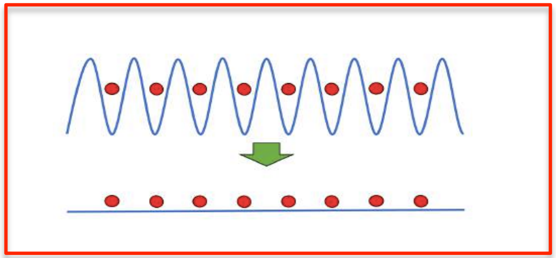
Non equilibrium dynamics of quantum systems – quantum quench and quantum work

Dynamics of bosons released
in optical traps



The Lieb-Liniger model

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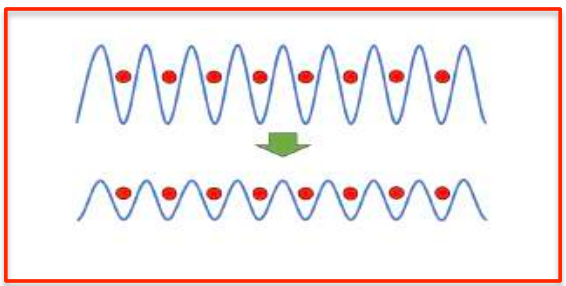

The sine-Gordon model

Colin Rylands
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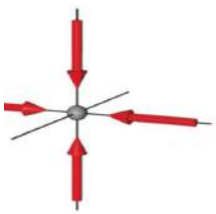
Low-dimensional emergent phenomena
in correlated systems and topological quantum matter
Tbilisi - June 2019

Non equilibrium dynamics of quantum systems - Quantum Quench of Optical Lattices

Nonequilibrium – the new frontier : Old and new questions

- Many experiments: cold atom systems, nano-devices, molecular electronics
- Isolated systems – effects not washed out by coupling to environment
- Fine control of parameters
- Many systems described by integrable Hamiltonians

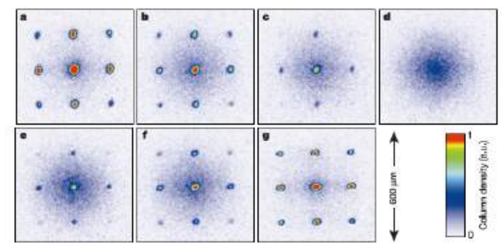
Quench protocol: release bosons in optical traps



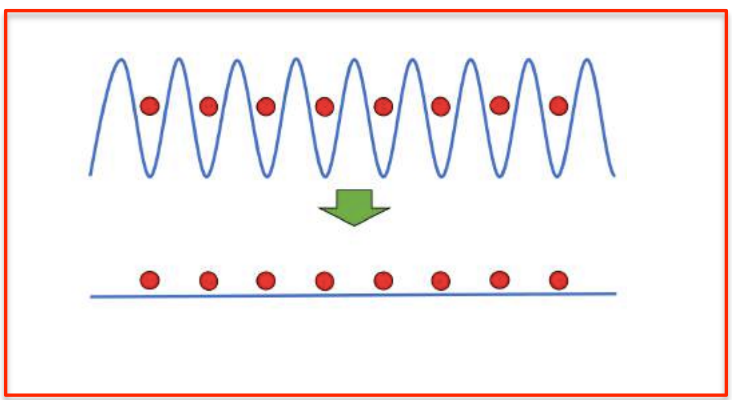
Bloch et al 2008

Greiner et al 2002

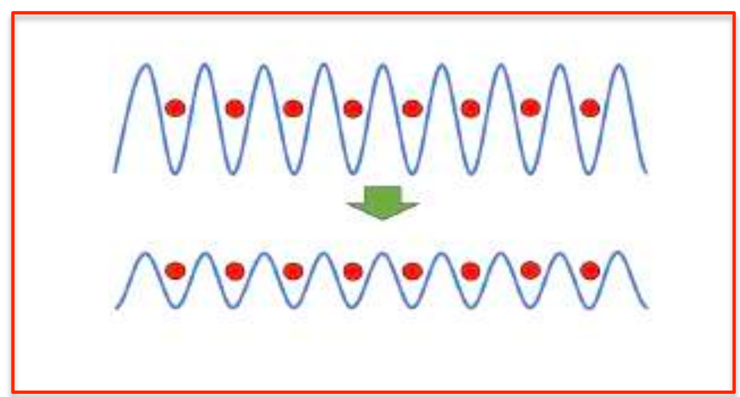
SF



Mott



The Lieb-Liniger model

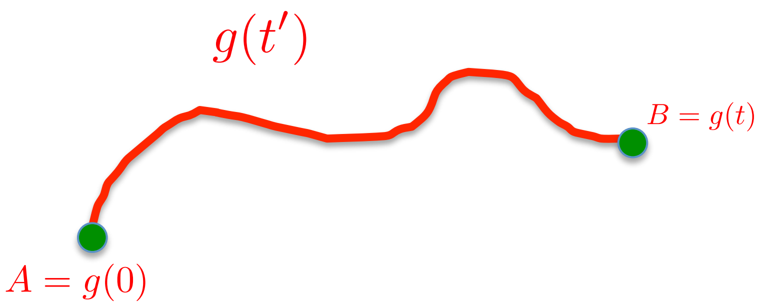


The sine-Gordon model

Time evolution - Quench protocol in isolated systems

Quench protocol

- **Isolated system** with Hamiltonian $H(t) = H[g(t)]$ depends on "work parameter" $g(t)$
- Initial state, $|\Phi_i\rangle$, typically ground state of $H(0) = H[A]$
- Evolve initial state under $H(t') = H[g(t')]$ from $t' = 0$ to $t' = t$ where



$$|\Phi_i, t\rangle = T e^{-i\int_0^t H(t')dt'} |\Phi_i\rangle \quad \Longrightarrow \quad |\Phi_i, t\rangle = e^{-iHt}|\Phi_i\rangle$$

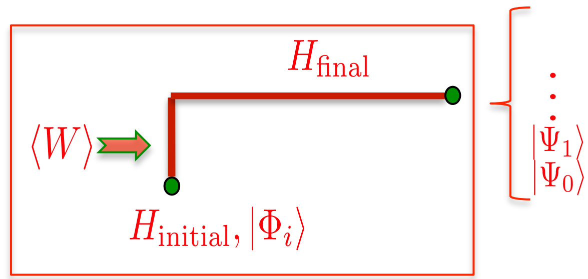
Sudden quench

- Process depends on initial state and on Hamiltonian
- Local characteristics: calculate evolution of local observables, correlations ..
- Global characteristics: **quantum work,** spread of entanglement ..

