Superconductivity vs non-Fermi liquid at a Quantum Critical Point

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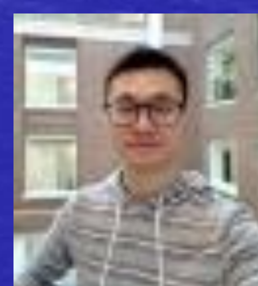
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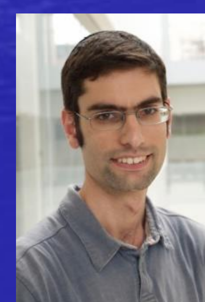
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Multiple solutions for the pairing gap at T=0

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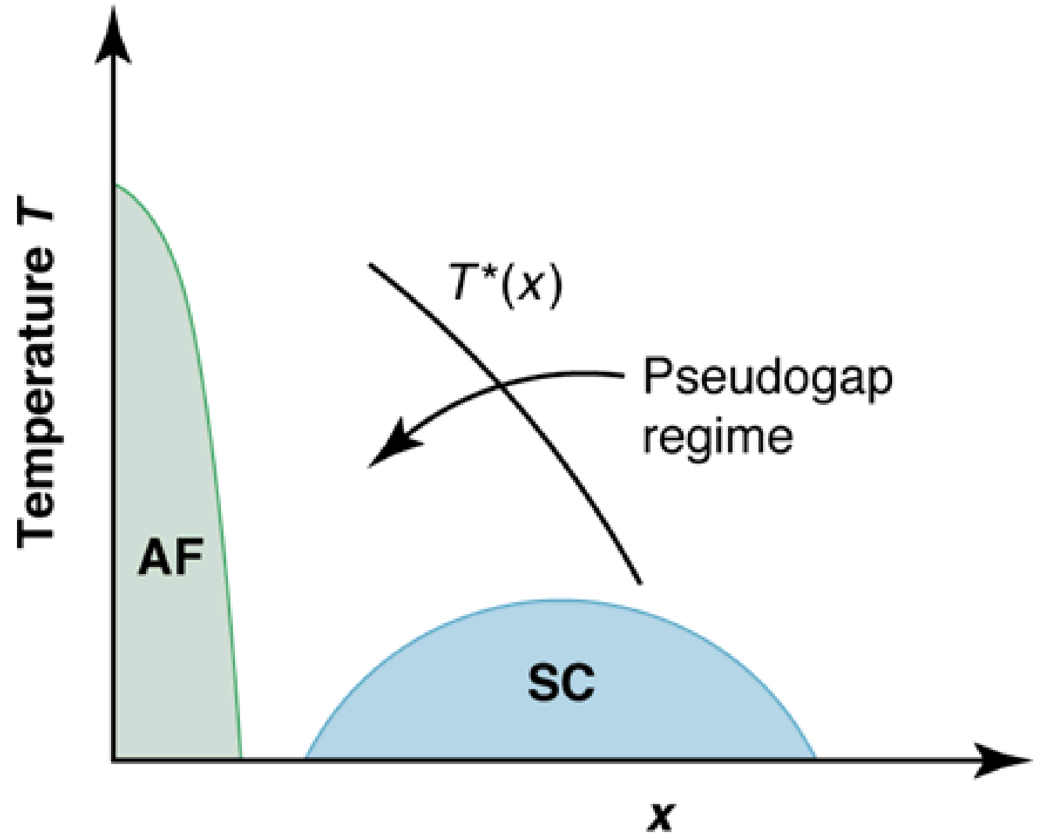
Key motivation:

to understand the phenomenon called the pseudogap



Two key lines of thoughts:

- Pseudogap (at least, some part of it) is a failed superconductor (order is destroyed by phase fluctuations, but the gap persists)

- Pseu
some … loop currents)
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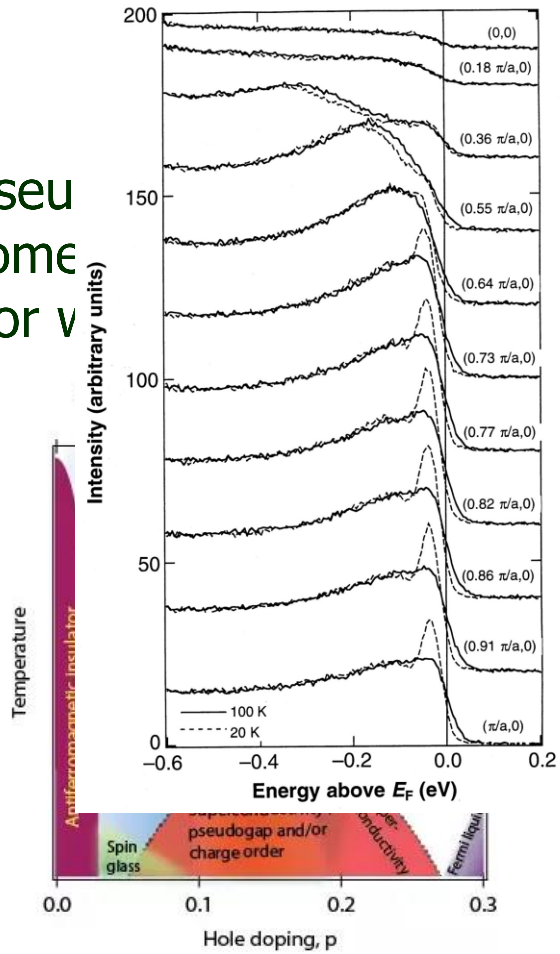
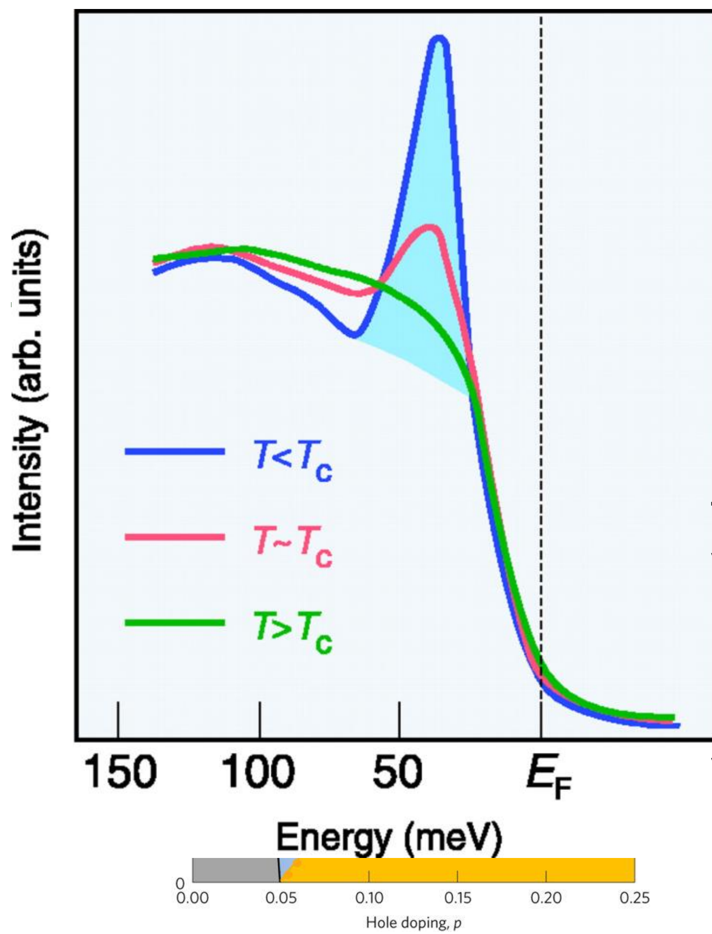




