Electric activity of different magnetic textures

D. I. Khomskii

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- **Polarization in frustrated Mott insulators**
- **Monopoles and dipoles in spin ice**
- **Dipoles at spin fragmentation**
- **Dipoles on spin spirals (both ways)**
- **Dipoles on spin waves**
- **Magnetic monopoles in magnetoelectrics**
- **Electric activity of skyrmions**

Electronic Orbital Currents and Polarization in Mott Insulators

L.N. Bulaevskii, C.D. Batista, M. Mostovoy and D. Khomskii

PRB **78**, 024402 (2008)

D. Khomskii J.Phys.-Cond. Mat. **22**, 164209 (2010)

Mott insulators

$$H = -\sum_{ij\sigma} t_{ij}(c_{i\sigma}^{+}c_{j\sigma} + c_{j\sigma}^{+}c_{i\sigma}) + \frac{U}{2}\sum_{i}(n_i - 1)^2,$$

Standard paradigm: for U>>t and one electron per site electrons are localized on sites. All charge degrees of freedom are frozen out; only spin degrees of freedom remain in the ground and lowest excited states

$$H_S = \frac{4t^2}{U}(S_1 \cdot S_2 - 1/4).$$

Not the full truth!

For certain spin configurations there exist in the ground state of strong Mott insulators **spontaneous electric currents** (and corresponding orbital moments)!

For some other spin textures there may exist a **spontaneous charge redistribution**, so that $\langle n_i \rangle$ is not 1! This, in particular, can lead to the appearance of a spontaneous **electric polarization** (a purely ***electronic mechanism of multiferroic behaviour***)

These phenomena, in particular, appear in frustrated systems, with **scalar chirality** playing important role

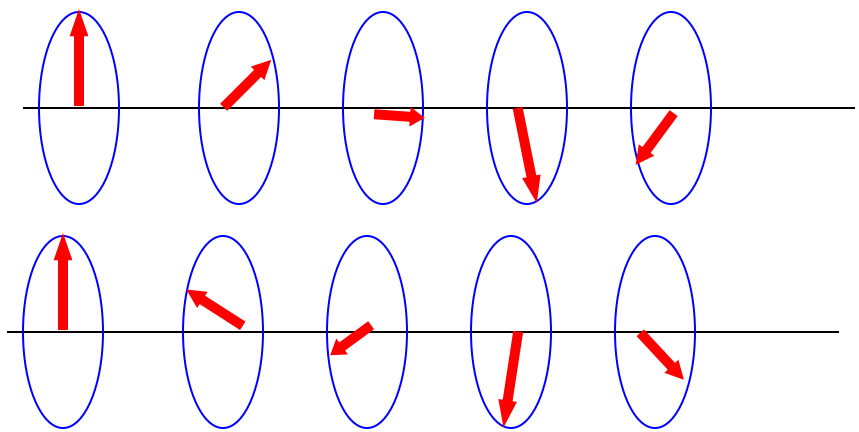
Often complicated ground states; sometimes $\langle \vec{S}_i \rangle = 0$ ⟶

⟶ spin liquids

Some structures, besides $\langle \vec{S}_i \rangle$, are characterized by:

Vector chirality

$$[\vec{S}_i \times \vec{S}_j]$$

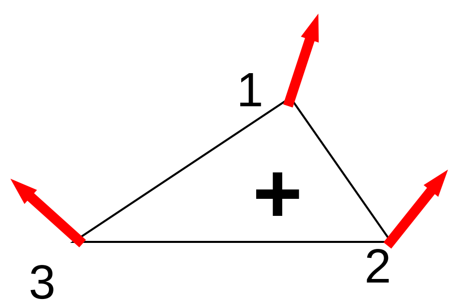


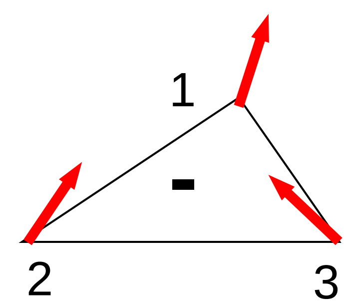
Scalar chirality

$$\chi_{123} = \vec{S}_1 [\vec{S}_2 \times \vec{S}_3]$$

- solid angle

χ may be + or - :





But what is the scalar chirality physically?

What does it couple to?

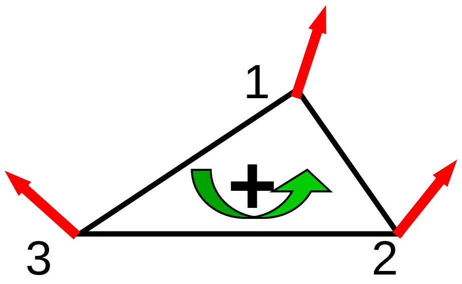
How to measure it?

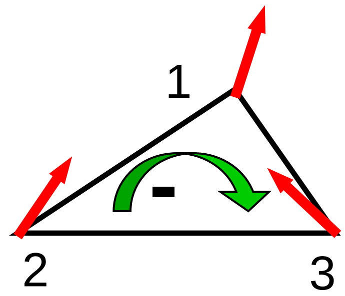
Breaks time-reversal-invariance T and inversion P - like currents!

$\chi_{123} \neq 0$ means spontaneous circular electric current

$j_{123} \neq 0$ and orbital moment $L_{123} \neq 0$

$$\boldsymbol{L_{123} \propto j_{123} \propto \chi_{123}}$$





Couples to magnetic field:

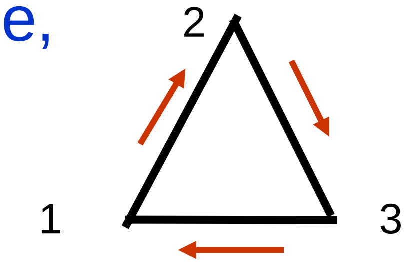
$$-\vec{L}\vec{H} \sim -\chi H$$

Spin current operator and scalar spin chirality

- Current operator for Hubbard Hamiltonian on bond ij:

$$I_{ij} = \frac{iet_{ij}r_{ij}}{\hbar r_{ij}} \sum_{\sigma} (c_{i\sigma}^{+} c_{j\sigma} - c_{j\sigma}^{+} c_{i\sigma}).$$

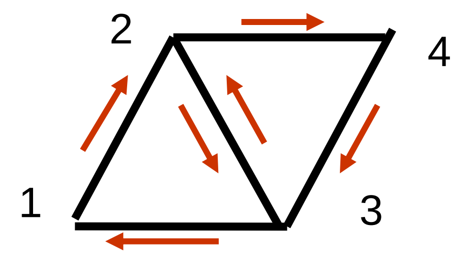
- Projected current operator: odd # of spin operators, scalar in spin space. For smallest loop, triangle,



$$I_{S,12}(3) = \frac{r_{ij}}{r_{ij}} \frac{24et_{12}t_{23}t_{31}}{\hbar U^{2}} [S_1 \times S_2] \cdot S_3.$$

- Current via bond 23

$$I_{S,23} = I_{S,23}(1) + I_{S,23}(4).$$



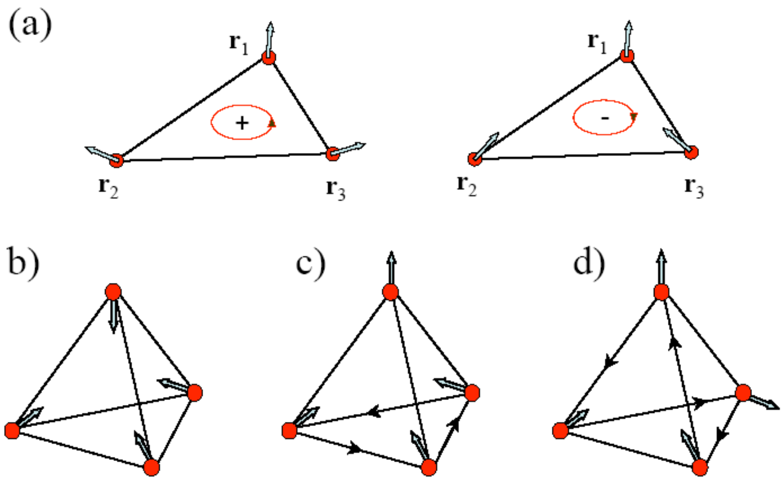
- On bipartite nn lattice I_S is absent.

