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BETWEEN ONE AND INFINITE DIMENSIONS: SURPRISES OF ANDERSON LOCALIZATION

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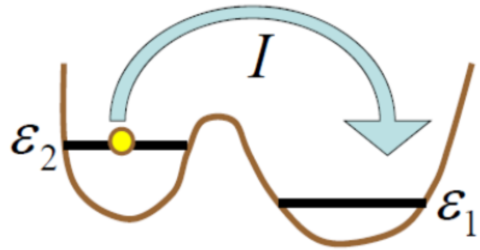
Two-site Anderson model = Two-well potential



$$\hat{H} = \begin{pmatrix} \varepsilon_1 & I \\ I & \varepsilon_2 \end{pmatrix}$$

Hamiltonian

Diagonalization: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Leftarrow \begin{pmatrix} \psi_{11} \\ \psi_{21} \end{pmatrix}, \begin{pmatrix} \psi_{12} \\ \psi_{22} \end{pmatrix}$

$$|\varepsilon_2 - \varepsilon_1| >> I$$

$$\psi_{1,1}, \psi_{2,2} \approx 1$$

$$\psi_{1,2}, \psi_{1,2} = O\left(\frac{I}{|\varepsilon_2 - \varepsilon_1|} \right)$$

Off-resonance

Eigenfunctions are close to the original on-site wave functions

$$|\varepsilon_2 - \varepsilon_1| << I$$

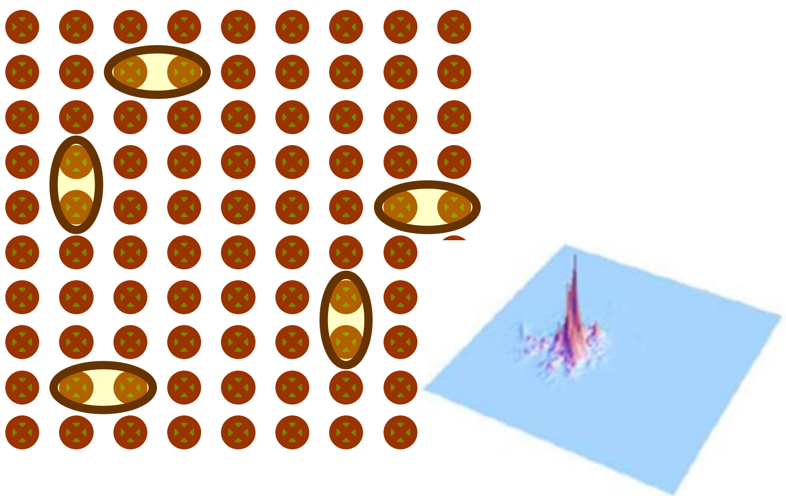
$$\psi_{\alpha,\beta} \approx \pm \frac{1}{\sqrt{2}}$$

Resonance

In both bonding and anti-bonding eigenstates the probability is equally shared between the sites