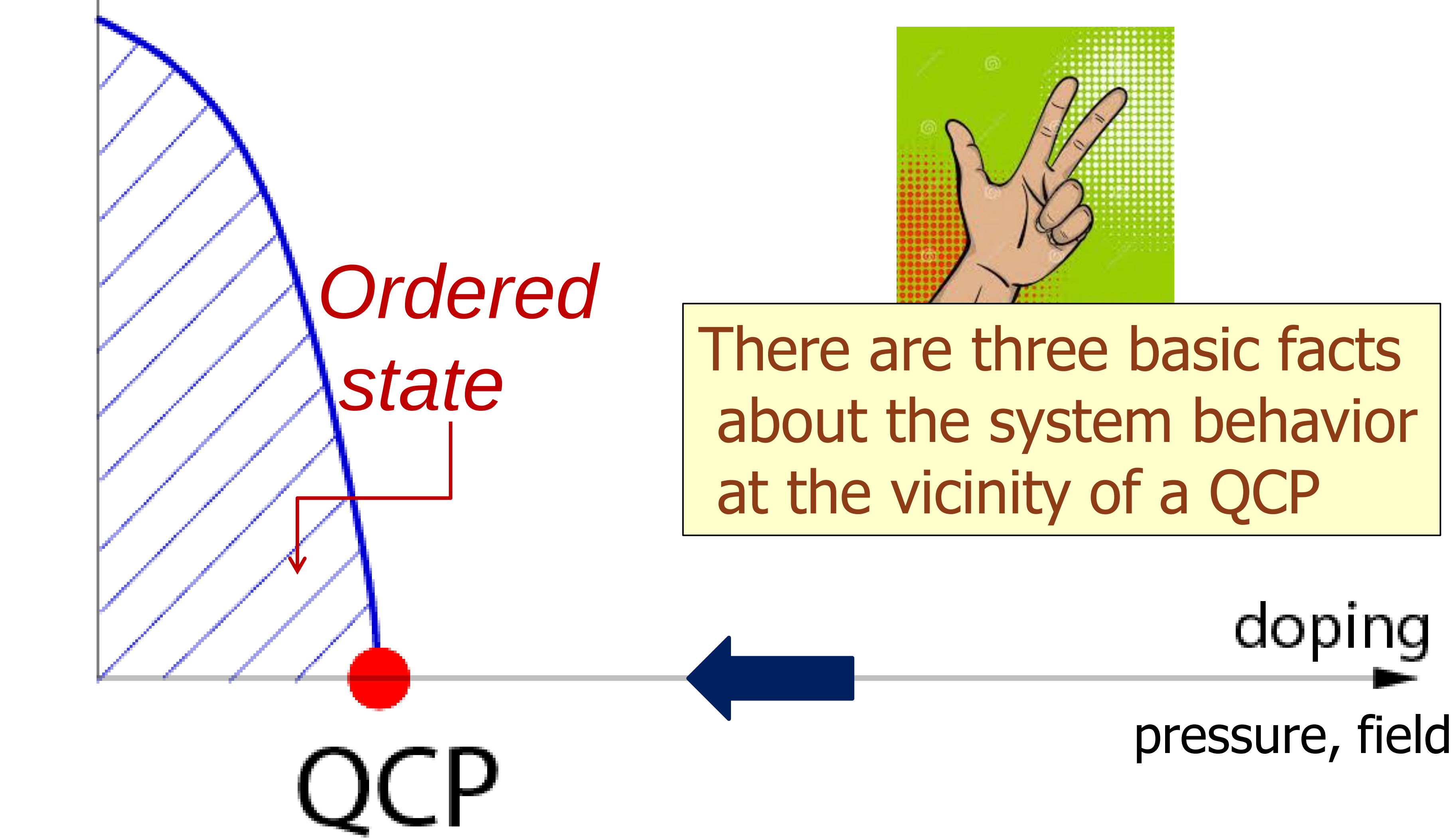
Taillefer,
Varma, Sachdev,
R-H He et al
(Stanford/Berkeley
collaboration),
Vishik et al ….

The key goal of this particular study is to understand the structure of the superconducting state at T=0 in a clean metal, at the onset of a spin/charge order (AFM, Ising-nematic…)

Can gap fluctuations destroy SC order at T=0 in a clean system?

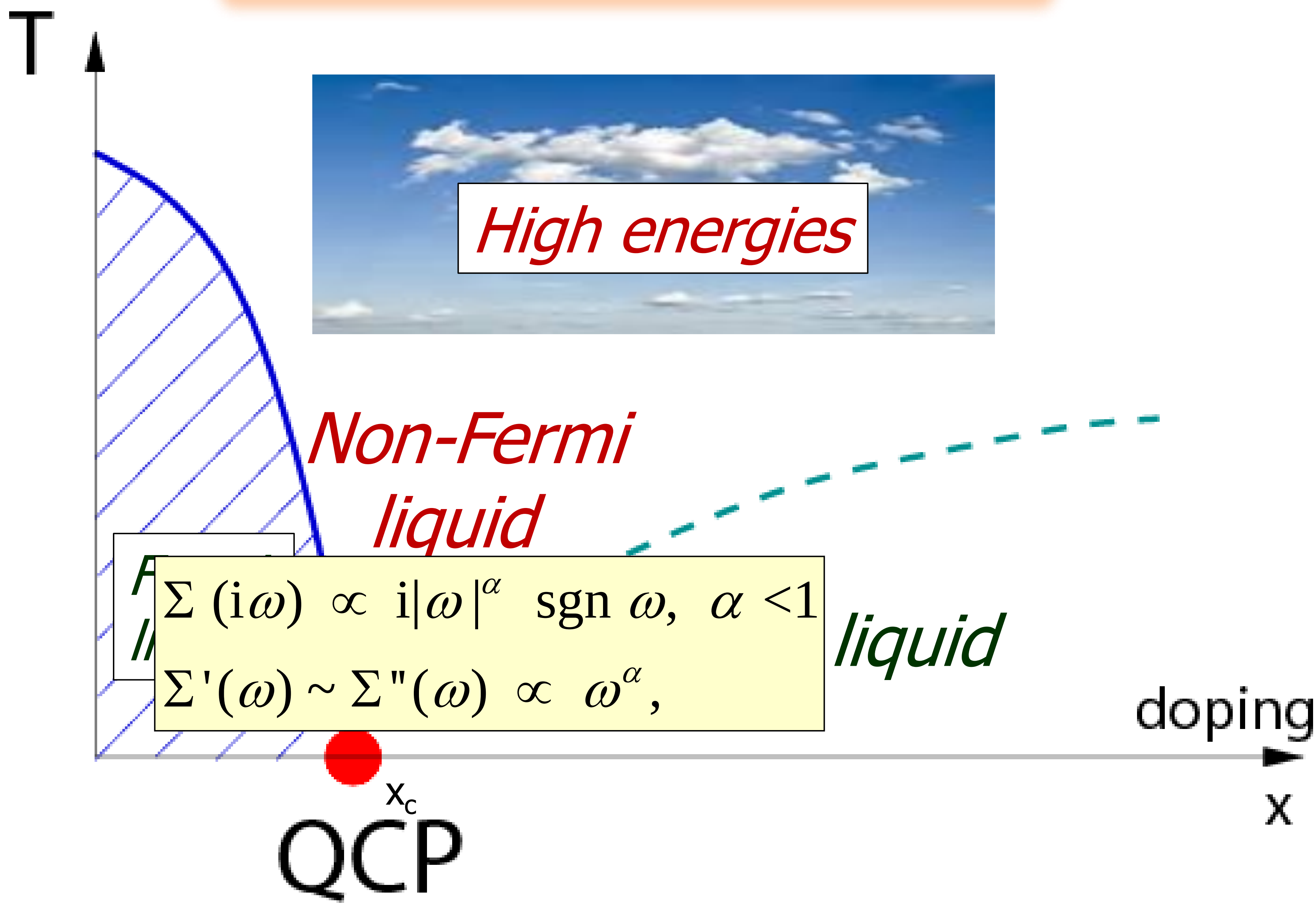
The set:

Consider a situation when a system of itinerant fermions approaches an instability towards some electronic order



There are three basic facts about the system behavior at the vicinity of a QCP

Basic fact #1: Non-FL at a QCP



$$\Sigma\,(i\omega)\ \propto\ i|\omega|^{\alpha}\ \operatorname{sgn}\omega,\ \ \alpha<1$$

$$\Sigma'(\omega)\sim\Sigma''(\omega)\ \propto\ \omega^{\alpha},$$

Basic fact #2: strong attraction

If we only consider pairing interaction
(i.e., neglect non-FL self-energy)