Orbital currents in the spin ordered ground state $\langle \vec{S}_i \rangle \neq 0$

- Necessary condition for orbital currents is nonzero average chirality

$$\chi_{12,3} = [S_1 \times S_2] \cdot S_3, \qquad \langle \chi_{ij,k} \rangle \neq 0.$$

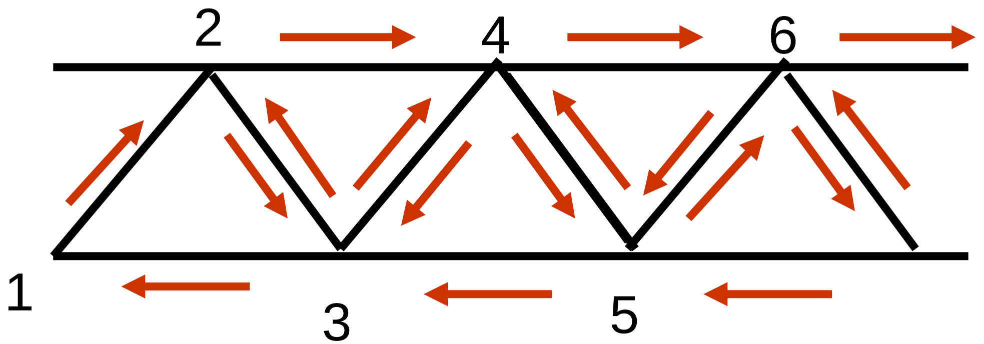
- It may be inherent to spin ordering or induced by magnetic field

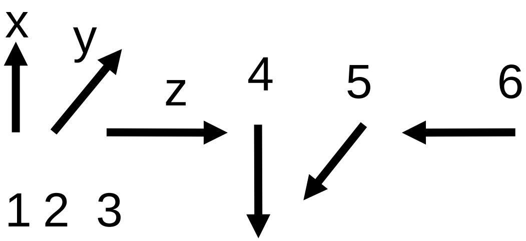


Triangles with $\pm$ chirality

Tetrahedra with [111] anisotropy

Boundary and persistent current





$$\chi = const$$

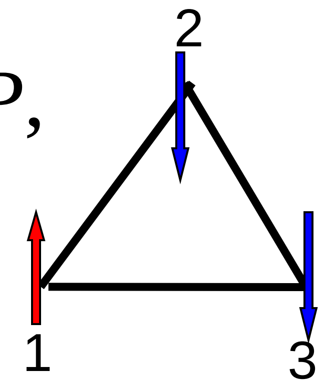
Boundary current in
gaped 2d insulator

Spin-dependent electronic polarization

Charge operator on site i: $$Q_i = e\sum_{\sigma} c^{+}_{i\sigma} c_{i\sigma}.$$

Projected charge operator $$n_{S,i} = P e^{S} n_i e^{-S} P,$$

$$\langle n_1 \rangle = 1 + \delta n_1 = 1 - 8\left(\frac{t}{U}\right)^3 \left[\mathbf{S}_1(\mathbf{S}_2 + \mathbf{S}_3) - 2\mathbf{S}_2\mathbf{S}_3\right]$$

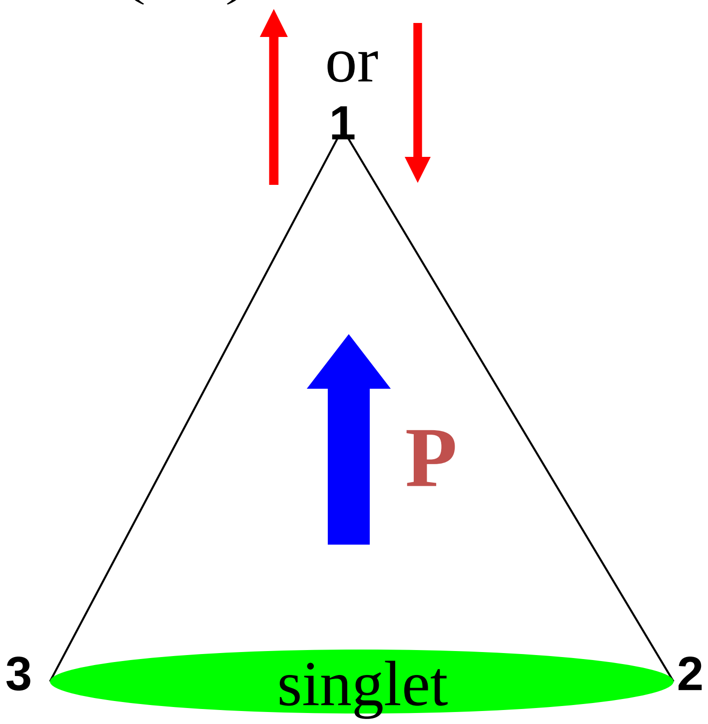


Polarization on triangle $$\vec{P}_{123} = e\sum_{i=1,2,3} n_{S,i}\vec{r}_i, \qquad \sum_i n_{S,i} = 3.$$

Charge on site *i* is sum over triangles at site *i*.

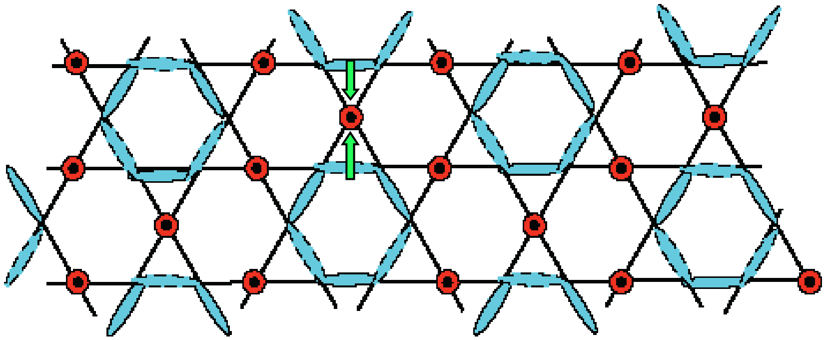
Electronic polarization on triangle

$$\langle n_1 \rangle = 1 + \delta n_1 = 1 - 8\left(\frac{t}{U}\right)^3 \left[\mathbf{S}_1(\mathbf{S}_2 + \mathbf{S}_3) - 2\mathbf{S}_2\mathbf{S}_3\right]$$



Purely electronic mechanism of multiferroic behavior!