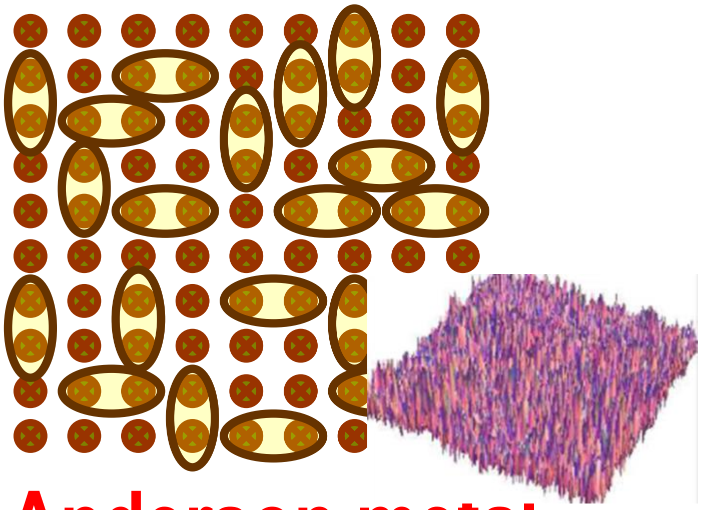
Hybridization

Delocalization from localization: proliferation of resonances



Anderson insulator

Few isolated resonances



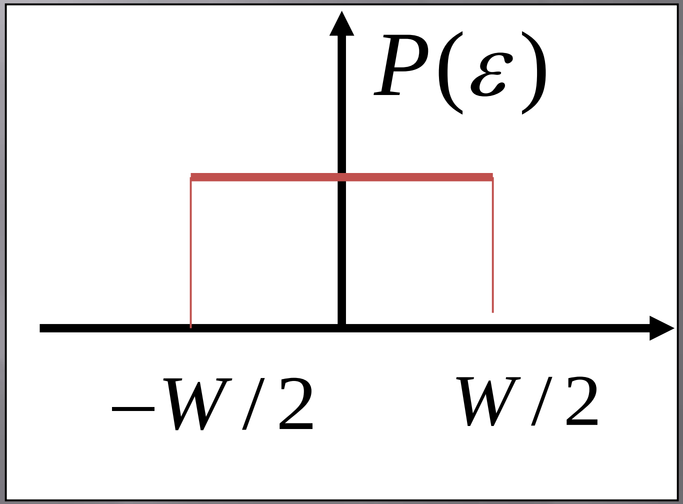
Anderson metal

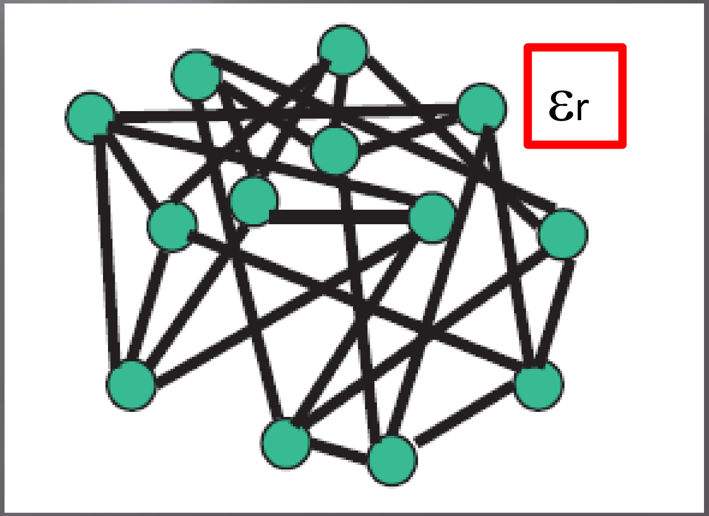
Many resonances and they overlap

Anderson model of localization

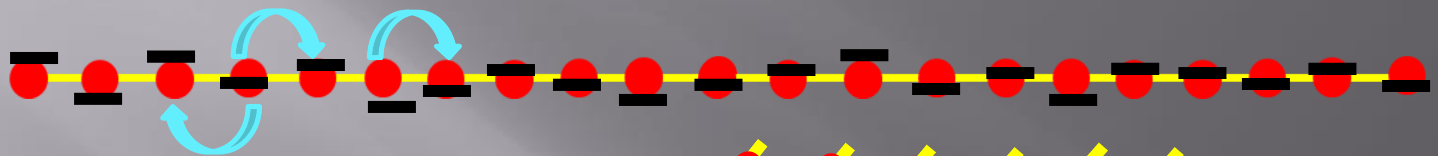
$$H = -I \sum_{<r,r'>} c_r^+ c_{r'} + \sum_r \varepsilon_r n_r$$