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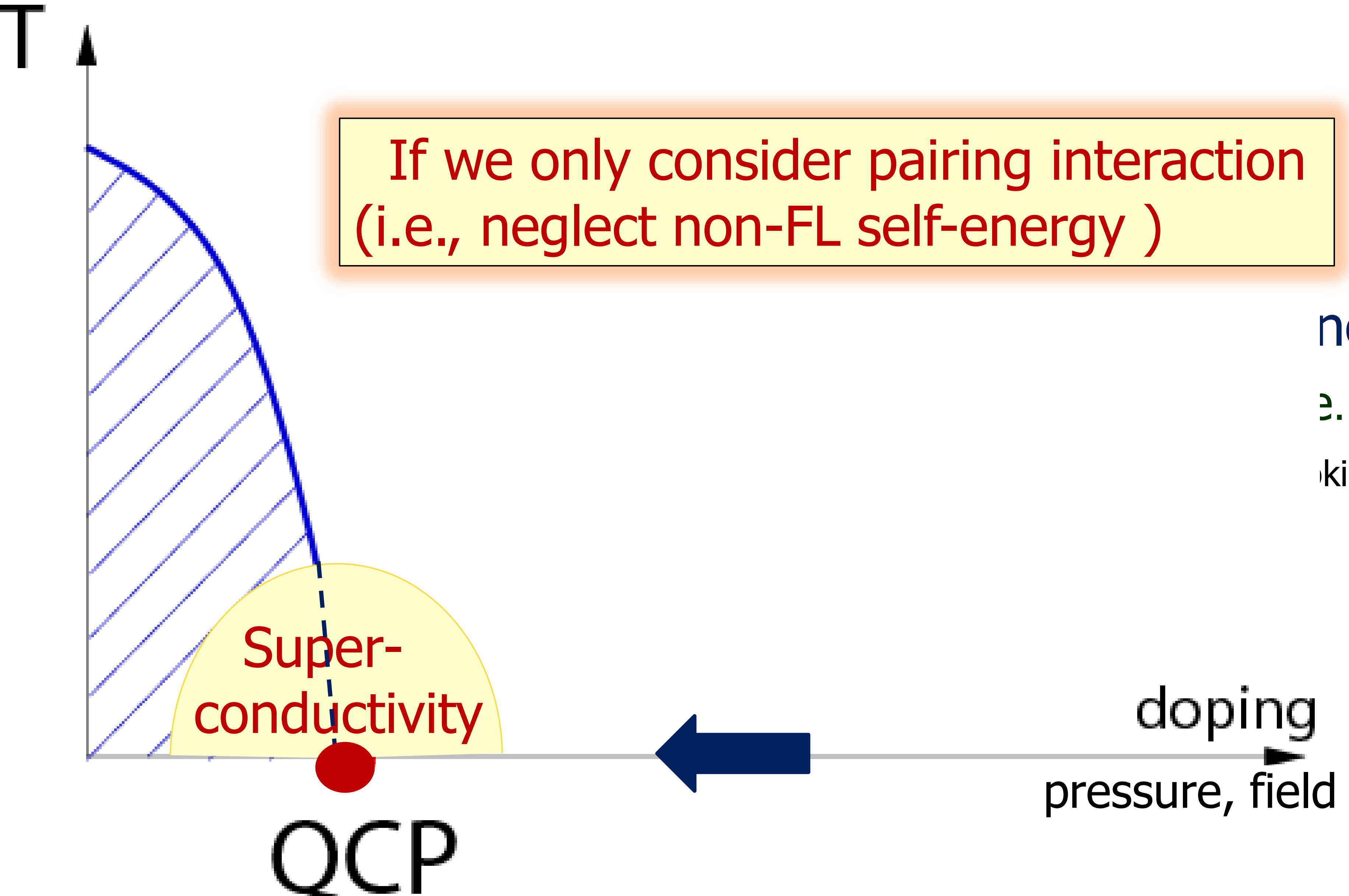


Basic fact #3: competition

SC

NFL

Superconductivity and NFL compete with each other

- Strong fermionic self-energy kills the Cooper logarithm
- Superconductivity gaps low-energy excitations and restores fermionic coherence

Superconductivity and a non-FL come from the same "source", and characteristic energy is the same for both

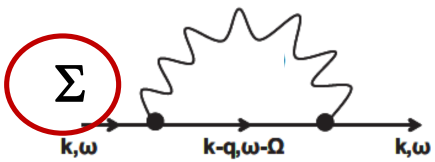
We consider a particular class of itinerant fermionic systems, in which soft bosons are slow compared to fermions

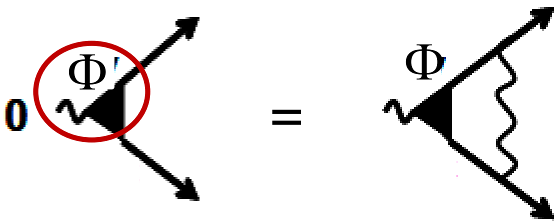
- $v_F >> v_{\text{coll. modes}}$ this works for el-phonon interaction, and/or when bosons are Landau overdamped collective modes

This condition sets up a set of rules how to analyze superconductivity near a QCP, when the interaction is smaller or, at most, comparable to the bandwidth

Eliashberg theory

pairing symmetry





$\Phi\,(k,\omega) = \Phi(\omega) * f_\Phi\,(\theta_k)$

$\Sigma(k,\omega) = \Sigma(\omega)$

θ_k is the angle along the FS

The pairing problem reduces to the competition between $\Sigma(\omega)$ and $\Phi\,(\omega)$.

For spin-singlet superconductivity

Pairing vertex

$$\Phi(\Omega_m) = \pi T \sum_{\omega \neq \Omega} \frac{\Phi(\omega_m)}{\sqrt{(\omega_m + \Sigma(\omega_m))^2 + \Phi^2(\omega_m)}} \chi_L(\omega_m - \Omega_m)$$

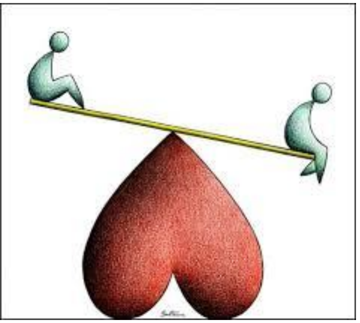
Self-energy

$$\Sigma(\Omega_m) = \pi T \sum_{\omega \neq \Omega} \frac{\omega_m + \Sigma(\omega_m)}{\sqrt{(\omega_m + \Sigma(\omega_m))^2 + \Phi^2(\omega_m)}} \chi_L(\omega_m - \Omega_m)$$

Superconducting gap $\Delta(\Omega_m) = \Omega_m \dfrac{\Phi(\Omega_m)}{\Omega_m + \Sigma(\Omega_m)}$

$\chi_L(\Omega)$ is a bosonic propagator, integrated along the Fermi surface (an analog to $\alpha^2 F$ for el-phonon interaction)

In a Fermi liquid, $\chi_L(\Omega)$ is a constant at small Ω.
At a QCP, $\chi_L(\Omega) = (g/\Omega)^\gamma$, $\gamma > 0$ diverges at $\Omega=0$



This is the problem of a kind that theorists love and hate: no small or large parameter (coupling g is the only scale)

SC:
- Convergence in the ultra-violet
- No Cooper logarithm

$Tc = g\ \Psi(\gamma)$, but can be zero, if NFL wins

NFL: $\boxed{\Sigma(\omega_m) \sim (\omega_m)^{1-\gamma}\ g^{\gamma}}$ in the normal state:

Quantum-critical (NFL) — Fermi gas ("high energies")

0 — $\Sigma(\omega_m) > \omega_m$ — g — $\Sigma(\omega_m) < \omega_m$ — ω_m

g is the upper edge of NFL behavior

Tendency to SC and tendency to NFL are comparable in strength

This is well known in 1D:

Tsvelik, Gogolin, Nersesyan, Oshikawa, Japaridze, Andrei, Wiegmann, ...

