(0)}{e} = \frac{1}{e} \int_0^{t_F} \nabla \mu \, dr
\]

\[
j = 0 \implies V = p \left( \frac{\mu_s(0)}{2e} - \frac{\mu_s(t_F)}{2e} \right)
\]

\[
\mu_s(t_F) = 0 \implies V = \frac{p \mu_s(0)}{2e}
\]
Problem of a wide measuring contact

When \( \mu_s \) is variable, which \( \mu_s \) should be substituted into \( V = \frac{p\mu_s(0)}{2e} \)?
**Generation of loop electric currents**

**Theorem:**
For $\nabla p \times \nabla \mu^s \neq 0$ circular electric current is generated in the F/N structure.

**Physics:** EMF is concentrated near the spin injection point. The circuit is completed through the bulk.
Consequence #1

Voltage depends on the thickness of the F-layer, and on the measurement point

“spin fountain”

Bazaliy, Ramazashvili, APL 110, 092405 (2017)
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pure spin current injection
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"spin fountain"

pure spin current injection
Consequence #2
How far do the spins propagate along the F/N interface?

$\ln(\mu_S) \text{ (arb. units)}$

$x \text{ (nm)}$

$t_F \to \infty$

$t_F = 250 \text{ nm}$

Bazaliy, Ramazashvili, APL 110, 092405 (2017)
Long-range propagation in finite-thickness overlayers

\[ \lambda_c = \frac{t_F}{\pi} \left( 1 + \frac{R t_N}{t_F} + \ldots \right) \]
Long range spin propagation

\[ \mu_s(x) \sim \exp(-x/\lambda) \ (x \to \infty) \]

\[ \mu_s(x) \sim \exp(-x/\lambda') \ (x \to \infty) \]

\[ \mu_s(x) \sim x^{-4} \ (x \to \infty) \]
Conclusions-I

1. Pure spin currents injected into electrically “floating” regions produce loop electric currents

2. These currents lead to long-range spin propagation along the F/N interface

Bazaliy, Ramazashvili, APL 110, 092405 (2017)
Amplification of pure spin current?


Spins are injected into a normal metal element and propagate to the other terminal.

Claim: Geometrical directionality of the conductor’s shape enhances spin current flowing along the arrow.
General lumped element
• Spin currents at two terminals are not the same

• Spin potentials do not enter current expressions as differences

\[ J_s = J_s(\mu_{s1}, \mu_{s2}, V_1 - V_2) \]
Spin potential is a vector

Spin potential has information about energy difference AND the direction of unpaired spins

\[ \vec{\mu}_s = (\mu^{sx}, \mu^{sy}, \mu^{sz}) \]
Lumped element description for non-conserved currents

**Electronics element**

\[ V_1 \quad R, \quad G = 1/R \quad V_2 \]

\[ I = G(V_1 - V_2) \]

**Spintronics element**

\[ \hat{G} \]

- 7 potentials: \( (V_1 - V_2), \mu_{1x}^s, \mu_{2x}^s, \mu_{1y}^s, \mu_{2y}^s, \mu_{1z}^s, \mu_{2z}^s \)
- 7 x 7 matrix:
  \[ I^a = G^{ab} \mu^b \]
- 7 currents: \( I, I_1^{sx}, I_2^{sx}, I_1^{sy}, I_2^{sy}, I_1^{sz}, I_2^{sz} \)
**Diffusive elements**

**Bulk diffusive equations**

\[
\begin{align*}
    j^e_i &= -\frac{\sigma}{e^2} \left( \nabla_i \mu^e + \frac{1}{2} \rho \nabla_i \mu^s \right), \\
    j^{s\alpha}_i &= -\frac{m^{\alpha} \sigma}{e^2} \left( \frac{1}{2} \nabla_i \mu^s + \rho \nabla_i \mu^e \right)
\end{align*}
\]

**Boundary conditions with strong ferromagnets**

\[
\begin{align*}
    \mu^a(N)\big|_S &= \mu^a(F)\big|_S \\
    j^e_i (N) n_i \big|_S &= j^e_i (F) n_i \big|_S, \\
    m^\alpha j^{s\alpha}_i (N) n_i \big|_S &= m^\alpha j^{s\alpha}_i (F) n_i \big|_S
\end{align*}
\]
Reciprocity theorem for diffusive elements

\[
\begin{align*}
I & \quad G & \quad C & \quad (V_1 - V_2) \\
I^s & \quad C^T & \quad S = S^T & \quad \mu^s
\end{align*}
\]