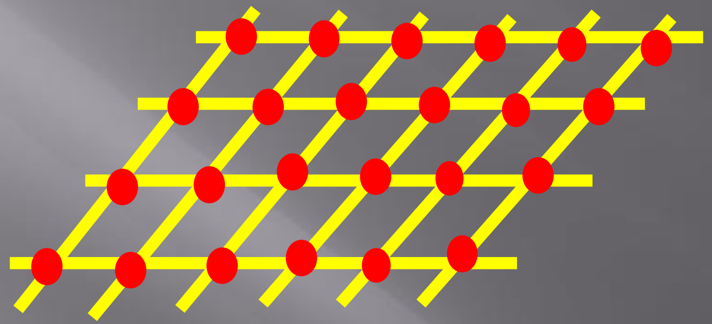
Disorder strength W



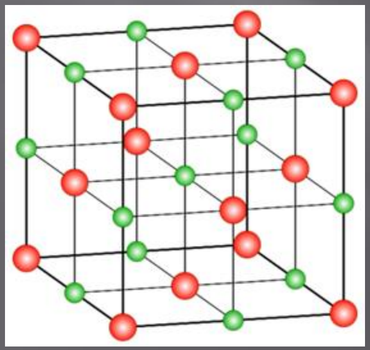


Role of dimensionality



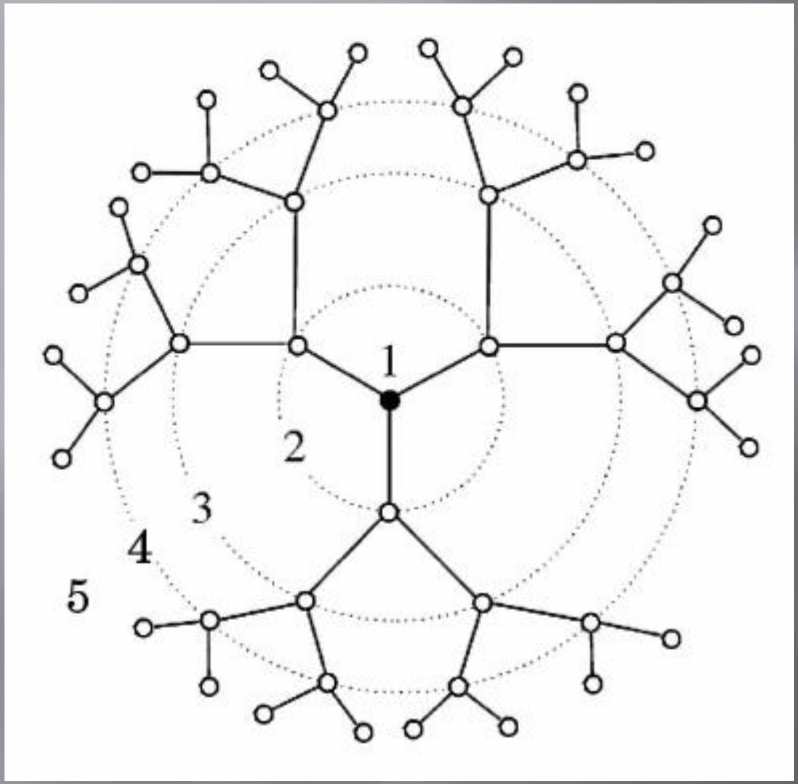


1D and 2D systems: all states localized at any disorder



3D systems: mobility edge and localization transition

Cayley tree



No loops like in 1D but $N(r)=K^r$ like in infinite dimensions

Coordination number finite like in finite dimensions

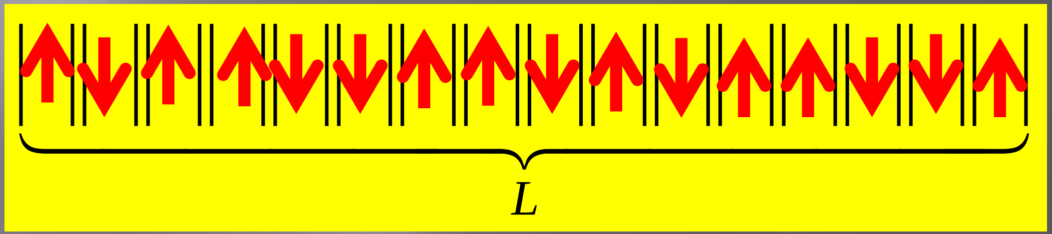
Why interesting?

Many-Body Hilbert space

basis state in Many-Body Hilbert/Fock Space

One-dimensional spin-chains

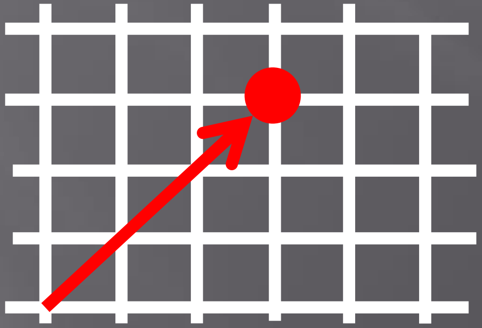
$$\Psi_b = | s_1^z, s_2^z, \dots\ s_L^z \rangle, \quad Dim = 2^L$$



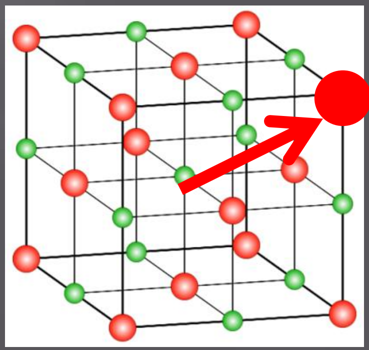
Exponentially large size of Hilbert space = INFINITE DIMENSIONALITY