Examples $\chi_L(\Omega) = (g/\Omega)^\gamma$ Our story is chiefly about 2D

Near a nematic transition in 2D

$$\chi(q,\Omega_m) = \frac{1}{m^2 + q^2 + \alpha \frac{|\Omega_m|}{q}}$$

$$\chi_L(\Omega_m) = \int \chi(q,\Omega_m)\, dq \propto \Omega_m^{-1/3}, \; m=0$$

$$\gamma = 1/3$$

Lee, Bonesteel, McDonald, Nayak; Millis, Altshuler, Ioffe, Metlitski, Mross, Sachdev, Senthil, Berg, Kivelson, Fradkin, Oganesyan, Lederer, Trebst, Metzner, Pepin, Efetov, Maslov, Klein, Raghu, A.C..

Near an AFM transition in 2D (hot spot model)

$$\chi(q,\Omega_m) = \frac{1}{m^2 + (q-Q)^2 + \alpha |\Omega_m|}$$

$$\chi_L(\Omega_m) = \int \chi(q,\Omega_m)\, dq \propto \Omega_m^{-1/2}, \; m=0$$

$$\gamma = 1/2$$

Millis, Sachdev, Varma, Zaanen, Georges, Tremblay, Abanov, Finkelstein, Schmalian, Metlitski, Y. Wang Rosch, Berg, Fernandes, Tsvelik, Ferraz, S-S Lee, Di Castro, Castellani, Grilli, Caprara.. (CDW), A.C.

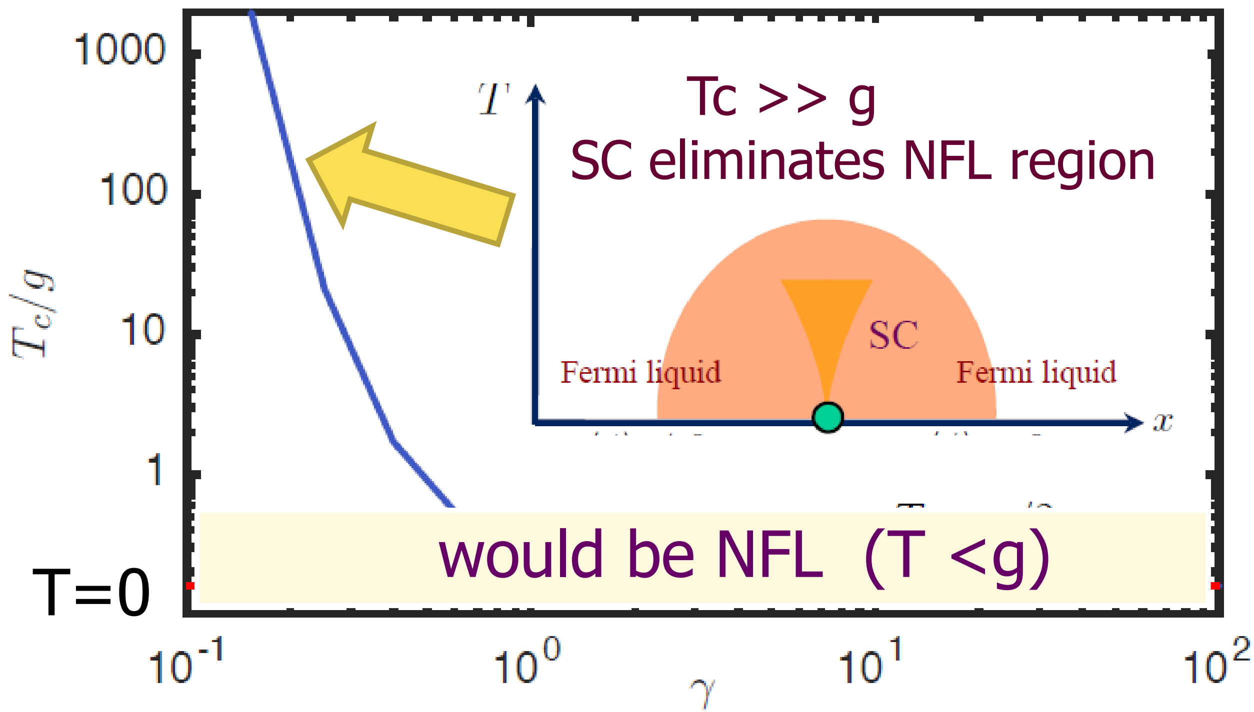
Small γ — Son, Raghu, Torroba, Senthil, Mross, Metlitski, Sachdev, Moon, Schmalian, A.C.....

$\gamma=1.2$ — Lee, Miao, Kotliar, A.C (application to Fe-based)

$\gamma=2$ — Electron-phonon interaction at vanishing Ω_D — Carbotte, Marsiglio, Combescot, Scalapino, Ranninger, Mazin, Dolgov, Maksimov, Kivelson.....

$\gamma=0.4$ or $\gamma=2$ — Dispersion-less fermions interacting with an Einstein phonon via random Yukawa coupling — I. Esterlis and J. Schmalian, Y. Wang A. Altman and A. Kamenev

Let's solve numerically for Tc



Metlitski et al

Superconductivity wins!
Tc is non-zero for all critical theories

A comment about self-energy

Pairing vertex

$$\Phi\,(\Omega_m) = \pi T \sum_{\omega\neq\Omega} \frac{\Phi(\omega_m)}{\sqrt{(\omega_m + \Sigma(\omega_m))^2 + \Phi^2(\omega_m)}} \chi_L(\omega_m - \Omega_m)$$

Self-energy

$$\Sigma(\Omega_m) = \pi T \sum_{\omega\neq\Omega} \frac{\omega_m + \Sigma(\omega_m)}{\sqrt{(\omega_m + \Sigma(\omega_m))^2 + \Phi^2(\omega_m)}} \chi_L(\omega_m - \Omega_m)$$

$\Sigma(\Omega_m)$ is the self-energy without thermal contribution

The full self-energy $\Sigma_{full}(\Omega_m) = \Sigma_{thermal}(\Omega_m) + \Sigma(\Omega_m)$

$$\Sigma_{thermal}(\Omega_m) = \pi T \frac{\Omega_m + \Sigma(\Omega_m)}{\sqrt{(\Omega_m + \Sigma(\Omega_m))^2 + \Phi^2(\Omega_m)}} \chi_L(0)$$

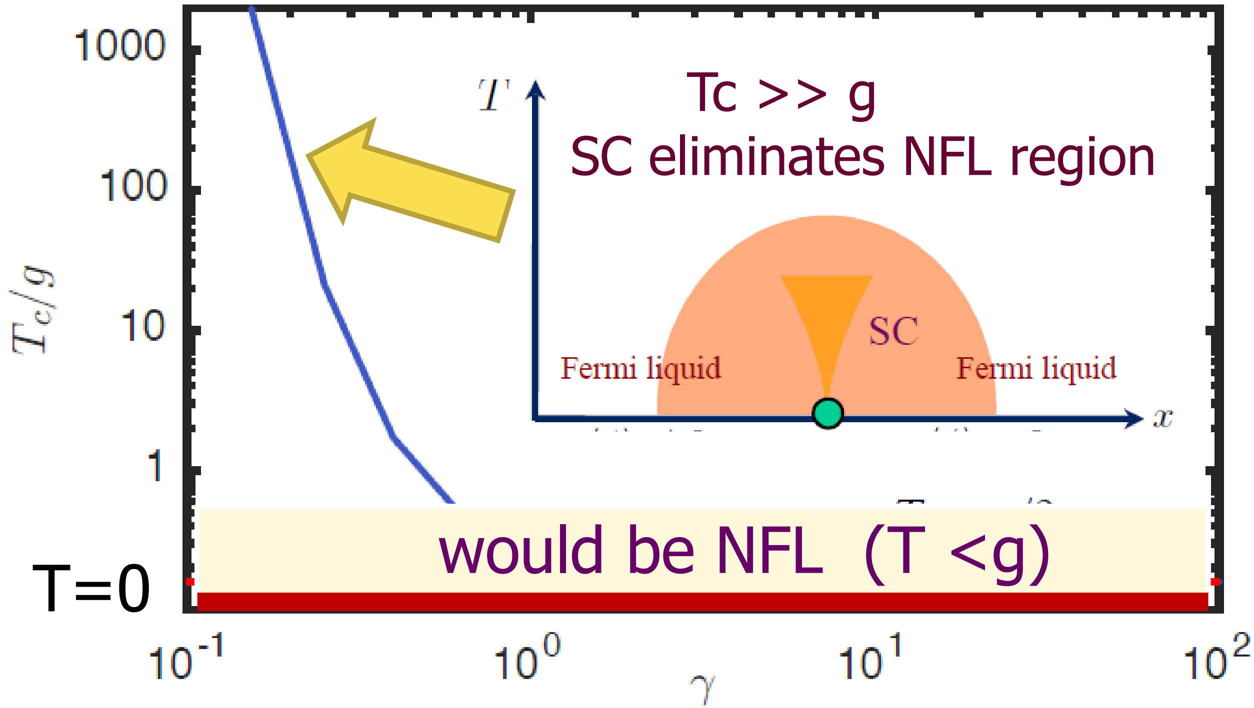
accounts for precursors to SDW and linear in T resistivity