Charges on kagome lattice

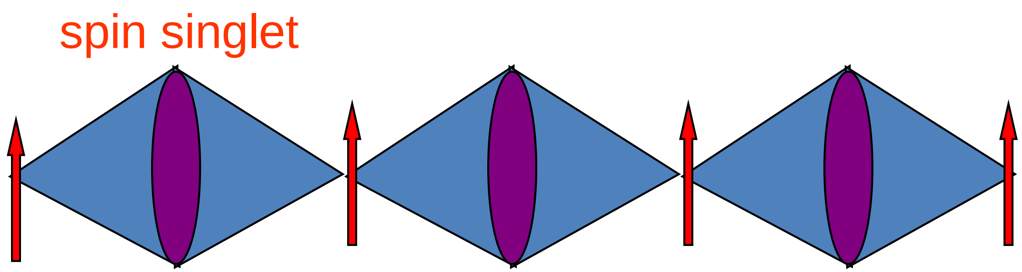


1/3 magnetization plateau:

Charge ordering for spins 1/3 in magnetic field: spin-driven CDW

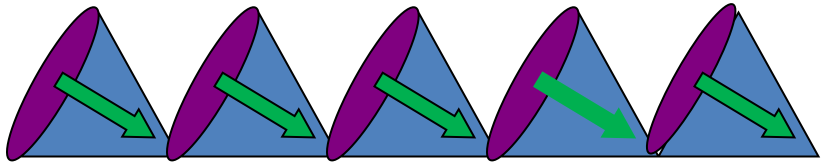
- Typical situation at the magnetization plateaux!

- Diamond chain (azurite $Cu_3(CO_3)_2(OH)_2$)

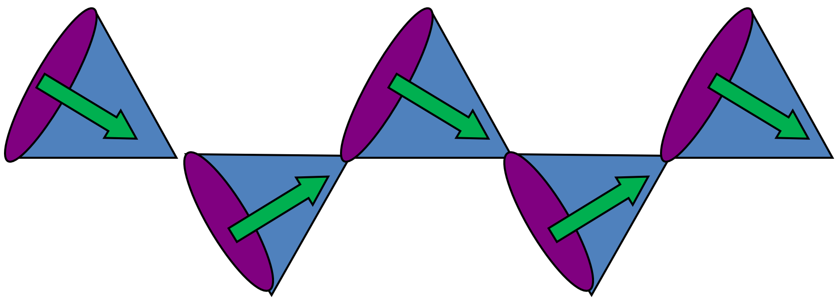


-will develop S-CDW

- Saw-tooth (or delta-) chain



Net polarization



Net polarization

$Cu_2(AsO_4)(OH)\cdot 3H_2O$ (euchroite)

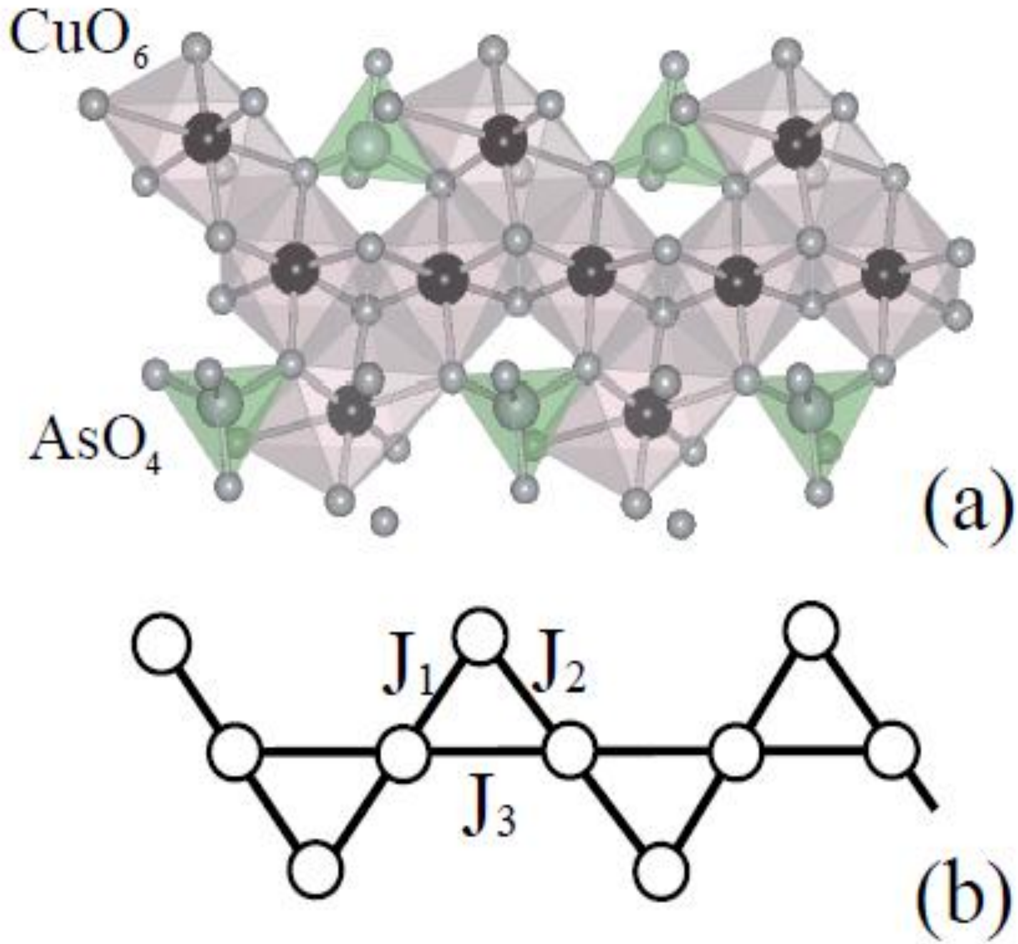
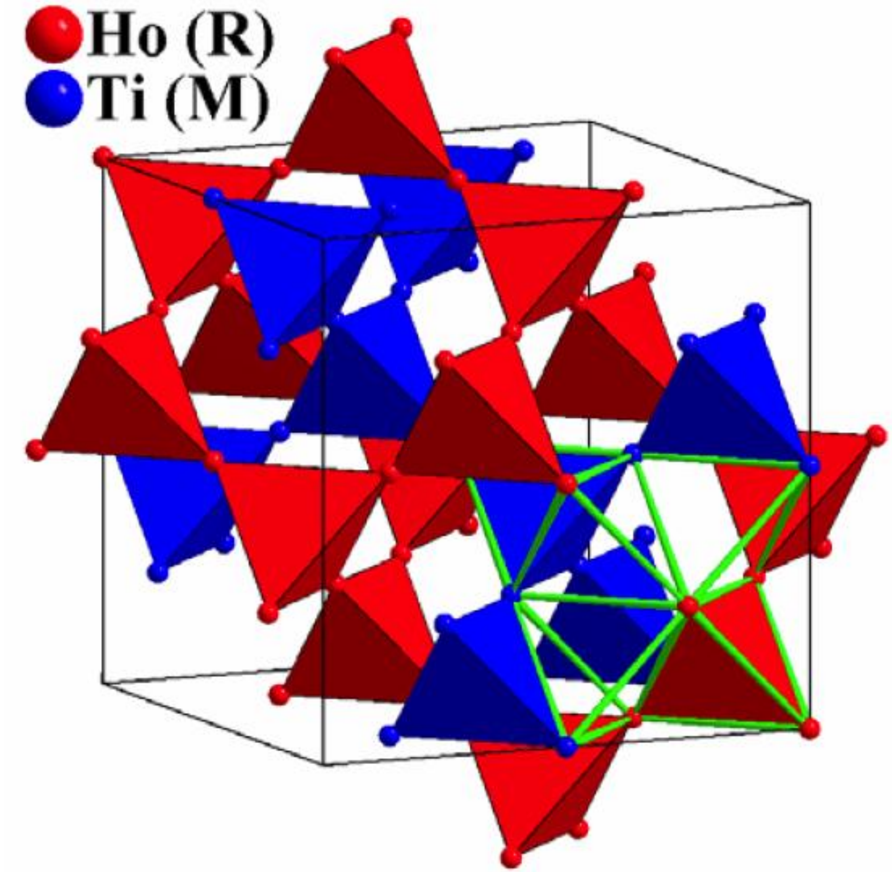