Basis states of a single-particle on a lattice

$$\Psi_b = | \vec{n} \rangle, \quad Dim = L^d$$



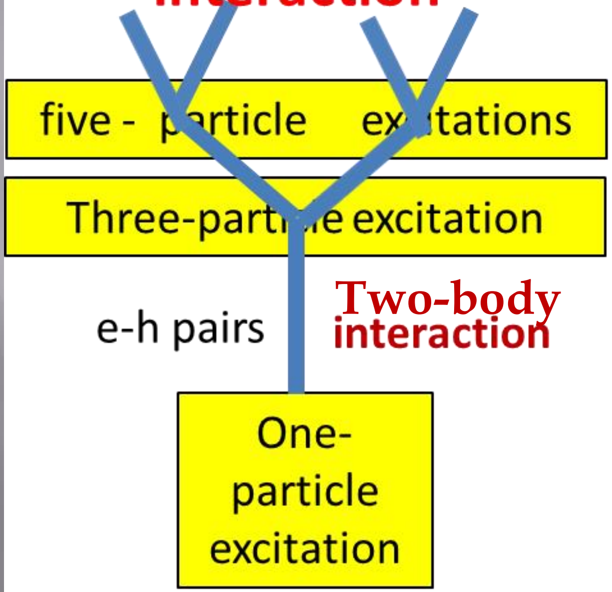
d=2



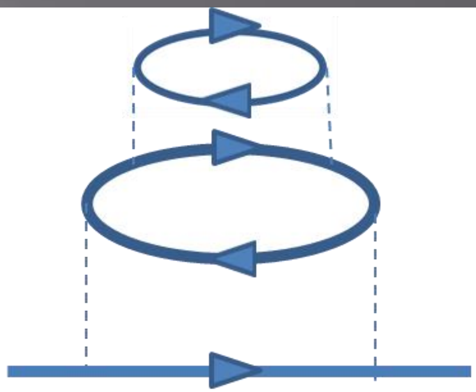
d=3

Tree structure of Fock/Hilbert space of interacting systems

Tree-like structure of many-body interaction

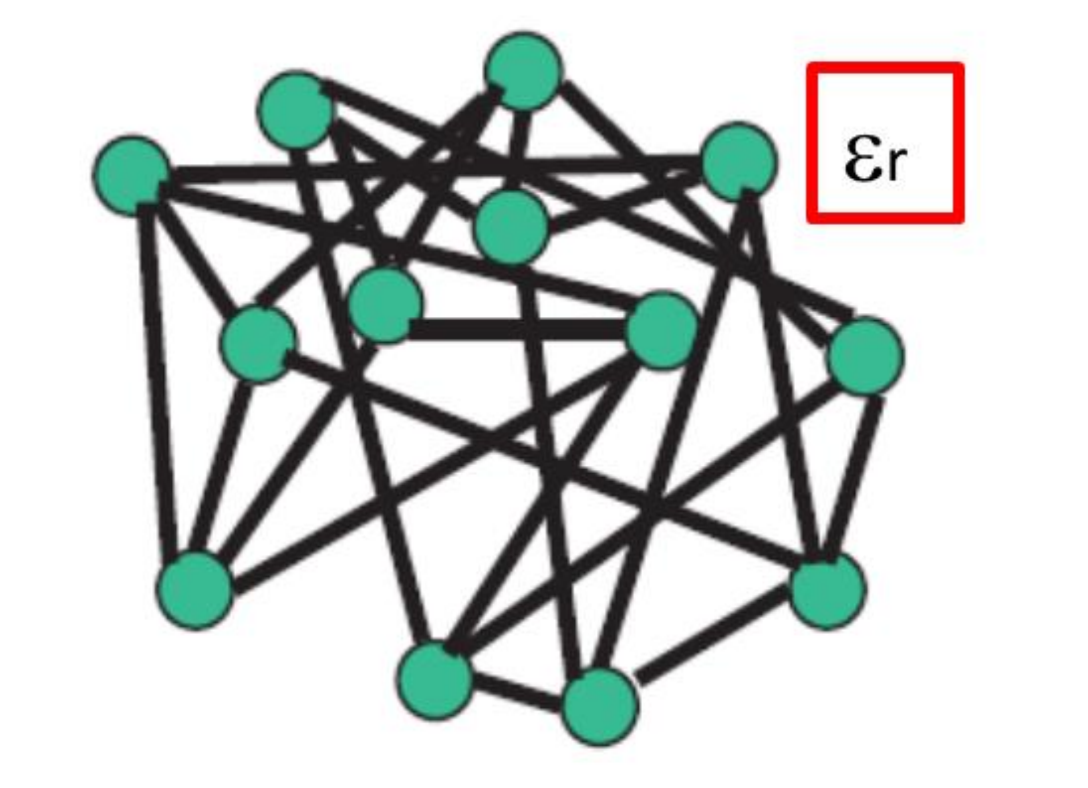


Altshuler, Gefen, Kamenev, Levitov , 1997



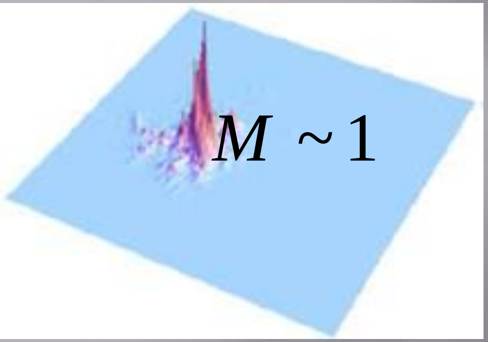
Basko, Aleiner, Altshuler, 2005

…but no boundary



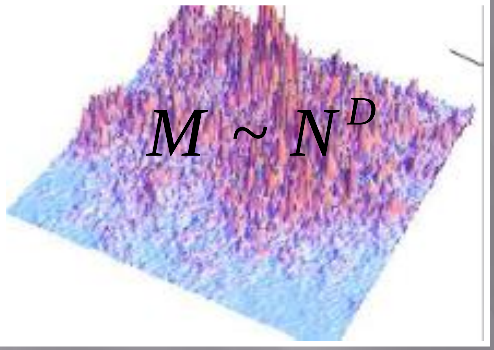
Random regular graph (RRG)

LOCALIZED, EXTENDED ERGODIC AND EXTENDED NON-ERGODIC PHASES



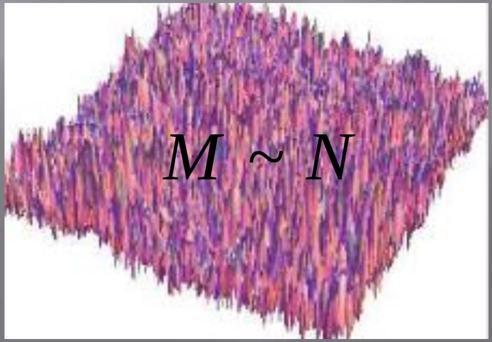
Finite number of occupied sites in thermodynamic limit

D=0



Infinite number of occupied sites but zero fraction of all sites in the thermodynamic limit