$$\Sigma\,(\Omega_m) = \pi T \sum_{\omega\neq\Omega} \frac{\omega_m + \Sigma(\omega_m)}{\sqrt{(\omega_m + \Sigma(\omega_m))^2 + \Phi^2(\omega_m)}} \chi_L(\omega_m - \Omega_m)$$

$$\Sigma\,(\Omega_m) \propto (\Omega_m)^{1-\gamma}$$

accounts for NFL

Let's solve numerically for Tc

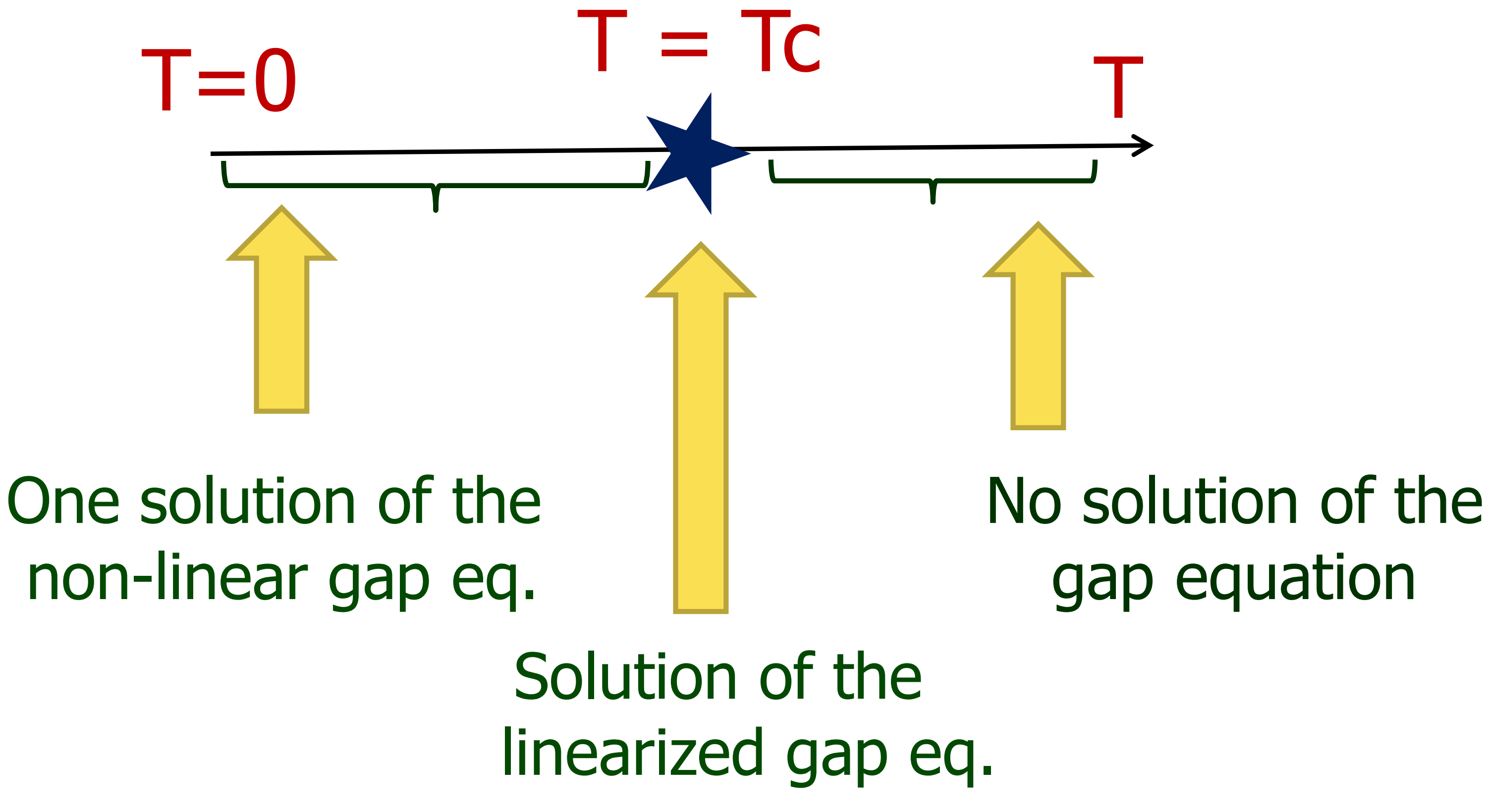


Metlitski et al

It looks, we should expect a conventional BCS -like behavior at T=0, at least at γ <<1

Conventional wisdom

(borrowed from studies of SC in non-critical systems)



At a QCP, our calculations show qualitatively different behavior at T=0:

an infinite set of solutions for the pairing gap, including the solution of the linearized gap eqn.

- Linearized gap equation at T=0

BCS (γ =0, upper cutoff Λ)

$$\boxed{\Phi\,(\Omega_m) = \lambda \int_0^{\Lambda} \frac{d\omega_m}{|\,\omega_m\,|}\,\Phi(\omega_m)}$$

$$\boxed{\Phi\,(\Omega_m) = \Phi}$$

$$\boxed{1 = \lambda \underbrace{\int_0^{\Lambda} \frac{d\omega_m}{|\,\omega_m\,|}}_{\text{diverges}}}$$

Obviously, no solution at T=0.

- Linearized gap equation at T=0, our case of $\gamma > 0$

At small frequencies, Ω_m <<

$$\Phi(\Omega_m) = \frac{1-\gamma}{2}\int d\omega_m \frac{\Phi(\omega_m)}{\underbrace{|\omega_m|^{1-\gamma}}_{\text{fermionic self-energy}}\ \underbrace{|\omega_m-\Omega_m|^{\gamma}}_{\text{gapless boson}}}$$

Scale invariant gap equation:

Neglect bare ω_m

$$\Phi(\Omega_m) = |\Omega_m|^{a}$$

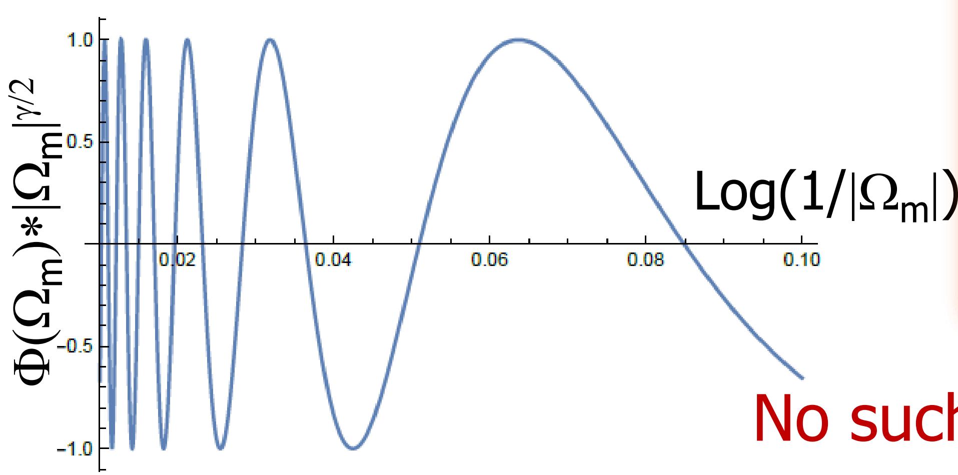
leftover from the bare interaction

$$\boxed{a = \gamma/2 \pm 2i\beta}$$

$$\Phi(\Omega_m) = \frac{1}{|\Omega_m|^{\gamma/2}}\left(C\,|\Omega_m|^{2i\beta} + C^{*}\,|\Omega_m|^{-2i\beta}\right)$$
$$= \frac{1}{|\Omega_m|^{\gamma/2}}\cos\left(\beta\log\left(\Omega_m^2+\phi\right)\right)$$