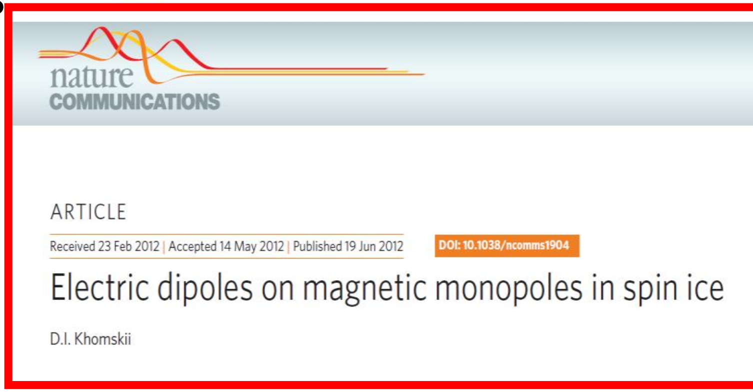
Figure 1. (a) Structure of euchroite, (b) schematic view of the delta chain.

Monopoles and dipoles in spin ice

Pyrochlore: Two interpenetrating metal sublattices



$Ho_2Ti_2O_7$


nature COMMUNICATIONS

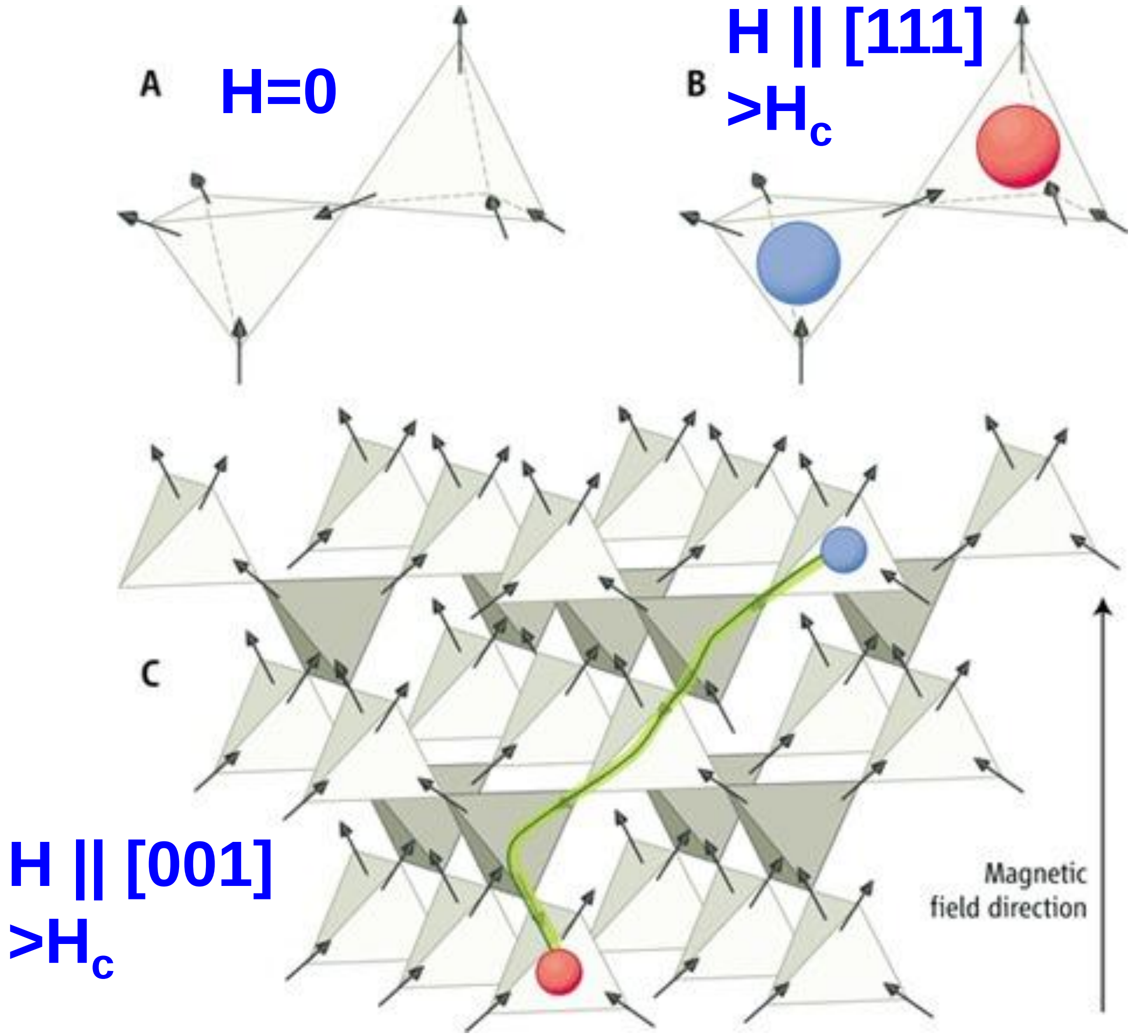
ARTICLE

Received 23 Feb 2012 | Accepted 14 May 2012 | Published 19 Jun 2012 DOI: 10.1038/ncomms1904

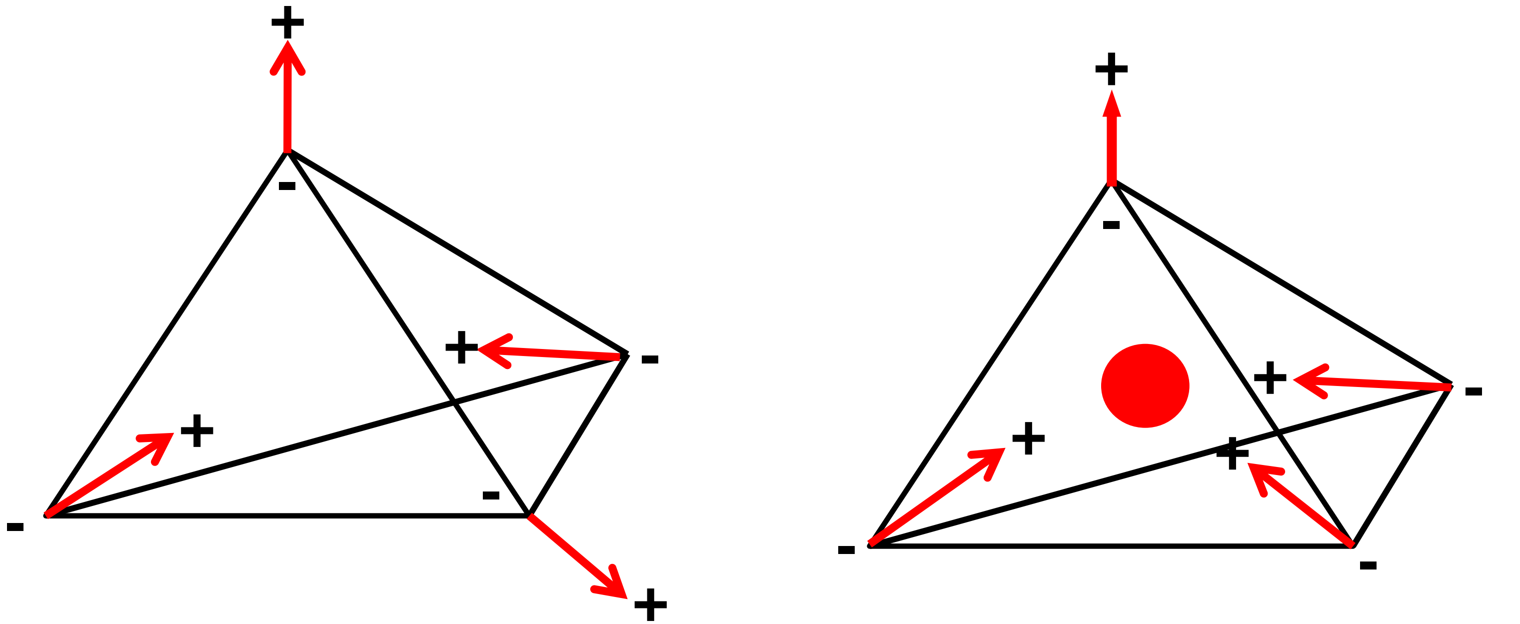
Electric dipoles on magnetic monopoles in spin ice