Nonequilibrium Thermodynamics -Work done in a quench

- During the quench energy is pumped into the system - $W = E_{\rm final} - E_{\rm initial}$: **work** is done

  First law of thermo' $dE = dQ + dW$ but system isolated: $dQ = 0$, so $dE = dW$

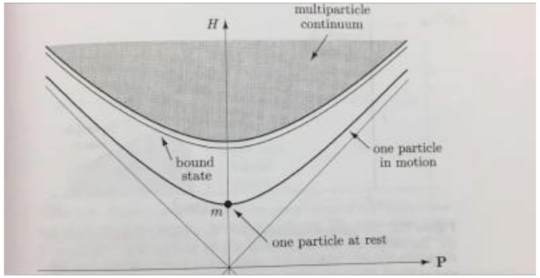


Work – *random variable* : involves two measurements - initial and final energies,
- at initial time - yielding $|\Phi_i\rangle$ and ϵ_i with probability $P_i = 1$
- at final time - yielding $|\Psi_n\rangle$ and E_n with probability $P_{i\to n} = |\langle \Psi_n | \Phi_i \rangle|^2$

- For a sudden quench the work distribution: *(Talkner et al '07 "Not an observable")*

$$P(W) = \sum_n \delta\left(W - (E_n - \epsilon_i)\right) |\langle \Psi_n | \Phi_i \rangle|^2$$

Nonequilibrium Thermodynamics -Work done in a quench



Silva, Gambassi, Palmai, Sotiriadis, Mussardo, Calabrese, Goold …. : '08 –'18

- $P(W)$ **has the form of the spectral function:**

- Defined for $W \geq \delta E = E_0 - \epsilon_i$

- A delta function at threshold $W = \delta E$ weighted by the fidelity $\mathcal{F} = |\langle \Psi_0 | \Phi_i \rangle|^2$ transition from the initial state to the ground state of H_f

- If H_f is gapped and translation invariant - continuum of excited states into which $|\Phi_i\rangle$ can transition with $W = 2m + \delta E$ lowest threshold for the continuum.

- Power like behavior at threshold $P(W) \sim \theta(W - \delta E - 2m)(W - \delta E - 2m)^\alpha$

- Similarly the four-particle emission continuum threshold $W = 4m + \delta E$

- It there are bound states m_b then delta function may appear at $W = m_b + \delta E$

- **Work distribution directly measureable**

- ***Reversible and irreversible processes, entropy production, spread of entanglement***

- ***Fluctuation theorems*** : *Jarzynski '97* $\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$ *Cohen-Galavotti '95* $\frac{P(S)}{P(-S)} = e^{\frac{S}{k_B}}$ *Crooks '99* $\frac{P_F(W)}{P_B(-W)} = e^{\frac{W - \Delta F}{T}}$ *and more.*

  - relate equilibrium and nonequilibrium, relate forward and backward evolution

Nonequilibrium Dynamics – Loschmidt amplitude

- Claim: Work distribution is related to Loschmidt Amplitude (*Talkner et al 2007*)

$$\mathcal{G}(t) = \langle \Phi_i | \, e^{-iHt} \, | \Phi_i \rangle$$

$$\Longrightarrow \quad P(W) = \int_{-\infty}^{\infty} \frac{\mathrm{d}t}{2\pi} e^{iWt + i\epsilon_i t} \mathcal{G}(t)$$

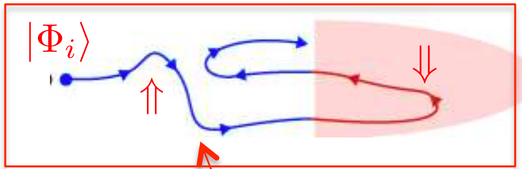
Related concepts:

Return Probability
Survival probability
Persistence
Scrambling

- The Loschmidt amplitude probes the full Hilbert space of states

$$\mathcal{G}(t) = \langle \Phi_i | \, e^{-iHt} \, | \Phi_i \rangle = \sum_n e^{-iE_n t} |\langle \Psi_n | \Phi_i \rangle|^2$$

starting from and weighted by overlaps with initial state $|\Phi_i\rangle$



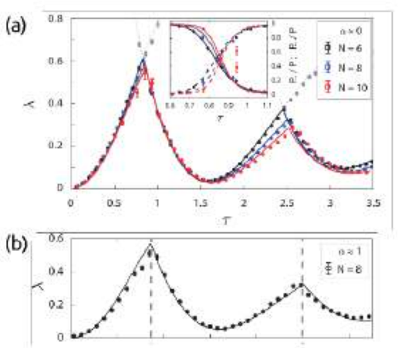
$e^{-iHt} \, |\Phi_i\rangle$

- May exhibit singularities Dynamical Quantum Phase Transitions DQPT *(Heyl ’18)* some similarities to thermodynamic phase transitions, Fisher &Lee-Yang zeroes

$$\lambda(t) = \lim_{N \to \infty} \frac{1}{N} \log |\mathcal{G}(t)|^2$$

DQPT depends on initial state, not only on the Hamiltonian

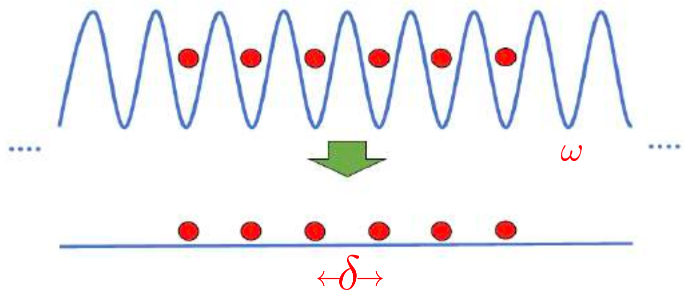
Observation of DQPT in Ising model



Jurevic et al ‘17

Quench of an optical lattice: *gapless scenario*

- **Quench protocol**: Release N neutral bosons from a deep trap $\bar{x}_j = j\delta,\ j = 1 \cdots N$

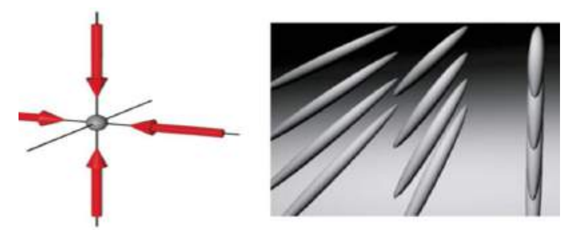


$$|\Psi_i\rangle = \int \mathrm{d}^N x \prod_{j=1}^{N} \left[\frac{m\omega}{\pi}\right]^{\frac{1}{4}} e^{-\frac{m\omega}{2}(x_j - \bar{x}_j)^2} b^\dagger(x_j)|0\,\rangle$$

- Bosons well described by the Lieb-Liniger Hamiltonian

$$H_{LL} = \frac{1}{2m}\int \mathrm{d}x\,[\,-b^\dagger(x)\partial_x^2 b(x) + c\,b^\dagger(x)b(x)b^\dagger(x)b(x)\,]$$

I. Bloch et al



Coupling: $c > 0$ repulsive, $c < 0$ attractive

- Allow them to evolve $|\Psi(t)\rangle = e^{-itH_{LL}}|\Psi_i\rangle$ **Localized peaks broaden and bosons begin to collide**

- How to calculate evolution? Use partition of the unity?

$$|\Psi_i(t)\rangle = e^{-itH_{LL}}|\Psi_i\rangle = e^{-itH_{LL}}\sum_n |n\rangle\langle n|\Psi_i\rangle = \sum_n e^{-iE_n t}|n\rangle\langle n|\Psi_i\rangle$$

- But what are the eigenstates $|n\rangle$ and how to calculate ***overlaps***?