Spin-electric conductance matrix is symmetric
(28 independent parameters)
Abdullah et al. were comparing (a) and (c). Yet (a) and (b) have identical spin conductance. The arrow of opposite direction “enhances” spin conductance by the same measure. Case (d) may have even larger spin conductance.
Onsager reciprocity

\[ x_i \quad \text{— thermodynamic variables} \]

\[ S = S_0 - \frac{1}{2} A_{ij} x_i x_j. \]

\[ Y_i = A_{ij} x_j \quad \text{— thermodynamic forces} \]

\[ \dot{x}_i = -\Gamma_{ij} x_j = -\gamma_{ij} Y_j \]

**Time-reversal invariance \( \Rightarrow \quad \gamma_{jj} = \pm \gamma_{ij} \)**

\[ S = S_0 - A_{t\sigma; t'\sigma'} \delta N_{t\sigma} \delta N_{t'\sigma'} \]

Can prove that forces are chemical potentials

\[ \delta \hat{N}_{t\sigma} = -\gamma_{t\sigma; t'\sigma'} \mu_{t'\sigma'} \]

\[ \delta \hat{N}_{t\sigma} = I_{t\sigma} \]
**Diffusive reciprocity vs. Onsager reciprocity**

Diffusive reciprocity:

\[ \hat{G}(M_i) = \begin{pmatrix} G & C \\ D^T & \hat{S} \end{pmatrix} \]

Onsager:

\[ \hat{G}(-M_i) = \begin{pmatrix} G & -D \\ -C^T & \hat{S}^T \end{pmatrix} \]

\[ C = D^T \]

For \( M = 0 \):

\[ S_{ij} = S_{ji} \]

\[ C = -D^T \]
Resolution of the apparent paradox

**Normal metal:** $C = D = 0$ both diffusive and Onsager reciprocity relations are valid.

**Limitations of diffusive reciprocity:** Metal with spin-orbit interaction
Particle current along $y$ produces spin current along $x$.

But you can look at it differently, and then the spin current flows along $z$!

So where does the spin current flow?!
Spin current is a tensor

\[ j_{\alpha i} = \langle s_{\alpha} v_i \rangle = \begin{pmatrix} s_x v_x & s_x v_y \\ s_y v_x & s_y v_y \end{pmatrix} \]

After averaging over all possible incoming spin directions

\[ \delta j_{\alpha i} = v \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \]

\[ \delta j_{\alpha i} = v \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} \]

\[ \delta j_{\alpha i} = v \begin{pmatrix} \sin \alpha \cos \alpha & -\sin^2 \alpha \\ \cos^2 \alpha & -\sin \alpha \cos \alpha \end{pmatrix} \]

After averaging over all possible incoming spin directions

\[ j_{\alpha i} = \begin{pmatrix} 0 & -J \\ J & 0 \end{pmatrix} \]
Diffusion equations in Pt

\[ \dot{j}^e_i = -\frac{\sigma}{\epsilon^2} \left( \nabla_i \mu^e + \alpha \epsilon_{ij\alpha} \nabla_j \mu^{s\alpha} \right) \]
\[ \dot{j}^{s\alpha}_i = -\frac{\sigma}{\epsilon^2} \left( \nabla_i \mu^{s\alpha} - \alpha \epsilon_{i\alpha j} \nabla_j \mu^e \right) \]

Equations are now coupled by spin-orbit interaction

As a result of equation coupling, the proof of diffusive reciprocity breaks down. Yet starting from new equations it is possible to prove the Onsager reciprocity.

\[ \hat{G}(M_i) = \begin{pmatrix} G & C \\ D^T & \hat{S} \end{pmatrix} \]

For \( M = 0 \)
\[ C = -D^T \]

\[ S_{ij} = S_{ji} \quad \rightarrow \quad \text{“Spin ratchets” are still impossible in Pt wires} \]
Spin current in Rashba 2DEG

In 2DEG with Rashba spin-orbit interaction there is a spin current in equilibrium. What can it mean? Is the definition of this current even meaningful?

\[ H_{SO} = \alpha [s \times p] \cdot \hat{z} \]

\[
\begin{align*}
\dot{j}_+ &= \begin{pmatrix} 0 & -J_+ \\ J_+ & 0 \end{pmatrix} \\
\dot{j}_{(-)} &= \begin{pmatrix} 0 & -J_{(-)} \\ J_{(-)} & 0 \end{pmatrix} \\
\dot{j}_{tot} &= \begin{pmatrix} 0 & -J \\ J & 0 \end{pmatrix} \neq 0
\end{align*}
\]
Can equilibrium spin current be used in a “perpetual spin source”?