D.I. Khomskii

Excitations creating magnetic monopole (Castelnovo, Moessner and Sondhi)



M J P Gingras Science 2009;326:375-376

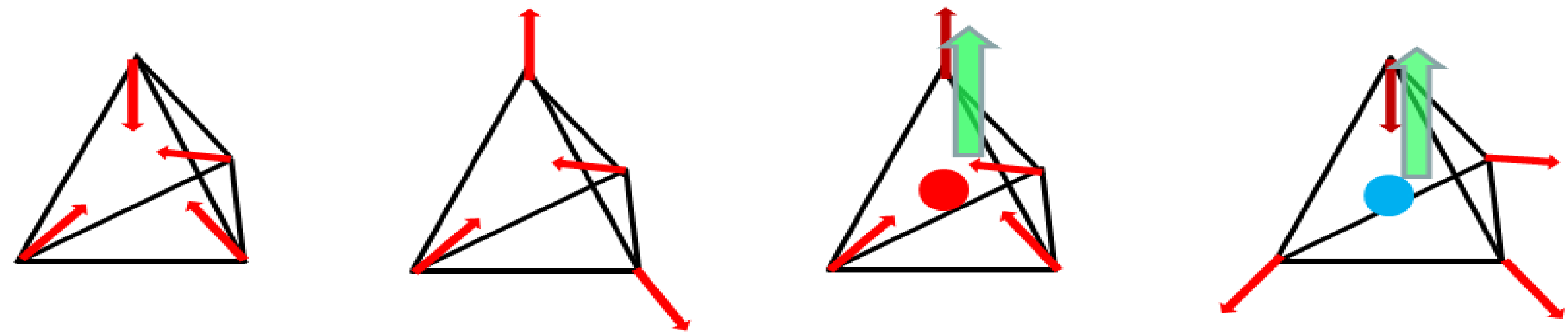


2-in/2-out: net magnetic charge inside tetrahedron zero

3-in/1-out: net magnetic charge inside tetrahedron ≠ 0 – monopole or antimonopole

★ Dipoles on tetrahedra:

D.I.Khomskii
Nature Comm. **3**, 904 (2012)



4-in or 4-out: **d=0**

2-in/2-out (spin ice): **d=0**

3-in/1-out or 1-in/3-out (monopoles/antimonopoles): **d** $\neq 0$

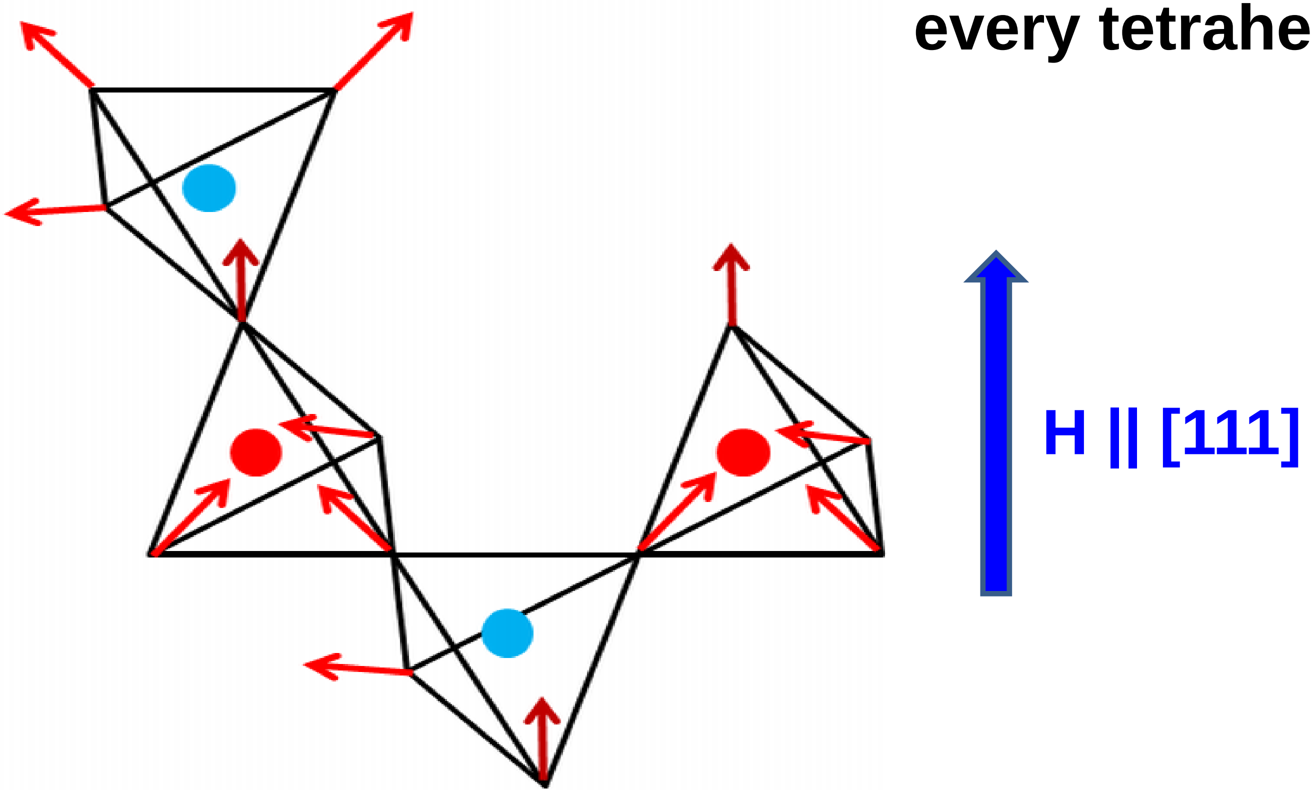
$$\langle n_1 \rangle = 1 + \delta n_1 = 1 - 8\left(\frac{t}{U}\right)^3 \left[\mathbf{S}_1(\mathbf{S}_2 + \mathbf{S}_3) - 2\mathbf{S}_2\mathbf{S}_3\right]$$

For 4-in state: from the condition $\mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4 = 0$ $\delta n_1 = 0$. Change of $\mathbf{S}_1 \rightarrow -\mathbf{S}_1$ (3-in/1-out, *monopole*) gives nonzero charge redistribution and **d** $\neq 0$.