0<D<1



Finite fraction of occupied sites

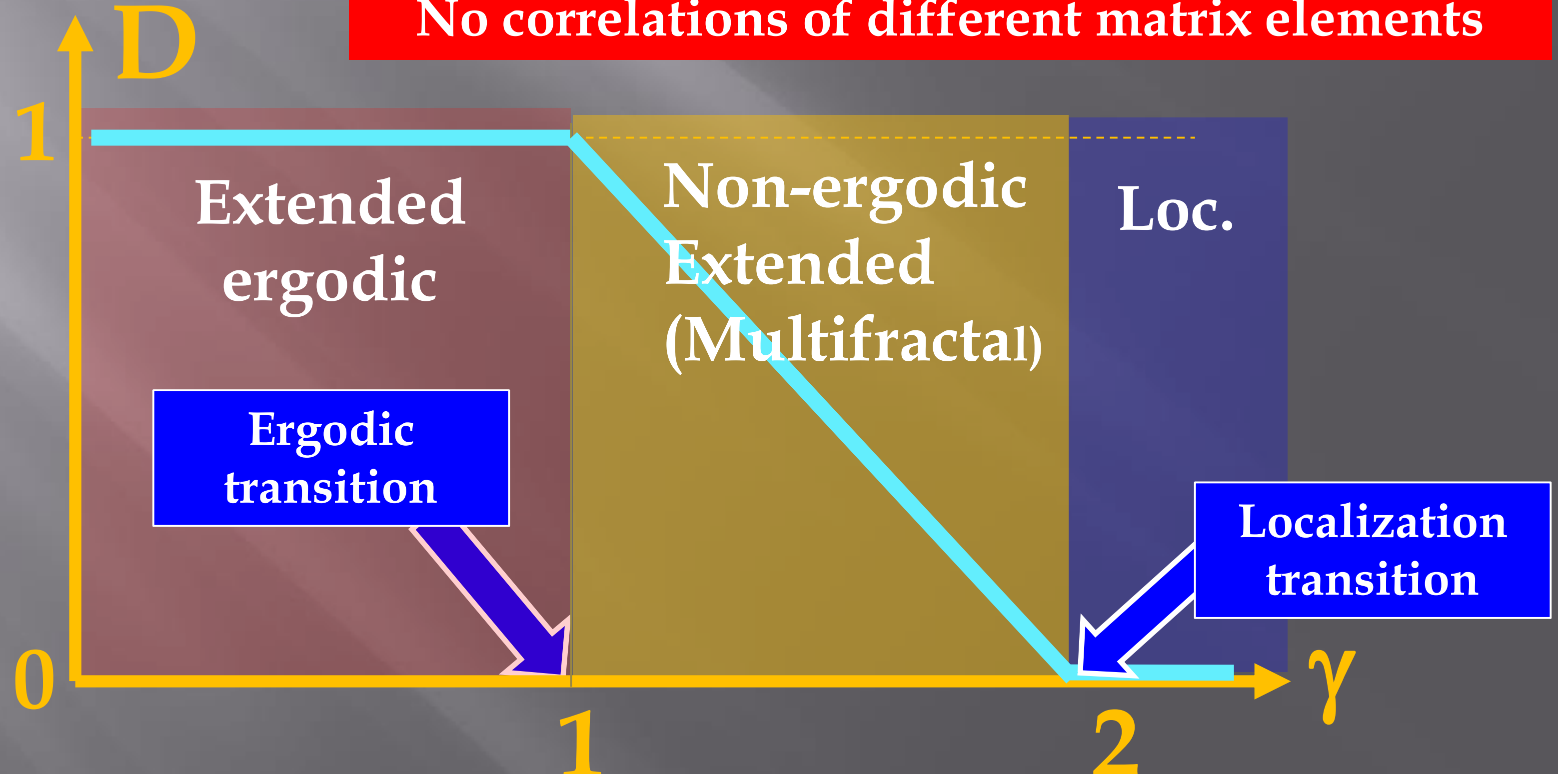
D=1

Non-ergodic extended phase and ergodic transition in RP RMT

V.E.K., I.M. Khaymvich,
E. Cuevas, M. Amini,
New J. Phys., v.17, 12202
(2015)

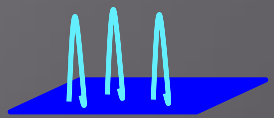
$$\langle | H_{nm} |^2 \rangle = \begin{cases} 1, & \text{if } n = m \\ \dfrac{\lambda^2}{N^{\gamma}}, & \text{if } n \neq m \end{cases}$$

No correlations of different matrix elements



Number of sites in resonance

$$P_{res} = N^{-\gamma}$$



$$\#\text{resonances} = \#\text{pairs} \times \text{probability} = N^{2-\gamma}$$

$$N^{2} \qquad N^{-\gamma}$$

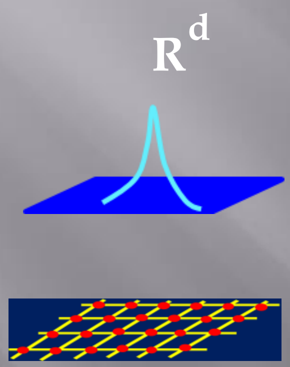
Competition of power laws

$$D = 2 - \gamma$$

The case of RRG

$$P_{res}(r) = \exp[-\lambda r]$$

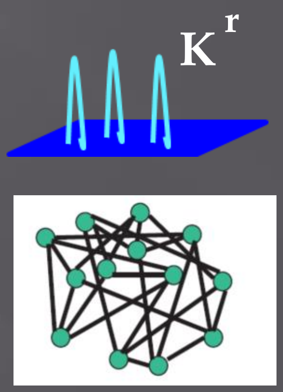
$$\#\ \text{pairs at a distance } r: \quad K^r = \exp[\ln K\ r]$$