Eigenstates and Yudson states

- The exact eigenstates of the Lieb-Liniger model *(Lieb-Liniger '63)*

$$|\{k_j\}\rangle = \int \mathrm{d}^N x \prod_{\substack{i,j=1\\ i<j}}^{N} \frac{k_i - k_j - ic\,\mathrm{sgn}(x_i - x_j)}{k_i - k_j - ic} \prod_{l=1}^{N} e^{ik_l x_l} b^\dagger(x_l)|0\rangle$$

For PBC the momenta satisfy $k_j = \frac{2\pi}{L} n_j - \frac{1}{L} \sum_{l \neq j} \varphi(k_j - k_l)$ with $\varphi(x) = 2\arctan(x/c)$

The integer quantum numbers $\{n_j\}$ determine the momenta $\{k_j\}$, denote eigenstates $|\{n_j\}\rangle$

- Claim *(Yudson '82, Goldstein, NA '12)* : Can rewrite the standard partition of the unity

$$I_N = \sum_{n_1 < \cdots < n_N} \frac{|\{n_j\}\rangle\langle\{n_j\}|}{\mathcal{N}(\{n_j\})}$$ as $$I_N = \sum_{n_1, \ldots, n_N} \frac{|\{n_j\}\rangle(\{n_j\}|}{\mathcal{N}(\{n_j\})}$$

normalization

$$\mathcal{N}(\{n\}) = \det\left[\delta_{jk}\left(L + \sum_{l=1}^{N} \varphi'(k_j - k_l)\right) - \varphi'(k_j - k_k)\right]$$

in terms of Yudson states $|\{n\}) = \int \mathrm{d}^N x\, \theta(x_1 > \cdots > x_N) \prod_l^N e^{ik_l x_l}\, b^\dagger(x_l)\, |0\rangle$

- This allows the computation of overlaps and therefore of time evolution of $|\Psi_i\rangle$:

$$|\Psi_i(t)\rangle = \left[\frac{4\pi\hbar}{m\omega}\right]^{\frac{N}{4}} \sum_{n_1, \ldots, n_N} \frac{e^{-\sum_{j=1}^{N}\left[\frac{\hbar}{2m\omega}(1+i\hbar\omega t)k_j^2 + ik_j\bar{x}_j\right]}}{\mathcal{N}(\{n_j\})} |\{n_j\}\rangle$$

Other approach:
Quench action
Perfetto *et al.* 1904.06259

The Loschmidt Amplitude

- The Loschmidt Amplitude:

$$\mathcal{G}(t) = \langle \Phi_i | \, e^{-iHt} \, |\Phi_i\rangle = \langle \Phi_i | \Phi_i(t)\rangle = \left[\frac{4\pi}{m\omega}\right]^{\frac{N}{2}} \sum_{n_1,\ldots,n_N} e^{-\frac{1}{m\omega}\left[1+i\frac{\omega}{2}t\right]\sum_{j=1}^{N} k_j^2} \frac{G(\{n\})}{\mathcal{N}(\{n\})}$$

with

$$G(\{n\}) = \det\left[e^{-ik_j(\bar{x}_j - \bar{x}_k) - i\theta(j-k)\varphi(k_j-k_k)}\right]$$

$$\mathcal{N}(\{n\}) = \det\left[\delta_{jk}\left(L + \sum_{l=1}^{N}\varphi'(k_j-k_l)\right) - \varphi'(k_j-k_k)\right]$$

- Exact for any c, N, L.

- Displays **recurrences** : $\tau = (1+2\rho/c)^2\, L^2/\pi\omega$

with $\rho = N/L$ and ω characterizing the initial state

The Loschmidt Amplitude (infinite volume)

Open system (partially filled lattice) $\rho = N/L \ll 1/\delta$

- ***Repulsive interactions*** Simplifies for strong repulsion $c \gg m\omega$

$$\mathcal{G}(t) = \frac{1}{\left[1 + i\frac{\omega}{2}t\right]^{\frac{N}{2}}} \sum_P (-1)^P e^{-\frac{\omega \alpha_P^2}{4\left(1+i\frac{\omega}{2}t\right)}}$$

with $\alpha_P^2 = m\delta_{\text{eff}}^2 \|P\|^2/2$

$\delta_{\text{eff}} = \left[1 + \frac{2}{c\delta}\right]\delta$ Effective increased distance due to repulsion

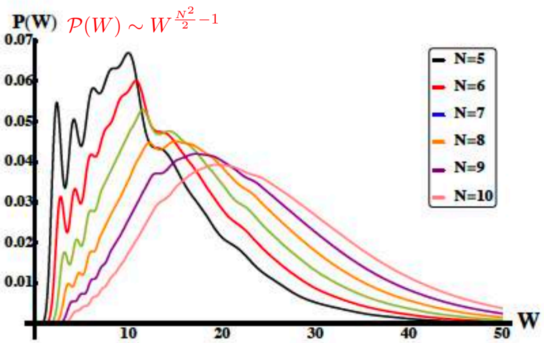
$\|P\|^2 = \sum_j^N (j - P(j))^2$ Measures how many particles were exchanged

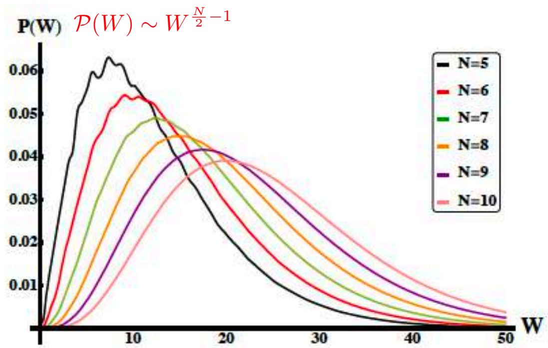
Sum over permutations corresponds to a sum over particles exchanging positions: e.g $\|P\|^2 = 2$ corresponds to a neighboring pair exchanging positions, $\|P\|^2 = 8$ could be 4 nearest neighbor exchanges or 1 next nearest neighbor exchange.