$\beta=\beta(\gamma)$ is a number,

ϕ is an arbitrary phase



The solution oscillates, and contains a free phase, as the consequence of translational invariance

No such freedom in a non-critical metal

Now keep bare ω_m

W ... the
b ... Ω_m

$$\Phi(\Omega_m) = \frac{1-\gamma}{2}\int d\omega_m \frac{\Phi(\omega_m)}{|\omega_m|^{1-\gamma}\;|\omega_m-\Omega_m|^{\gamma}}\frac{1}{1+|\omega_m/g|^{\gamma}}$$

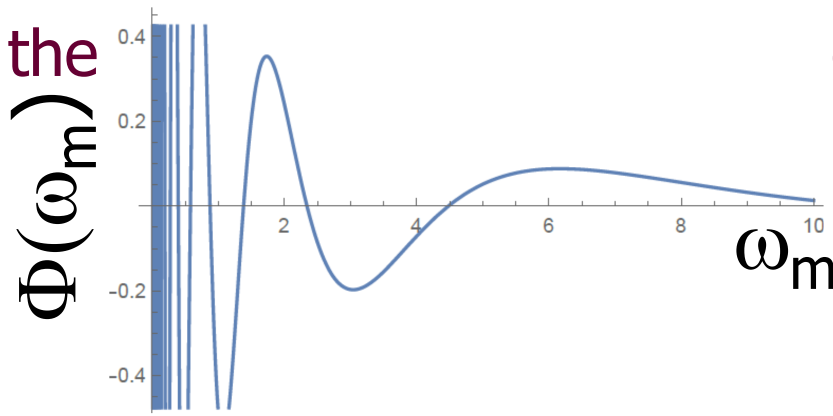
The existence of a free phase is crucial for this pricedure

Approximate procedure:

- introduce a hard cutoff at $|\Omega_m| \sim g$ (a boundary condition).
- choose the free phase to satisfy the boundary condition.

We went beyond this and found the exact solution of the full linearized gap equation at T=0 (it exists !)

This gives the ... gap equation at T=0



$$\Phi(\omega_m) \simeq \frac{1}{|\omega_m|^{\gamma/2}}\left(\frac{\Gamma(\frac{1}{2}+ik)\,\Gamma(\frac{3}{2}+ik)}{\Gamma(1+2ik)}|\omega_m|^{ik\gamma}\;{}_2F_1(\frac{1}{2}+ik, \frac{3}{2}+ik, 1+2ik, -|\omega_m|^{\gamma}) + \text{h.c}\right)$$

$$\frac{1-\gamma}{2} = \frac{\Gamma(\gamma(1+ik)/2)\Gamma(\gamma(1-ik)/2)}{\Gamma(\gamma)} + \Gamma(1\text{-}\gamma)\left(\frac{\Gamma(\gamma(1+ik)/2)}{\Gamma(1\text{-}\gamma(1-ik)/2)} + \frac{\Gamma(\gamma(1-ik)/2)}{\Gamma(1\text{-}\gamma(1+ik)/2)}\right)$$

Non-linear gap equation

A finite gap value sets another boundary condition on the solution of the linearized gap equation ($\Phi(\Omega_m) = \text{const}$ at $|\Omega_m| < \Delta$)

BCS: $\boxed{1 = \lambda \int_{\Delta}^{\Lambda} \frac{d\omega}{|\omega|}}$

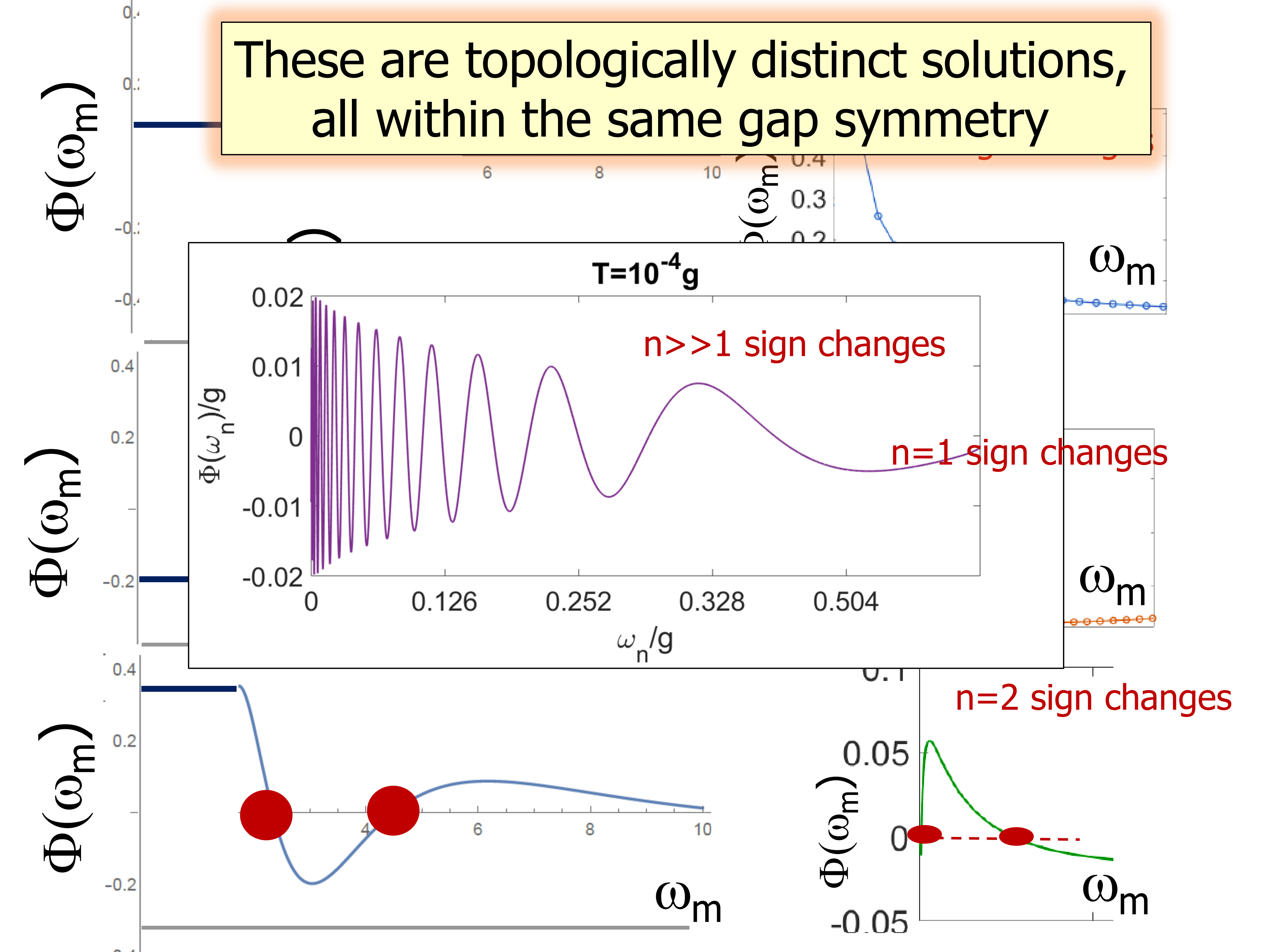
In our case: $\boxed{\sin\left(2\beta \log\left[\frac{g}{\Delta}\right]\right) = \text{const}}$