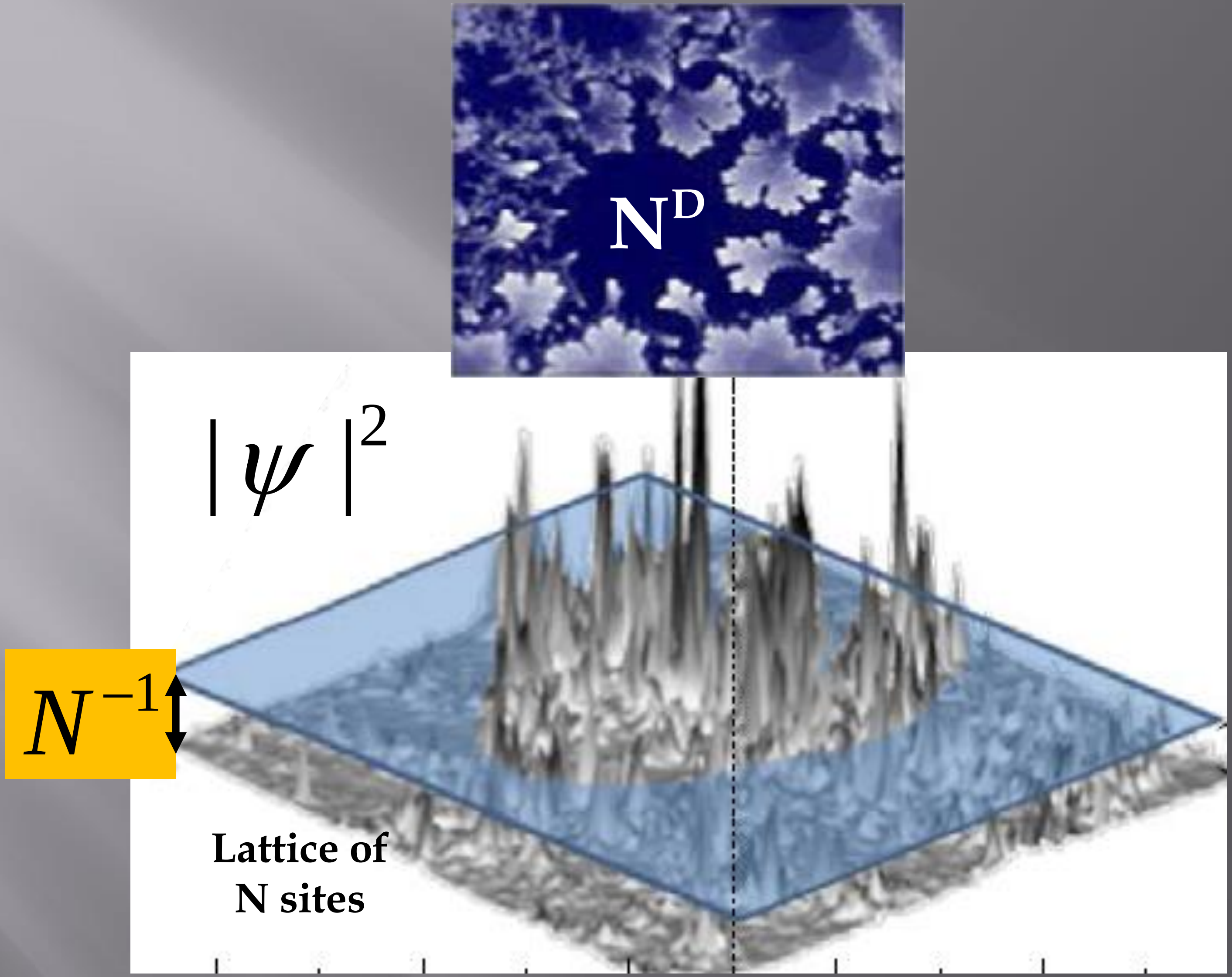
Competition of exponentials

$$\#\ \text{res} = \exp[(\ln K - \lambda(W))d] = N^{(1-\lambda/\ln K)}$$

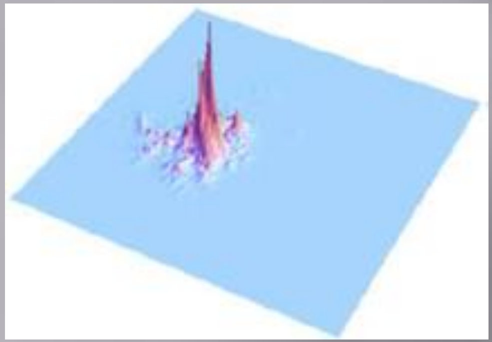
d=lnN/lnK is the graph diameter

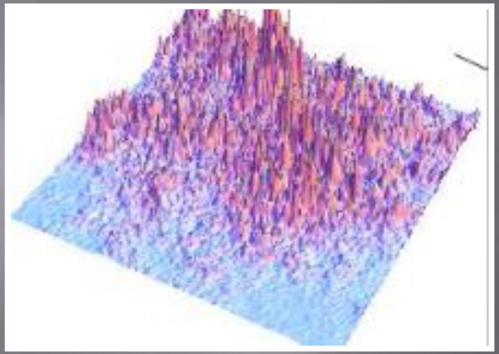


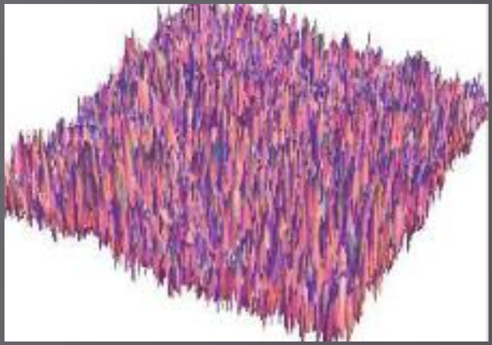
$$D = 1 - \lambda(W)/\ln K$$


N^D
$|\psi|^2$
N^{-1}
Lattice of
N sites

Fractal dimension D and Shannon entropy







Shennon entropy

$$-\left\langle \sum_i |\Psi_i|^2 \ln |\Psi_i|^2 \right\rangle = \ln N * \begin{cases} \mathrm{D} = 0, \text{ localized} \\ 0 < \mathrm{D} < 1, \text{ fractal} \\ \mathrm{D} = 1, \text{ ergodic extended} \end{cases}$$