In the presence of spin-orbit interactions, spins can rotate upon reflection. Spin current does not have to be continuous on the boundary.

Rashba SO metal  \[ J_{\alpha i}^s \neq 0 \]

Normal metal  \[ J_{\alpha i}^s = 0 \]

Rashba SO mechanically twists the film
Meaning of equilibrium spin currents

In mechanics, compressed rod in equilibrium carries a momentum flux

\[ F \rightarrow \Pi_{xx} \neq 0 \rightarrow -F \]

A hypothetical system above develops equilibrium momentum flux if the equilibrium length of the springs can be changed and made smaller than the length of the rod. Spring length here is the analogy of SO interaction.
Diffusive reciprocity and circuit theory

Spin-transport in multi-terminal normal metal-ferromagnet systems with non-collinear magnetizations

A. Brataas\textsuperscript{1,a}, Y.V. Nazarov\textsuperscript{2}, and G.E.W. Bauer\textsuperscript{2}

Landauer-Buttiker formula for conductance

Partial conductances for up/down spins

“mixing conductance”

\begin{align*}
G^\uparrow &= \frac{e^2}{\hbar} \left[ M - \sum_{nm} |t_{\uparrow}^{nm}|^2 \right] = \frac{e^2}{\hbar} \sum_{nm} |t_{\uparrow}^{nm}|^2, \\
G^\downarrow &= \frac{e^2}{\hbar} \left[ M - \sum_{nm} |t_{\downarrow}^{nm}|^2 \right] = \frac{e^2}{\hbar} \sum_{nm} |t_{\downarrow}^{nm}|^2, \\
G^{\uparrow\downarrow} &= \frac{e^2}{\hbar} \left[ M - \sum_{nm} r_{\uparrow}^{nm} (r_{\downarrow}^{nm})^\ast \right].
\end{align*}
Diffusive Circuit theory

\[
G_{ab}^{NN} = \begin{pmatrix}
-G & C_N^x & C_N^y & C_N^z \\
C_N^x & S_N^x & S_N^x y & S_N^x z \\
C_N^y & S_N^y & S_N^y z & S_N^z \\
C_N^z & S_N^z & S_N^z y & S_N^z
\end{pmatrix}
\]

10 parameters

\[
\hat{\mathcal{G}} = -\begin{pmatrix}
G_\uparrow + G_\downarrow & 0 & 0 & G_\uparrow - G_\downarrow \\
0 & 2Re[G_\uparrow \downarrow] & 2Im[G_\uparrow \downarrow] & 0 \\
0 & -2Im[G_\uparrow \downarrow] & 2Re[G_\uparrow \downarrow] & 0 \\
G_\uparrow - G_\downarrow & 0 & 0 & G_\uparrow + G_\downarrow
\end{pmatrix}
\]

4 parameters

Without spin relaxation

\((\tau_{s}^{-1} = 0)\)

Spin-inactive interface (\(\text{Im}[G_{\uparrow \downarrow}] = 0\))

Spin-active interface (\(\text{Im}[G_{\uparrow \downarrow}] \neq 0\))

CT: single contact

Reciprocity

No reciprocity

With spin relaxation

\((\tau_{s}^{-1} \neq 0)\)

CT: circuit with inner nodes providing spin relaxation

CT: circuit with inner nodes providing spin relaxation
**Question:** Is Onsager reciprocity violated in a coherent contact with non-zero \( \text{Im}(G_m) \)?
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**Answer:** Since ferromagnets are involved, Onsager relations connect different system \((M \text{ and } -M)\).
Conclusions-II

• Conductance matrix of diffusive elements is symmetric;
• This property is in addition to other symmetries of conductance matrix;
• “Directional geometry” does not influence spin transport;
• Bazaliy + Ramazashvili, PRB 99, 184443 (2019)