$\boxed{\Delta = \Lambda\, e^{-1/\lambda}}$

$\boxed{\Delta_n \sim g\, e^{-n\pi/(2\beta)}}$

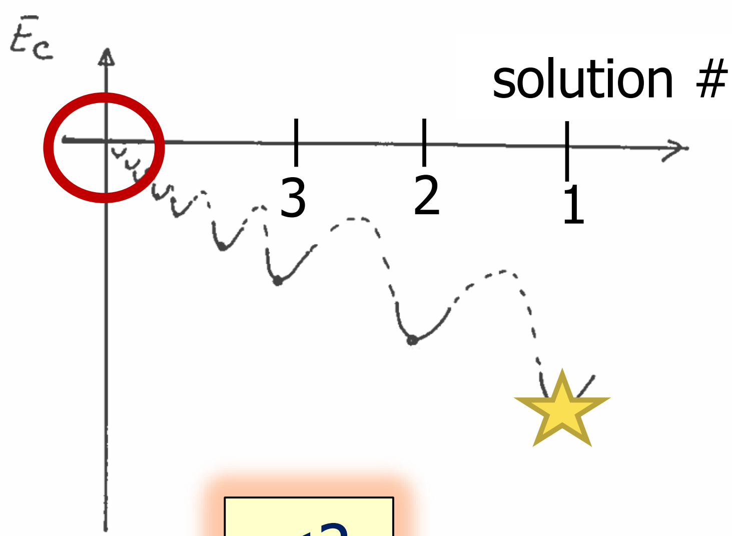
A single solution

An infinite, discrete set of solutions


These are topologically distinct solutions, all within the same gap symmetry
T=10^-4 g
n>>1 sign changes
n=1 sign changes
n=2 sign changes
Φ(ω_n)/g
ω_n/g
0.02
0.01
0
-0.01
-0.02
0
0.126
0.252
0.328
0.504
Φ(ω_m)
ω_m
0.4
0.2
-0.2
4
6
8
10
0.05
-0.05

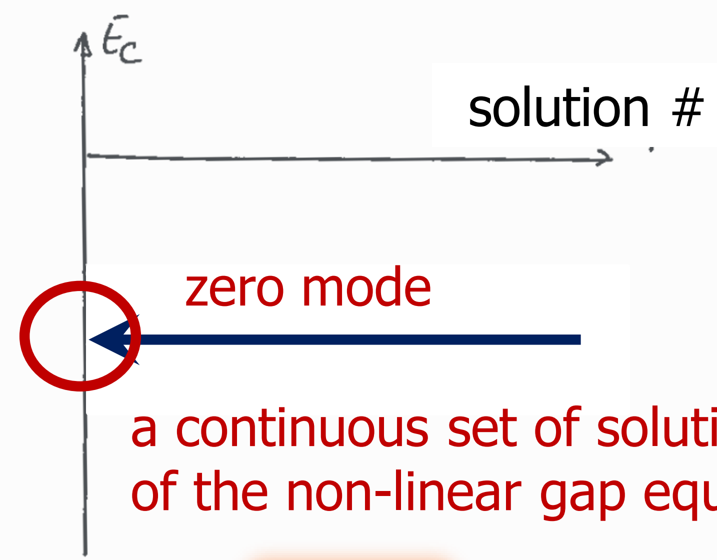
Why should we care about multiple solutions?

Condensation energy



$\gamma<2$

Condensation energy



zero mode

a continuous set of solutions
of the non-linear gap equation

$\gamma=2$

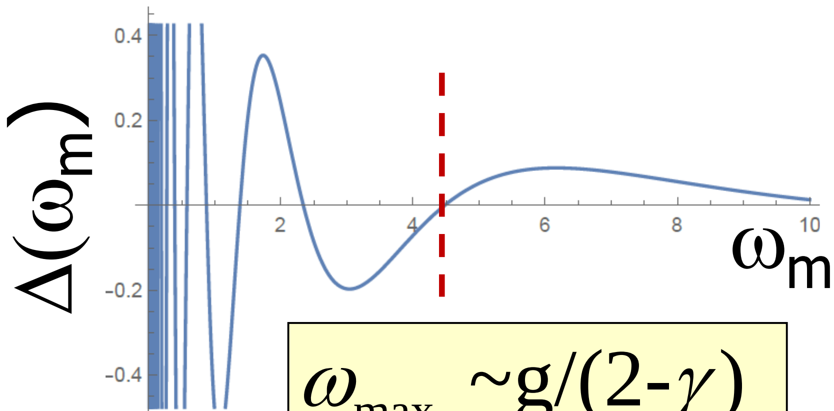
Po en
di nite T

Fluctuations of a phase of a single gap $\Delta(\omega)\, e^{i\phi}$ are weak at T=0
Their smallness is the applicability condition for Eliashberg theories

massless “longitudinal” gap fluctuations

$\gamma<2$

Condensation energy Ec for the solution of the linearized gap equation (the one with infinite number of zeros) vanishes

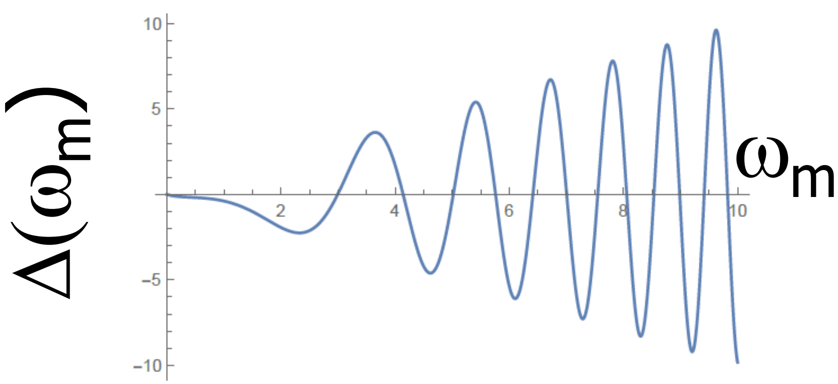


$\omega_{max} \sim g/(2-\gamma)$

$\gamma=2$

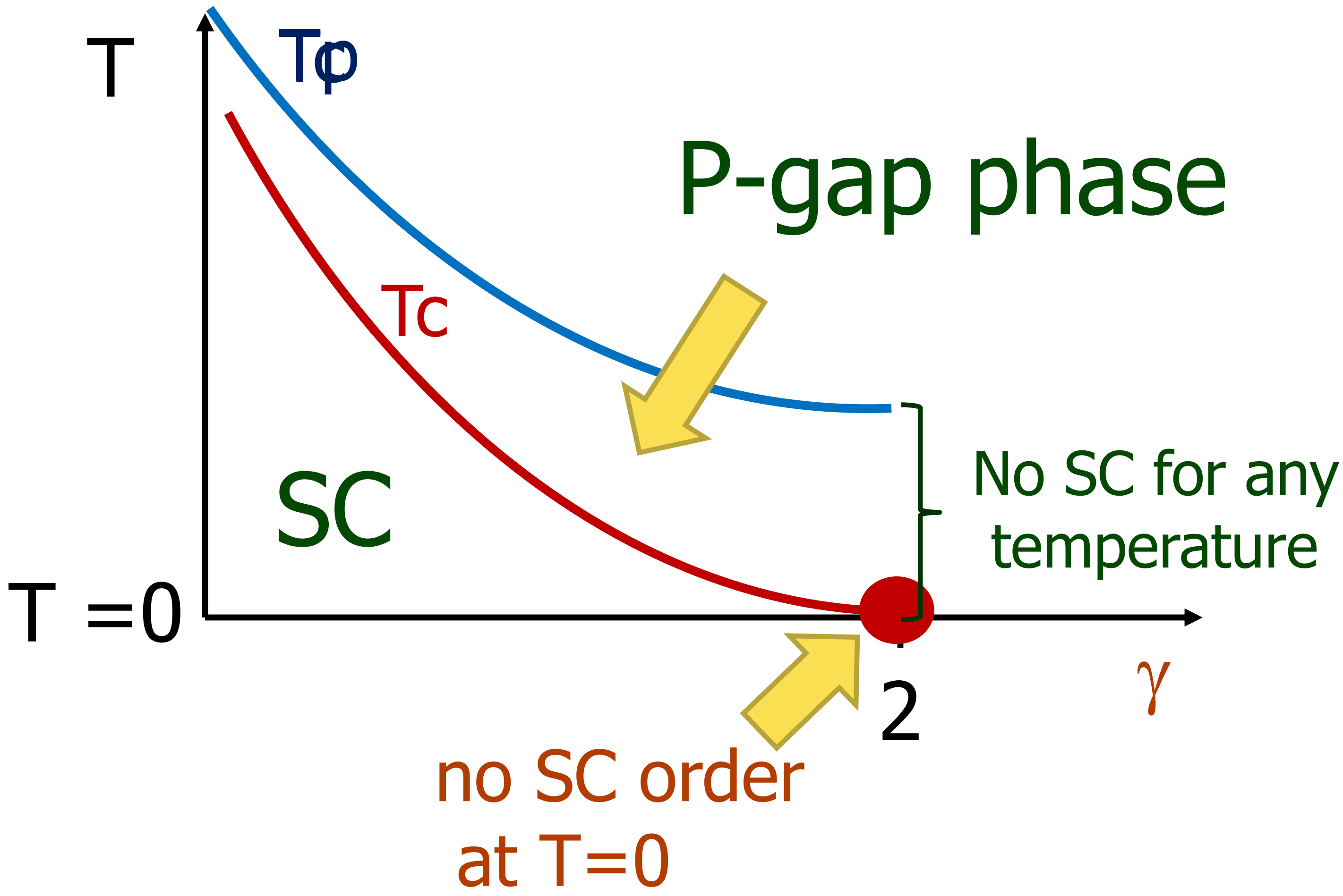
The exact solution of the linearized gap equation