Charge redistribution and dipoles are *even* functions of S; inversion of all spins does not change direction of a dipole: ⇒ Direction of dipoles on monopoles and antimonopoles is *the same*: e.g. ***from the center of tetrahedron to a "special" spin***

H || [111], >H_c

Monopoles/antimonopoles at every tetraheder, staggered



H Aoki et al., JPSJ 73, 2851 (2004)

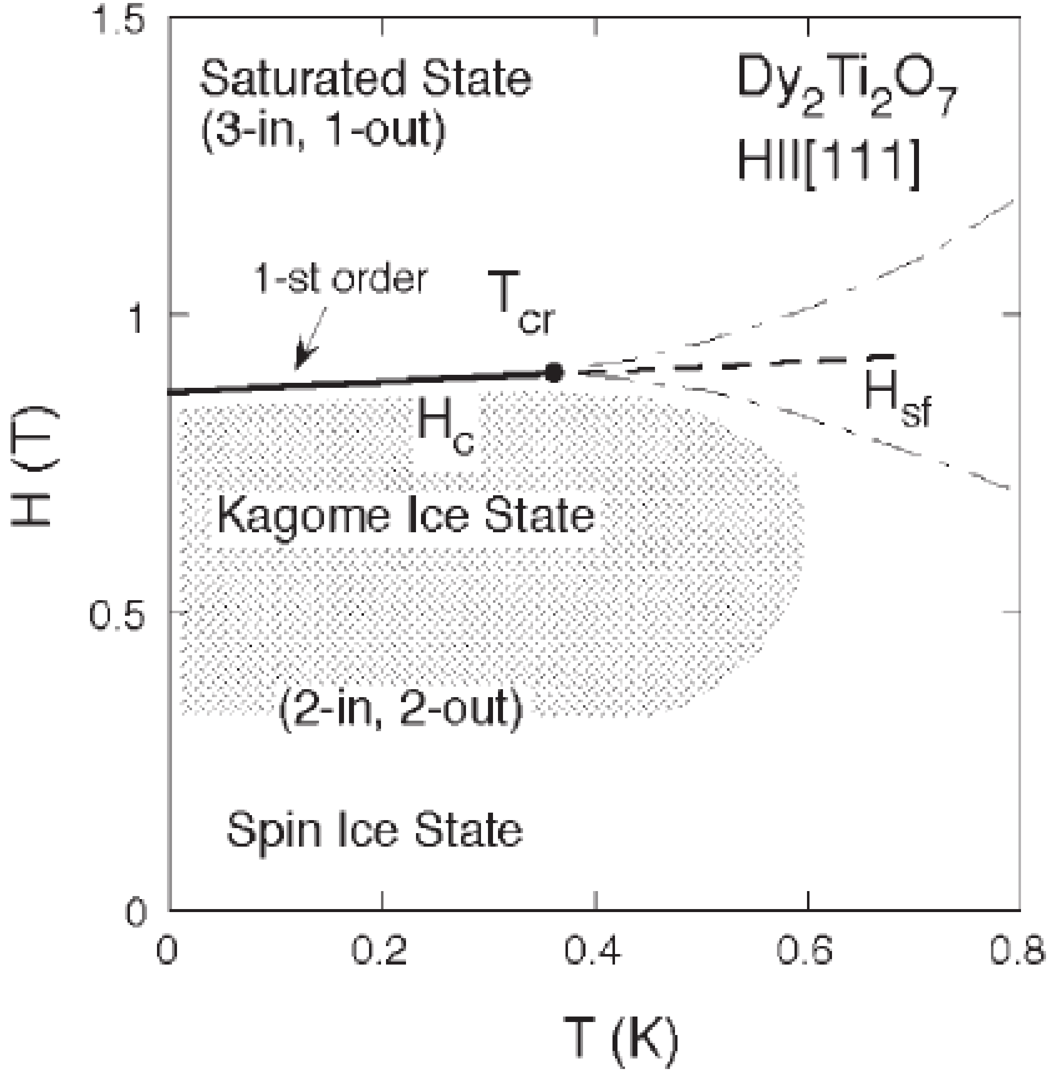


Fig. 1. Phase diagram of $Dy_2Ti_2O_7$ in a [111] magnetic field, determined by magnetization and specific heat measurements. The dashed line

Electric dipoles on magnetic monopoles

H ‖ [111]

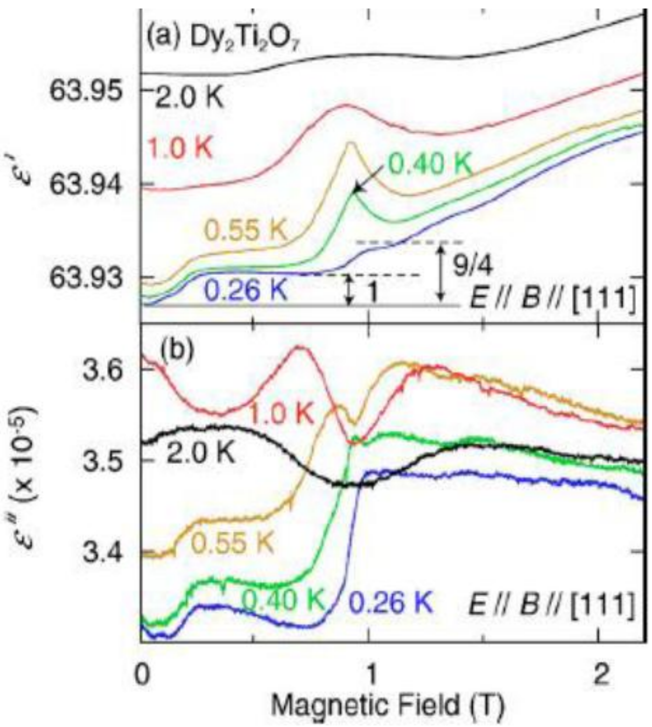
D.I. Khomskii, *Nature Comm.*, **3**, 1 (2012)

Experimental evidence:

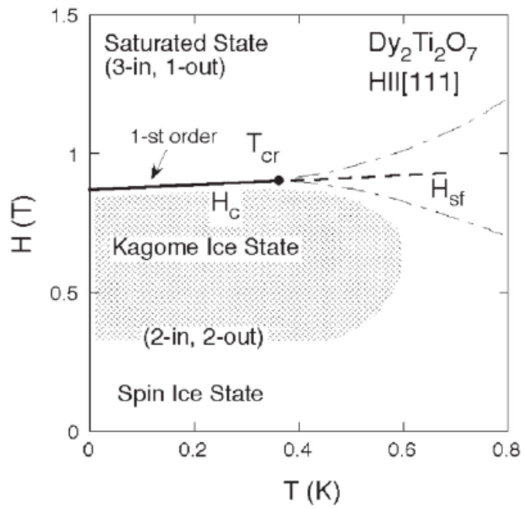
PHYSICAL REVIEW B 72, 144422 (2005)

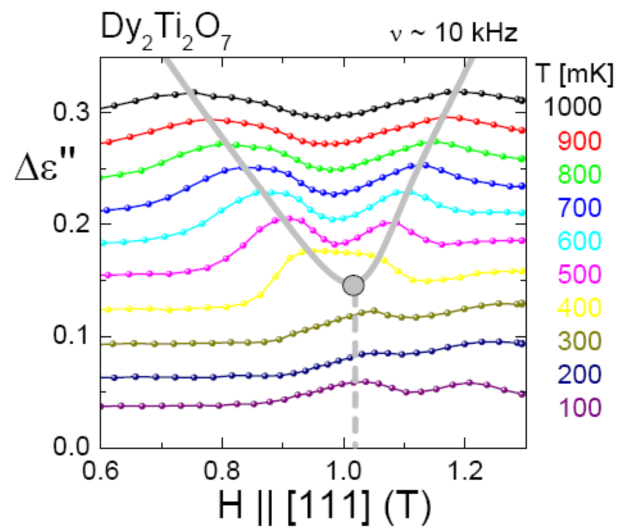
Magnetodielectric response of the spin-ice $Dy_2Ti_2O_7$