N is size of the system

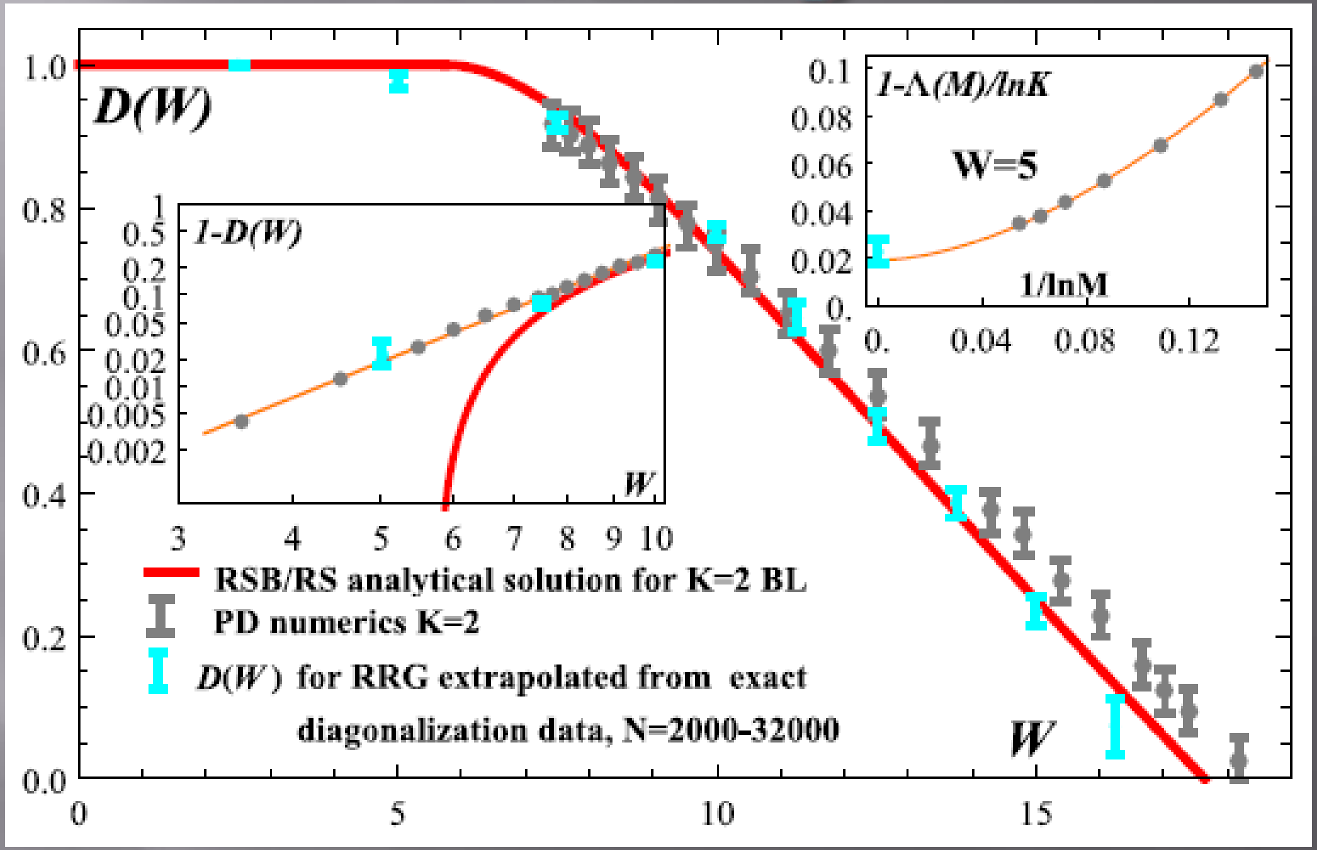
D is the Hausdorf dimension of eigenfunction support set

Fractal dimension D(q) and Renyi entropy

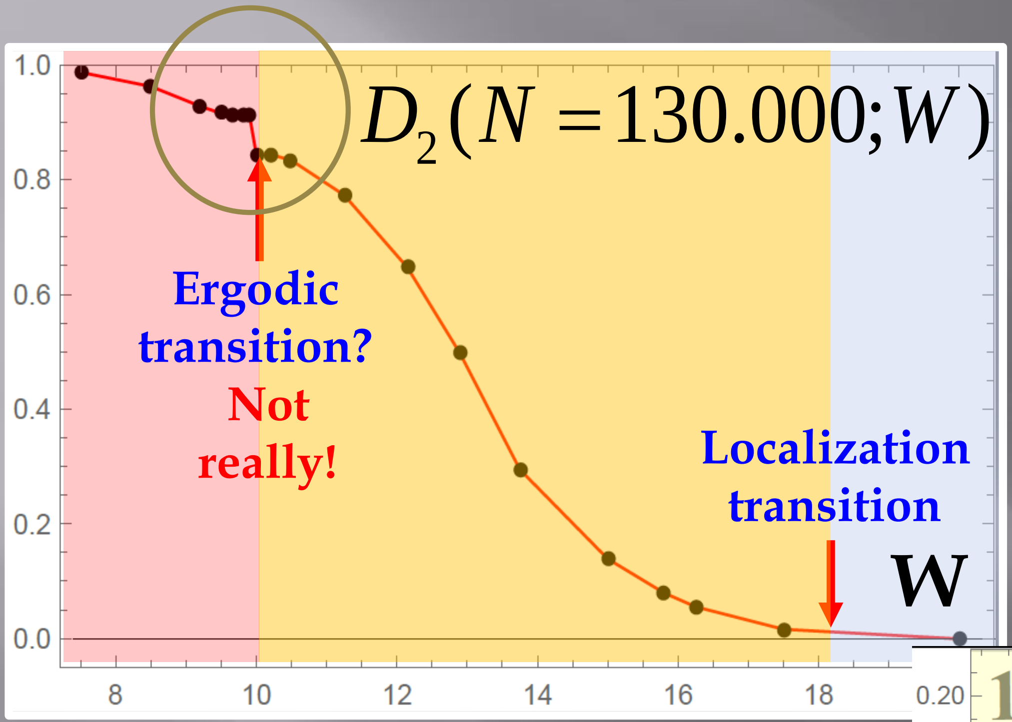
$$-\ln\left\langle \sum_i |\Psi_i|^{2q} \right\rangle = (q-1)D_q \ln N,$$

$$D_q = \begin{cases} 0, \text{ localized} \\ 0 < \mathrm{D}_q < 1, \text{ fractal} \\ 1, \text{ ergodic extended} \end{cases}$$