$$\Delta(\omega) = C\,\omega \sin\left[\beta\,(\log[\omega^2] - \omega^2)\right], \qquad \pi\beta \tanh[\pi\beta] = 1$$



Oscillations at all scales

Ec = 0*infinity -> const



T
Tp
P-gap phase
Tc
SC
No SC for any
temperature
T =0
2
γ
no SC order
at T=0

γ=2 is the case of electron-phonon interaction at vanishing Debye frequency ω_D and finite electron-phonon vertex g

$$\lambda = \frac{g^2}{\omega_D^2}$$

diverges

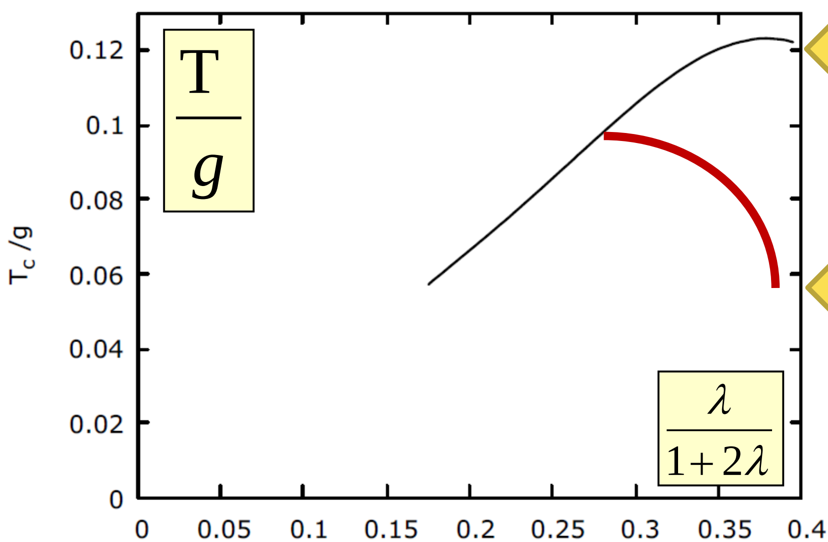
$$T_p \sim \omega_D \sqrt{\lambda}$$

Allen-Dynes formula

$$T_p = 0.18\ g$$

Marsiglio, Carbotte, Maximov, Combescot

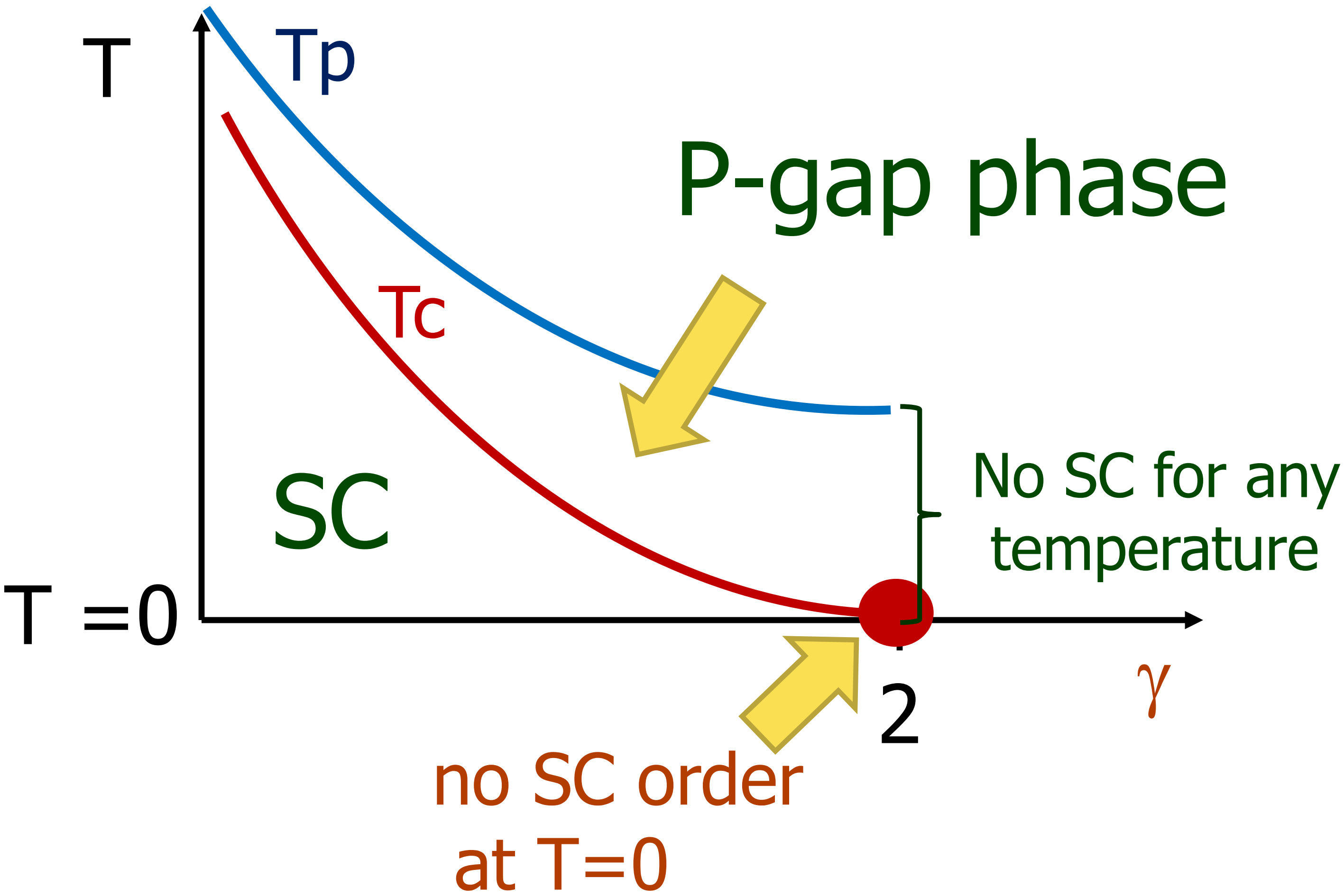
Esterlis, Kivelson



$$T_p = 0.12\ g$$

$$T_c \sim \omega_D$$

Conclusions:



THANK YOU