Masafumi Saito,[1] Ryuji Higashinaka,[1] and Yoshiteru Maeno[1,2,*]



[Aoki et al., JPSJ (2004)]





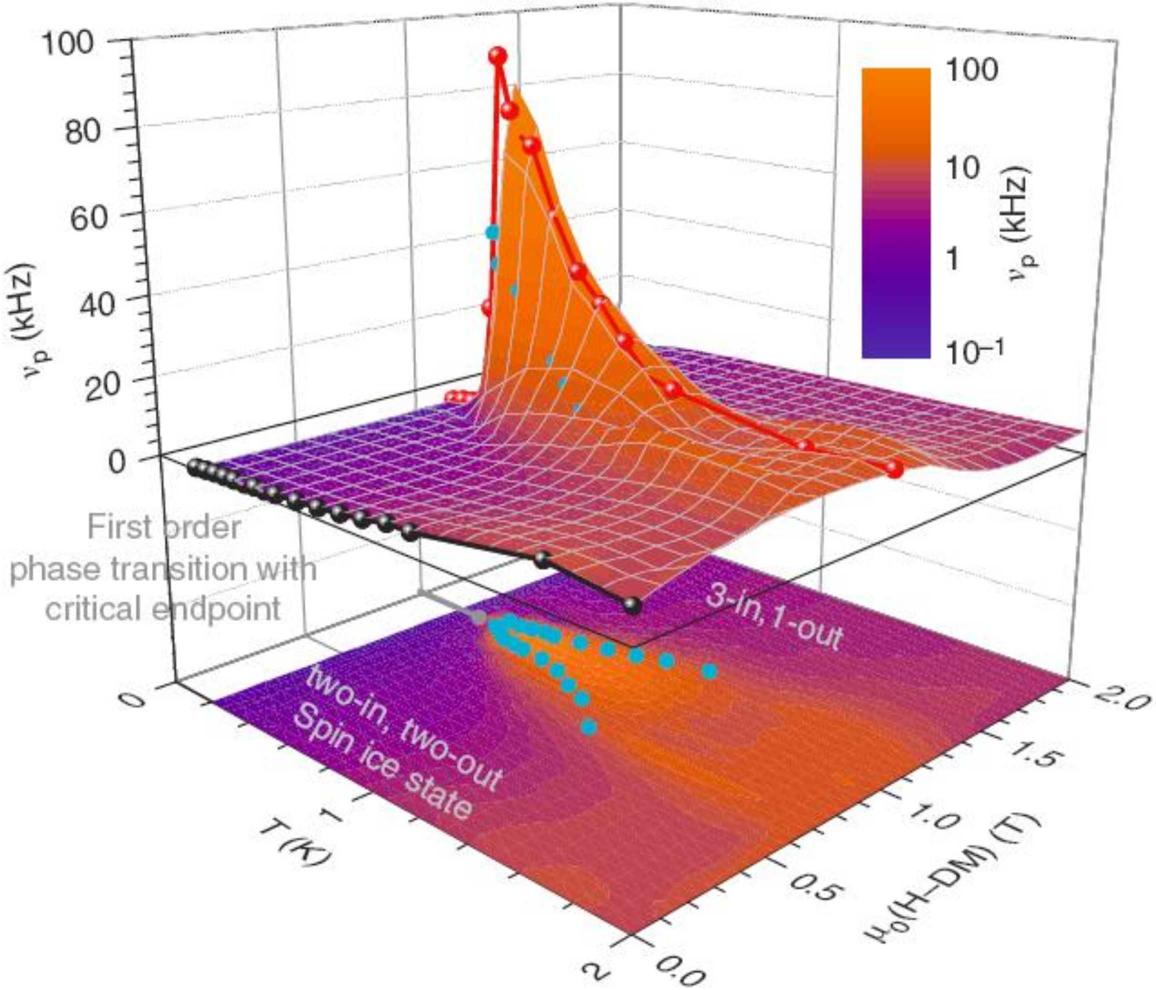
-> Anomalies at and above transition into monopole order

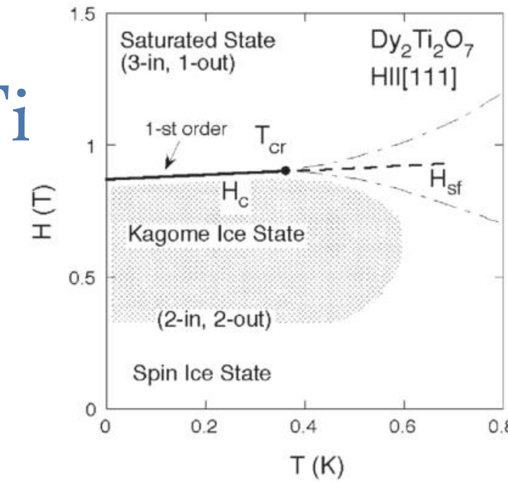
But: frequency dependent !

(large offset in ε' due to titanium sublattice)

-> dynamics ...

Monopole dynamics in Dy_2Ti





-> $\nu_p = 1/2\pi\tau$
recaptures features of
"monopole condensation"
-> slowing down outside
crossover lines

-> Critical **speeding up** above critical end-point !!!

[C.P.Grams, M. Valldor, M. Garst, J.H., Nature Commun. 5:4853 (2014)]

Spin fragmentation

PHYSICAL REVIEW X 4, 011007 (2014)

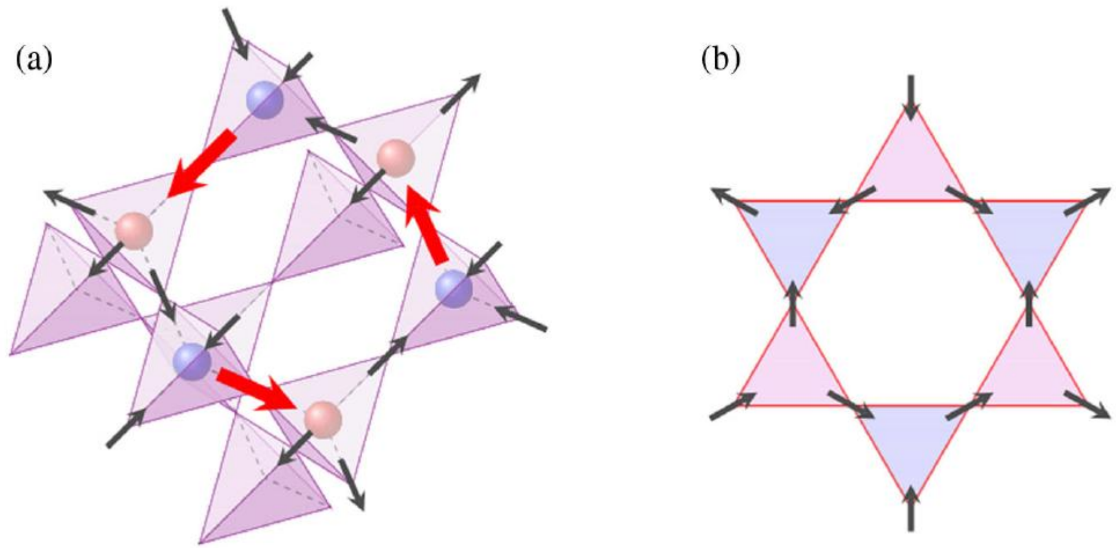
Magnetic-Moment Fragmentation and Monopole Crystallization