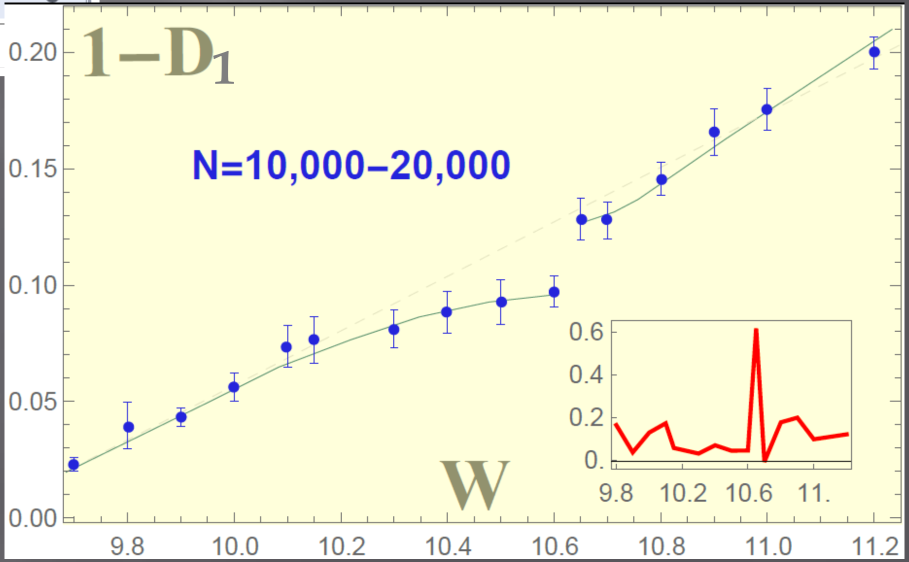
One-step replica symmetry breaking



Exact diagonalization on RRG

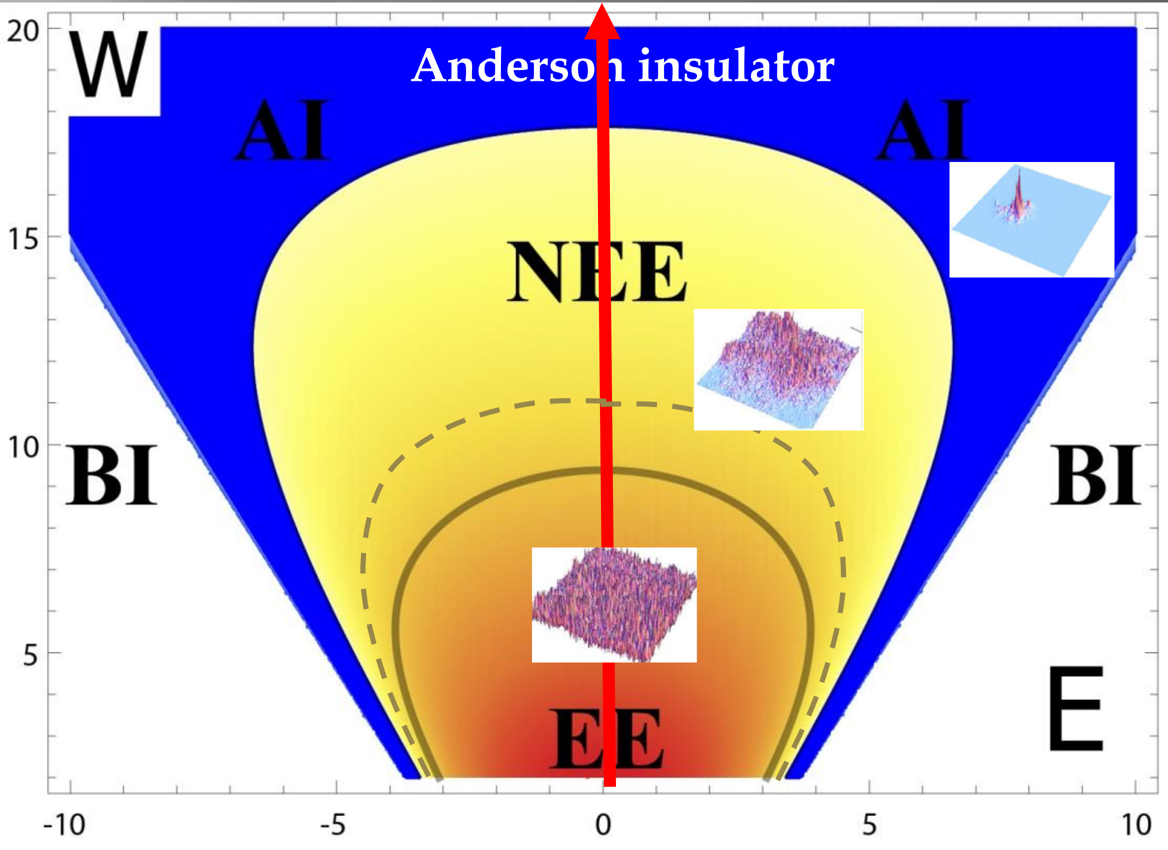


Something happens at W~10. Is that ergodic transition? Or a transition to another NEE phase?



Many open questions

Phase diagram on RRG



Why important?

- No Boltzmann statistics in MBL phase:

No equipartition of energy over degrees of freedom. No thermalization.

- The same is valid for NEE phase
- NEE =quantum glass?
- Slow (sub-diffusive) dynamics in NEE phase
- Bad metal
- No FDT: strongly enhanced noise