The Work Distribution (repulsive interactions, infinite volume)

- The work distribution is obtained from the Loschmidt amplitude

$$P(W) = \int_{-\infty}^{\infty} \frac{\mathrm{d}t}{2\pi} e^{iWt}\mathcal{G}(t) = \frac{e^{-\frac{2W}{\omega}}}{W}\left[\frac{2W}{\omega}\right]^{\frac{N}{2}} \sum_P (-1)^P \frac{J_{\frac{N-2}{2}}(2\sqrt{\alpha_P^2 W})}{[\alpha_P^2 W]^{\frac{N-2}{4}}}$$





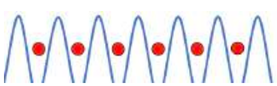
$5 \leq N \leq 10$

$\delta/m = 2, \;\; \omega = 10$

Work distribution for repulsive bosons

Work distribution for free bosons

- For large value of W (short times) no dependence on interaction (no overlaps initially)



- For small values of W strong effect of interactions: $\mathcal{P}(W) \sim W^{\frac{N^2}{2}-1}$ as opposed to: $\mathcal{P}(W) \sim W^{\frac{N}{2}-1}$
Indeed, $|\mathcal{G}(t)|^2 \to 1/t^{N^2}$ vs. $|\mathcal{G}(t)|^2 \to 1/t^{N}$ as $t \to \infty$. In 1-d even weak interaction have strong effect.

- Average work $\langle W \rangle = \int \mathrm{d}W \, W \mathcal{P}(W) = N\omega/4 = \langle \Psi_i | H | \Psi_i \rangle$

- Exponentiated work (yields all moments) $\langle e^{-\beta W} \rangle = \left(\frac{1}{1+\frac{\omega\beta}{2}} \right)^{\frac{N}{2}} \left[1 + \sum_{P \neq I} (-1)^P \frac{e^{-\frac{\omega \alpha_P^2}{2+\omega\beta}}}{\Gamma(N/2)} \right]$

cf Jarzynski equality

$$e^{-\beta \Delta F} = \langle e^{-\beta W} \rangle$$

The Work distribution (attractive interactions, infinite volume)

For $c < 0$ there are bound states of n -bosons, $n = 2, \cdots, N$
(momenta k form complex n- strings: $k_j = k + ijc/2, \; j = 1 \cdots n$

- Using a generalization of Yudson representation, we find

$$\mathcal{P}_{c<0}(W) = \mathcal{P}_{\text{unbound}}(W) + \mathcal{P}_{\text{bound}}(W)$$

- $\mathcal{P}_{\text{unbound}}(W)$ same expression as before with $c \to -c$, so $\delta_{\text{eff}} < \delta$ due to attraction.

Similar to super Tonks-Girardeau experiment, where one prepares system with large $c > 0$ and then suddenly quenches to large $c < 0$. Note, our expression valid for any c .

- $\mathcal{P}_{\text{bound}}(W)$ is due to the strings, n- string contributing $\mathcal{P}_{n-\text{bound}}(W) \propto |c|^{n-1} e^{-n|c|\delta}$

Transitions to states containing bound states are highly suppressed and in the true super-TG limit vanish entirely.

- For finite $c < 0$ new effects: bound states lower the energy and work distribution becomes non vanishing at negative values of W.

- Indeed, for a 2- string $\mathcal{P}_{\text{bound}}(W) \approx N\sqrt{\frac{2\pi\omega}{m}} \frac{e^{-|c|\delta - \frac{2W}{\omega}}}{\Gamma\left(\frac{N}{2}-1\right)} \left[\frac{2(W + \frac{|c|^2}{4m})}{\omega}\right]^{\frac{N}{2}-2}$, non vanishing for $-|c|^2/4m < W$

- There is a non zero probability that work can be extracted from the system. This does not violate the 2nd law of thermodynamics since $\langle W \rangle > 0$ *Jarzynski '11*

The Work distribution (repulsive interactions, finite volume)

Start from a **fully occupied lattice** – boundary effects important

As before:

$$P(W)=\int_{-\infty}^{\infty}\frac{\mathrm{d}t}{2\pi}e^{iWt}\mathcal{G}(t)=\frac{e^{-\frac{2W}{\omega}}}{W}\left[\frac{2W}{\omega}\right]^{\frac{N}{2}}\sum_P(-1)^P\frac{J_{\frac{N-2}{2}}(2\sqrt{\alpha_P^2W})}{[\alpha_P^2W]^{\frac{N-2}{4}}}$$

But boundary conditions enter when calculating $\alpha_P^2=m\delta_{\mathrm{eff}}^2\|P\|^2/2$

For instance when $\rho\ll\delta^{-1}$ the permutation $P=(23...N1)$ gives $\alpha_P^2=m\delta_{\mathrm{eff}}^2N(N-1)/2$
however with PBC it gives $\alpha_P^2=m\delta_{\mathrm{eff}}^2N/2$

- Region $W\sim\langle W\rangle$ not affected by BC, dominated by few exchanges of particles average work is as before.

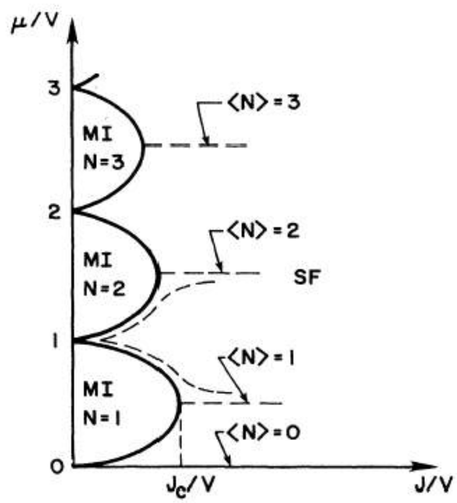
- Region $W\ll\langle W\rangle$ strongly affected, all permutation contribute

We have $\mathcal{P}(W)\sim W^{\frac{N}{2}}$ corresponding to $|\mathcal{G}(t)|^2\to 1/t^{N+2}$

As opposed to $\mathcal{P}(W)\sim W^{\frac{N^2}{2}-1}$ and $|\mathcal{G}(t)|^2\to 1/t^{N^2}$ as before.
The strongly interacting particles have no space to expand into unlike previously, resulting in slower decay of the echo.

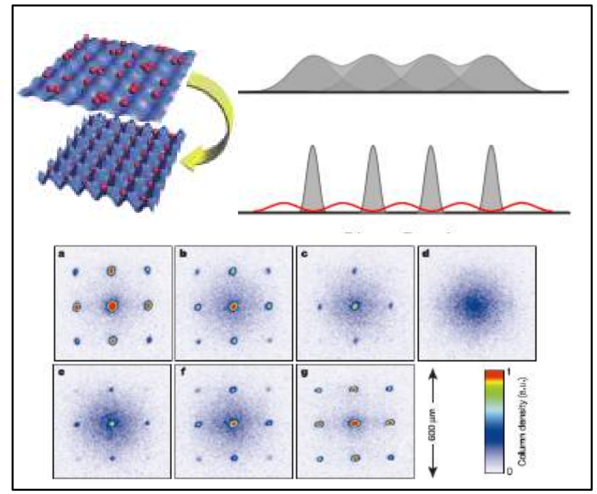
Quench of an optical lattice: *gapped scenario*

- Quench system from an initial Mott state, the ground state of very high barriers (BH model)



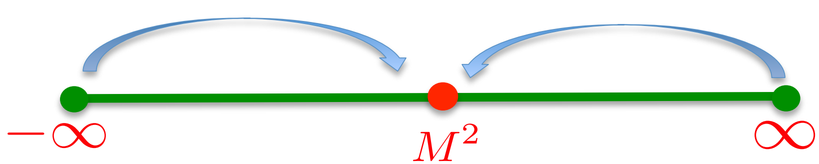
Fisher et al, 1989

Greiner et al '02 Bloch et al '08

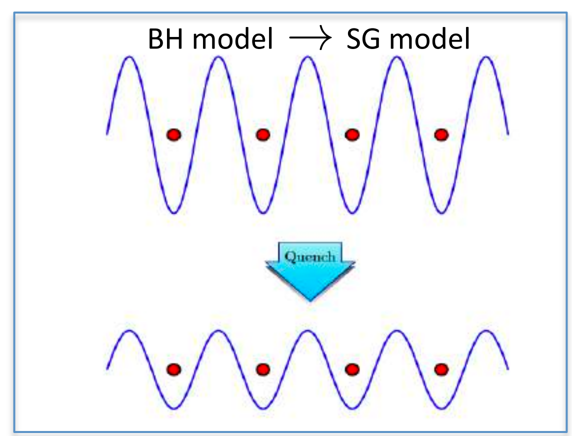


- Quench (*e.g.* SG as effective low-E theory of BHub)

$$H_{SG}(M^2 = \pm\infty, \beta) \rightarrow H_{SG}(M^2, \beta)$$



$$H_{SG}(M^2,\beta) = \frac{1}{2}\int \mathrm{d}x \left\{ \Pi^2(x) + [\partial\phi(x)]^2 - M^2 \cos[\beta\phi(x)] \right\}$$



Mott

SF

- Another realization: pair of coupled one-dimensional condensates of interacting atoms *(Gritsev et al '07)*

The Sine-Gordon / Massive Thirring model

- The Sine-Gordon model: **Quantum Integrable** (*Zamolodchikov^2 '1978)*

$$H_{SG}(M,\beta) = \frac{1}{2}\int \mathrm{d}x \left\{\Pi^2(x) + [\partial\phi(x)]^2 - M^2\cos[\beta\phi(x)]\right\}$$

- Low-energy effective action of many classical and quantum systems (e.g *Giamarchi book*)

Bose Hubbard model
Spin chains
Interacting bosons

Quantum impurity models
KT phase transitions

- It is equivalent to the massive Thirring model *(S. Coleman '78)*

$$H_{MTM}(m_0,g) = -i\int \Big(\psi_+^\dagger(x)\partial_x\psi_+(x) - \psi_-^\dagger(x)\partial_x\psi_-(x)\Big) + \\ + m_0\int \Big(\psi_+^\dagger(x)\psi_-(x) + \psi_-^\dagger(x)\psi_+(x)\Big) + 4g\int \psi_+^\dagger(x)\psi_-^\dagger(x)\psi_-(x)\psi_+(x)$$

Parameters are related in a non-universal way (depend on reno' scheme) $M,\beta \leftrightarrow m_0, g$

- Hamiltonian is **Quantum Integrable** (*Bergknoff and Thacker 1978*)

spectrum: solitons, anti-solitons of mass m for repulsive interaction $4\pi < \beta^2 < 8\pi$

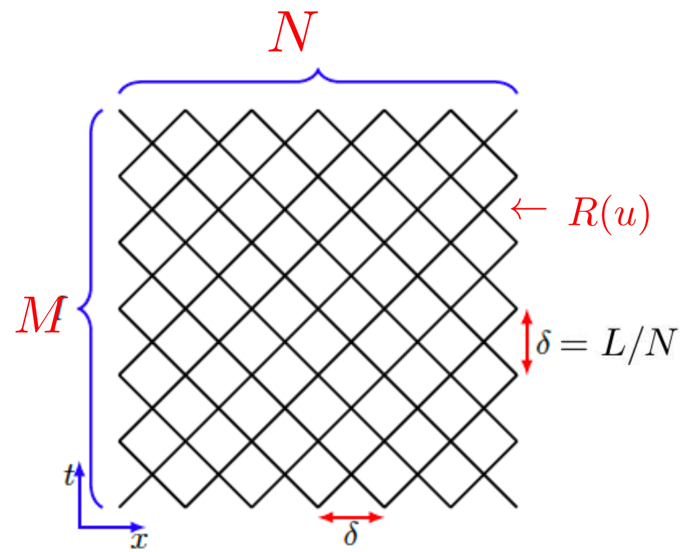
solitons, anti-solitons and breather bound states for attractive interactions $0 < \beta^2 < 4\pi$

- Non interacting limit: $g = 0,\ \beta^2 = 4\pi$ Luther-Emery point

The MTM model in space time lattice

- Represent the massive Thirring model on the light-cone lattice in space-time in terms of spins while preserving integrability (*Destri - De Vega '87)*: time evolution viewed as stat mech problem

- Left and right moving bare fermions evolve in a discretized Minkowski space time



- To each link emanating a vector space $V = \mathbb{C}^2$ with V_{2j} associated with left movers, V_{2j-1} right movers of j^{th} intersection.

- Jordan-Wigner $\boxed{\begin{array}{l} \sigma^+_{2j-1} \to \psi^\dagger_+(j\delta) \\ \sigma^+_{2j} \to \psi^\dagger_-(j\delta) \end{array}}$

- Each vertex associate matrix

$$\overset{V_{2j}\quad V_{2j+1}}{\times} = R_{2j,2j+2}(2\Theta) = \begin{pmatrix} \sinh(2\Theta+\eta) & 0 & 0 & 0 \\ 0 & \sinh(2\Theta) & \sinh(\eta) & 0 \\ 0 & \sinh(\eta) & \sinh(2\Theta) & 0 \\ 0 & 0 & 0 & \sinh(2\Theta+\eta) \end{pmatrix}$$

$\eta = i\gamma$ with $\gamma \sim \pi/2 + g$ or $\frac{\gamma}{\pi} = 1 - \frac{\beta^2}{8\pi}$

microscopic transition amplitudes

Θ rapidity cutoff for bare particles, related to bare mass $m_0 = \frac{4N}{L}\sin\gamma\, e^{-2\Theta}$, mass $m = \frac{4N}{L} e^{-\frac{\pi}{\gamma}\Theta}$

- Thermo limit $N, L \to \infty$ with $\delta = L/N$ fixed, continuum limit $\delta \to 0, \Theta \to \infty$, with m fixed

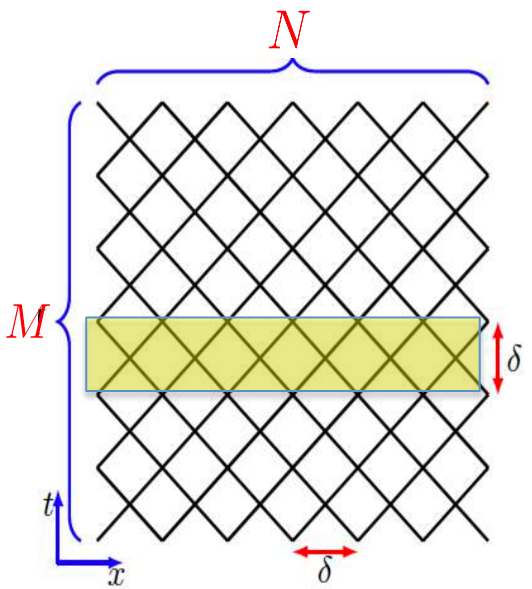
Time Evolution on the Lattice

- Time evolution by δ consist of a move to the left followed by a move to the right (*DDV ’87, ‘89, ‘92*)

$$e^{-iH\delta} = \left[\frac{\sinh(2\Theta+\eta)}{\sinh(2\Theta-\eta)}\right]^N \tau(-\Theta)\tau^{-1}(\Theta)$$

Where $\tau(u)$ is the **transfer matrix**

$$\tau(u) = \mathrm{Tr}_k\left[R_{k1}(u-\Theta)R_{k2}(u+\Theta)\dots R_{k2N-1}(u-\Theta)R_{k2N}(u+\Theta)\right]$$



- The time evolution over interval $t = M\delta$

$$e^{-iHt} = \lim_{N\to\infty}\left[\frac{\sinh(2\Theta+\eta)}{\sinh(2\Theta-\eta)}\right]^{NM}\left[\tau(-\Theta)\tau^{-1}(\Theta)\right]^M$$

- The states in this regularization scheme take the form *(Thacker ‘81)*

$$|\Psi\rangle = \sum_{\{j\}} a_{\{j\}} \prod_{l=1}^{N} \sigma^+_{j_l}|\Downarrow\rangle \xrightarrow[\text{continuum limit}]{} = \sum_{\{j\}} a_{\{j\}} \prod_{l=1}^{N} \psi^+_{a_{j_l}}(j_l\delta)|0\rangle$$

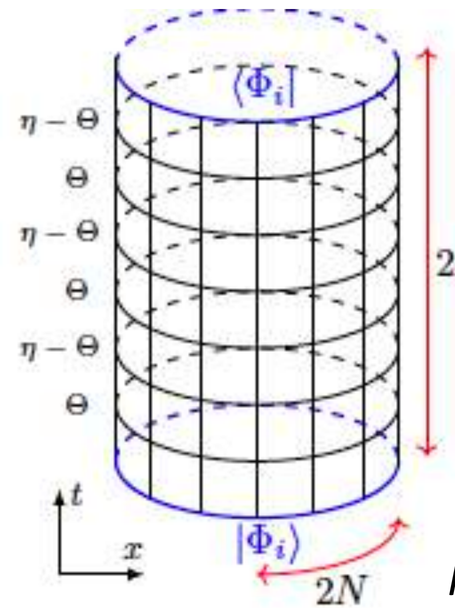
The Loschmidt Amplitude on the Lattice

- The survival amplitude to end up in state $|\Phi_i\rangle$ having propagated for time $t = M\delta$ on lattice of length $L = N\delta \to \infty$:

$$\mathcal{G}(t = M\delta) = \lim_{L=N\delta\to\infty} \left[\frac{\sinh(2\Theta+\eta)}{\sinh(2\Theta-\eta)}\right]^{NM} \langle\Phi_i| \left[\tau(-\Theta)\tau^{-1}(\Theta)\right]^{M} |\Phi_i\rangle$$

$$= \lim_{L\to\infty} \frac{\langle\Phi_i| \left[\tau(-\Theta)\tau(\Theta-\eta)\right]^{M} |\Phi_i\rangle}{\sinh^{2NM}(2\Theta-\eta)\sinh^{2NM}(\eta)}$$

used identity $\tau^{-1}(\Theta) \propto \tau(\Theta-\eta)$ *(Wang et al. 2017)*

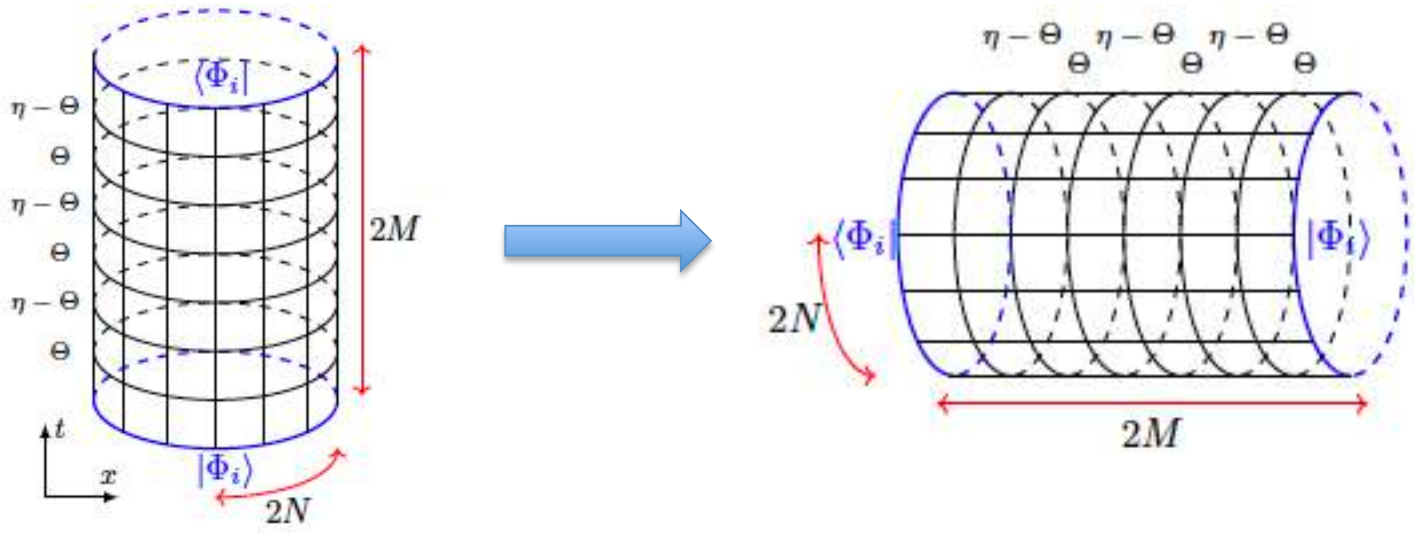


PBC

- Calculating the Loschmidt amplitude became a classical 2d lattice problem, with $\tau(-\Theta)\,\tau(\Theta-\eta)$ the vertical transfer matrix of an inhomogeneous 6-vertex model on a $2N \times 2M$ lattice, with periodic BC in the horizontal direction and $|\Phi_i\rangle$ initial and final boundary states.

Rotating space and time

- Rotating to a more convenient configuration (modular transformation)



The initial state becomes boundary conditions in the horizontal direction, the periodic boundary conditions becoming a trace in the time direction and parameters associated to vertical lines being exchanged with those associated to horizontal lines.

$$\mathcal{G}(t) = \lim_{L=N\delta\to\infty}\left[\frac{1}{\sinh(2\Theta-\eta)\sinh(\eta)}\right]^{2NM} \mathrm{Tr}\left[\langle\Phi_i|\otimes_{j=1}^{N}\left[T_{2j-1}(-\Theta)\times T_{2j}(\Theta)\right]|\Phi_i\rangle\right]$$

The trace being due to horizontal PBC

with the transfer matrix

$$T_j(u) = R_{j1}(u-\Theta)R_{j2}(u-(-\Theta+\eta))\dots R_{j2M-1}(u-\Theta)R_{j2M}(u-(-\Theta+\eta))$$

acting in the horizontal space.

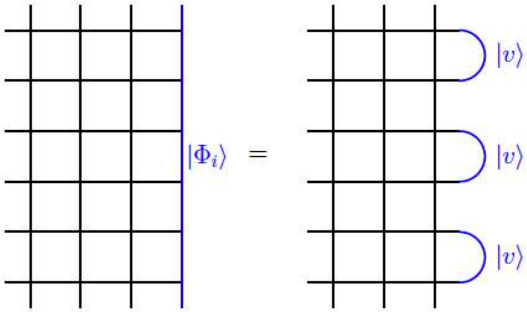
Some interesting initial states

- Consider initial states that can be written as the product of **two site states**

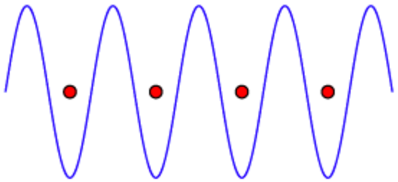
$$|\Phi_i\rangle = \otimes_{j=1}^{N} \frac{1}{\sqrt{\langle v_j | v_j \rangle}} |v_j\rangle$$

with

$$|v_j\rangle = c_1 |\uparrow\downarrow\rangle + c_2 |\downarrow\uparrow\rangle$$



The states $c_1 = \pm c_2$ correspond to the ground state and highest excited state of the Hamiltonian when the initial mass m_i is very large. In bosonic language the coefficient of the $\cos(\beta\phi(x))$ term is very large with the initial states $|\Phi_i\rangle$ corresponding to those which minimize or maximize this particular term.



- - In a low energy description of the superfluid-Mott transition these initial states correspond to having a very deep optical lattice with either positive of negative on site interaction

  - In a low energy description of a pair of coupled condensates these states correspond to an initial phase difference of 0 or π .

- Note: Quench process involves all eigenstates of the post-quench Hamiltonian which may result in the Sine-Gordon description being invalid.

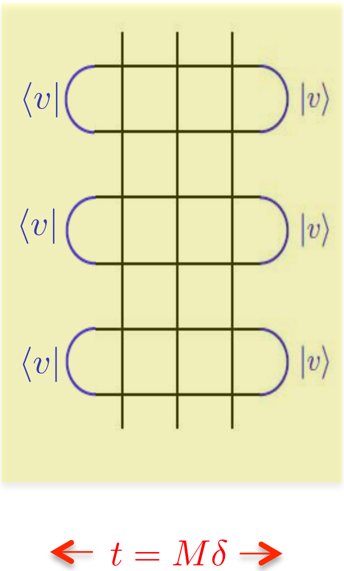
Loschmidt amplitude

- The Loschmidt amplitude takes the form

$$\mathcal{G}(t) = \lim_{N\to\infty} \left[\frac{1}{\sinh(2\Theta-\eta)\sinh(\eta)}\right]^{2NM} \mathrm{Tr}\left[\frac{\langle v|T(-\Theta)\otimes T(\Theta)|v\rangle}{\langle v|v\rangle}\right]^{N}$$

$$= \lim_{N\to\infty} \left[\frac{1}{\sinh(2\Theta-\eta)\sinh(\eta)}\right]^{2NM} \sum_j \left[\frac{\Lambda_j}{\langle v|v\rangle}\right]^{N}$$

$$= \lim_{N\to\infty} \left[\frac{1}{\sinh(2\Theta-\eta)\sinh(\eta)}\right]^{2NM} \left(\frac{\Lambda_{\max}}{\langle v|v\rangle}\right)^{N}$$



- Maximal eigenvalue dominates in the infinite volume limit $N \to \infty$

- If a level crossing occurs at some time t_c then a DQPT takes place.

Maximal Eigenvalue

- Need the maximal eigenvalue of the quantum transfer matrix $\langle v|\ T(u) \otimes T(-u)\ |v\rangle$ evaluated at $u = -\Theta$

Equivalent to finding the ground state energy of the XXZ model with open boundary conditions *(B. Pozsgay '13, Piroli, Pozsgay, Vernier '17)*:

- Claim: For an initial state $|v(\xi)\rangle$ specified in the continuum limit by $c_1 = e^{2\xi} c_2$ (i.e. $\xi = i\pi/2$ is ground state, and $\xi = 0$ highest excited state), the unique maximal eigenvalue is given by: $\Lambda_{\max} = \Lambda_\xi(-\Theta)$ **Book: Off Diagonal BA** Wang, Yang, Cao, Shi '17

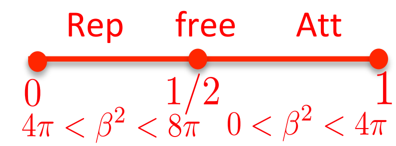
$$\Lambda_\xi(u) = \frac{\sinh(2u+\eta)}{\sinh(2u)} \sinh^2(u + (\xi - \eta/2)) \left[\sinh(u+\Theta)\sinh(u-\Theta+\eta)\right]^{2M} \prod_k^M \frac{\sinh(u-\lambda_k-\eta)}{\sinh(u-\lambda_k)} \frac{\sinh(u+\lambda_k-\eta)}{\sinh(u+\lambda_k)}$$
$$+ \frac{\sinh(2u-\eta)}{\sinh(2u)} \sinh^2(u - (\xi - \eta/2)) \left[\sinh(u-\Theta)\sinh(u+\Theta-\eta)\right]^{2M} \prod_k^M \frac{\sinh(u-\lambda_k+\eta)}{\sinh(u-\lambda_k)} \frac{\sinh(u+\lambda_k+\eta)}{\sinh(u+\lambda_k)}$$

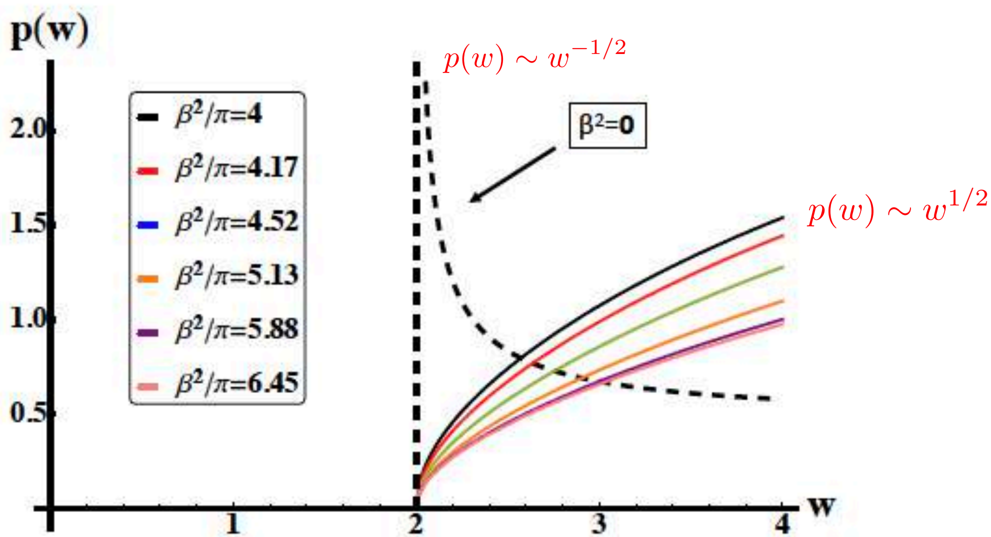
with the Bethe parameters $\{\lambda_j,\ j = 1 \dots M = t/\delta\}$ satisfying (for initial state $|v(\xi)\rangle$)

$$\frac{\sinh(2\lambda_j-\eta)}{\sinh(2\lambda_j+\eta)} \frac{\sinh^2(\lambda_j-(\xi-\eta/2))}{\sinh^2(\lambda_j+(\xi-\eta/2))} \left[\frac{\sinh(\lambda_j-\Theta)\sinh(\lambda_j+\Theta-\eta)}{\sinh(\lambda_j+\Theta)\sinh(\lambda_j-\Theta+\eta)}\right]^{2M} = -\prod_k^M \frac{\sinh(\lambda_j-\lambda_k-\eta)}{\sinh(\lambda_j-\lambda_k+\eta)} \frac{\sinh(\lambda_j+\lambda_k-\eta)}{\sinh(\lambda_j+\lambda_k+\eta)}$$

Work distribution at threshold: quench from ground state

Repulsive regime $4\pi < \beta^2 < 8\pi$





- The edge of the rescaled work distribution $p(w) = 4\pi P(W)/(mL\mathcal{F})$ for quench in the Sine-Gordon model, measured in units of mass from the ground state energy $w = (W - \delta E)/m$. The dashed line shows same quantity for non interacting bosons while solid lines are for different values of interaction strength. The solid black line is for a quench of free fermions.
- The interaction modifies the exponent from $-1/2$ to $1/2$.

Interactions - the repulsive regime

- For the quench from the ground state:

$$P(W) = \mathcal{F}\delta\left(W-\delta E\right) + \frac{\mathcal{F}L}{4\pi}\theta\left(W-\delta E-2m\right)\frac{\mathrm{Re}\left\{\mathcal{K}_{i\frac{\pi}{2}}\left(\frac{\gamma}{\pi}\mathrm{arcosh}\left[\frac{W-\delta E}{2m}\right]\right)\right\}}{\sqrt{\left(W-\delta E\right)^2/4m^2-1}}\left(W-\delta E\right)+\dots$$

- After expanding $\mathcal{K}$ about the edge singularity $W-\delta E\sim 2m$ we get that the edge exponent is 1/2 as compared to -1/2 for **non interacting bosons**.

- Starting from **fermions**, the effect of interactions on $P(W)$ near the threshold is negligible, since in this region the quench process is governed by transitions to excited states containing only two quasi-particles moving away from each other.

- More results on different initial state and attractive interactions (see paper)

Conclusions

- Quenches realizable in cold atoms experiments, work distribution measurable

- Calculated work distribution of a quench in a strongly interacting, gapless system.

- Studied bound state contributions to the work distribution.
  Showed they dramatically change the distribution and allow for negative values of work

- Showed that interactions strongly affect the universal edge exponents of the work distribution and also the long time decay of the Loschmidt echo

- Calculated work statistics for some quenches of the SG Hamiltonian – describes a sudden lowering of periodic potential

- Calculated for attractive and repulsive interactions, determined critical exponents at threshold

To do

- Connect to nonequilibrium thermodynamics: entropy production, fluctuation theorems

- Small and large systems - increase role of fluctuations (fluctuation theorems)

- Quench across critical points, defect production, Kibble-Zurek dynamics, scaling and universality

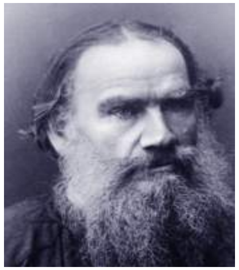
- Time dependent quenches: slow drives, Floquet

Non equilibrium dynamics of quantum systems

How to characterize a system out of equilibrium?

All equilibrium systems resemble each other, each out-of-equilibrium system is out-of-equilibrium in its own way.

L. N. Tolstoy, modernized



No general framework yet