M. E. Brooks-Bartlett,[1] S. T. Banks,[1] L. D. C. Jaubert,[2] A. Harman-Clarke,[1,3] and P. C. W. Holdsworth[3,*]

not, however, define the entire moment density, which can have two contributions through a Helmholtz decomposition. The first $\vec{M}_m$ is the gradient of a scalar potential and provides the magnetic charge; the second $\vec{M}_d$ is a dipolar field, can be represented as the curl of a vector potential, and is divergence free [21,22]:

$$\vec{M} = \vec{M}_m + \vec{M}_d = \vec{\nabla}\psi(r) + \vec{\nabla} \wedge \vec{Q}. \qquad (1)$$

The main point: one can have monopole crystal, with LRO, but simultaneously spins themselves disordered



Manifestation in neutron scattering: coexistence of diffuse background (due to disordered spins) and LRO signal of monopole crystal (pinch points)

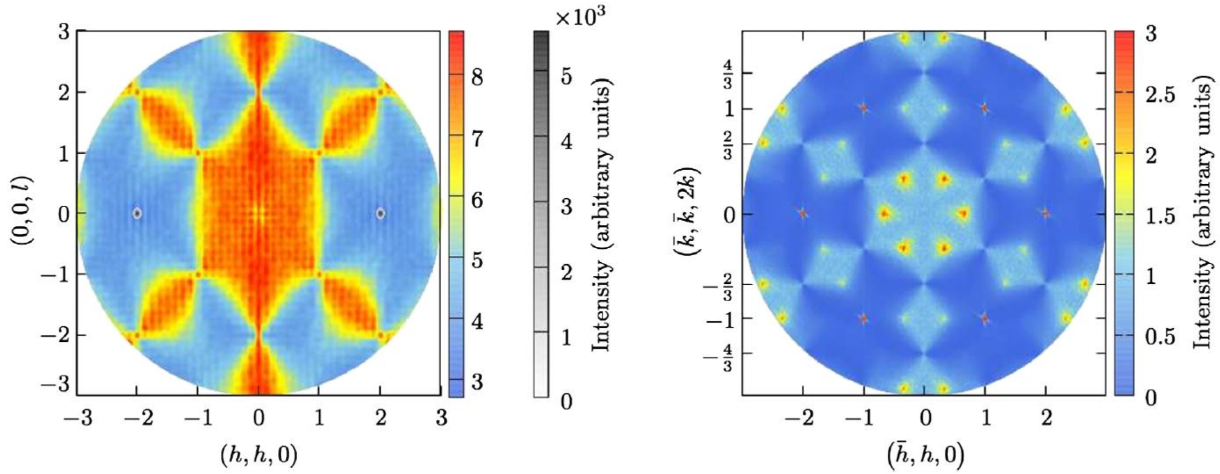


FIG. 4. Simulated unpolarized neutron-scattering structure factors $S(Q)$ for the pyrochlore monopole crystal (left) and for in-plane scattering from kagome ice (right). The pyrochlore $S(Q)$ has been calculated by averaging over 2000 distinct monopole-crystal ground

ARTICLE

Received 13 Jun 2015 | Accepted 23 Mar 2016 | Published 13 May 2016 DOI: 10.1038/ncomms11446 OPEN

Fragmentation of magnetism in artificial kagome dipolar spin ice

Benjamin Canals[1,2], Ioan-Augustin Chioar[1,2], Van-Dai Nguyen[1,2], Michel Hehn[3], Daniel Lacour[3], François Montaigne[3], Andrea Locatelli[4], Tevfik Onur Menteş[4], Benito Santos Burgos[4] & Nicolas Rougemaille[1,2]

LETTERS

PUBLISHED ONLINE: 4 APRIL 2016 | DOI: 10.1038/NPHYS3710

nature physics

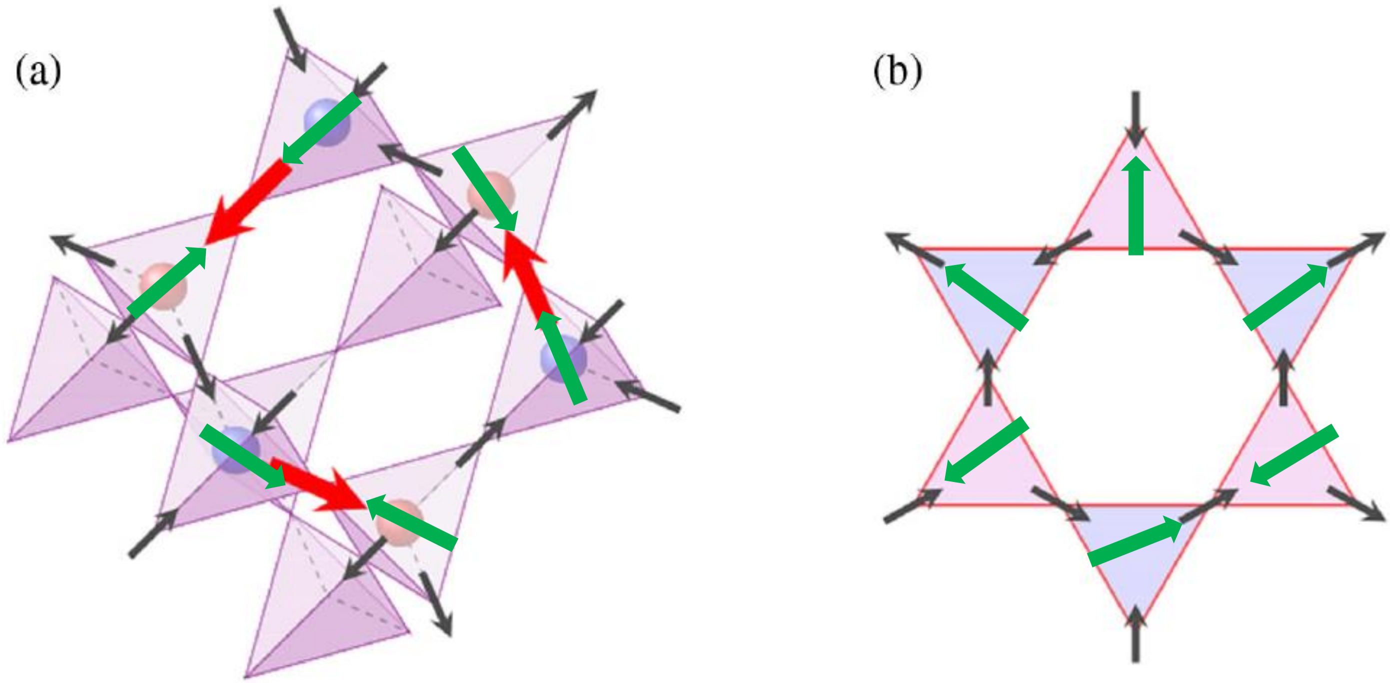
Observation of magnetic fragmentation in spin ice

S. Petit[1]*, E. Lhotel[2]*, B. Canals[2], M. Ciomaga Hatnean[3], J. Ollivier[4], H. Mutka[4], E. Ressouche[5], A. R. Wildes[4], M. R. Lees[3] and G. Balakrishnan[3]

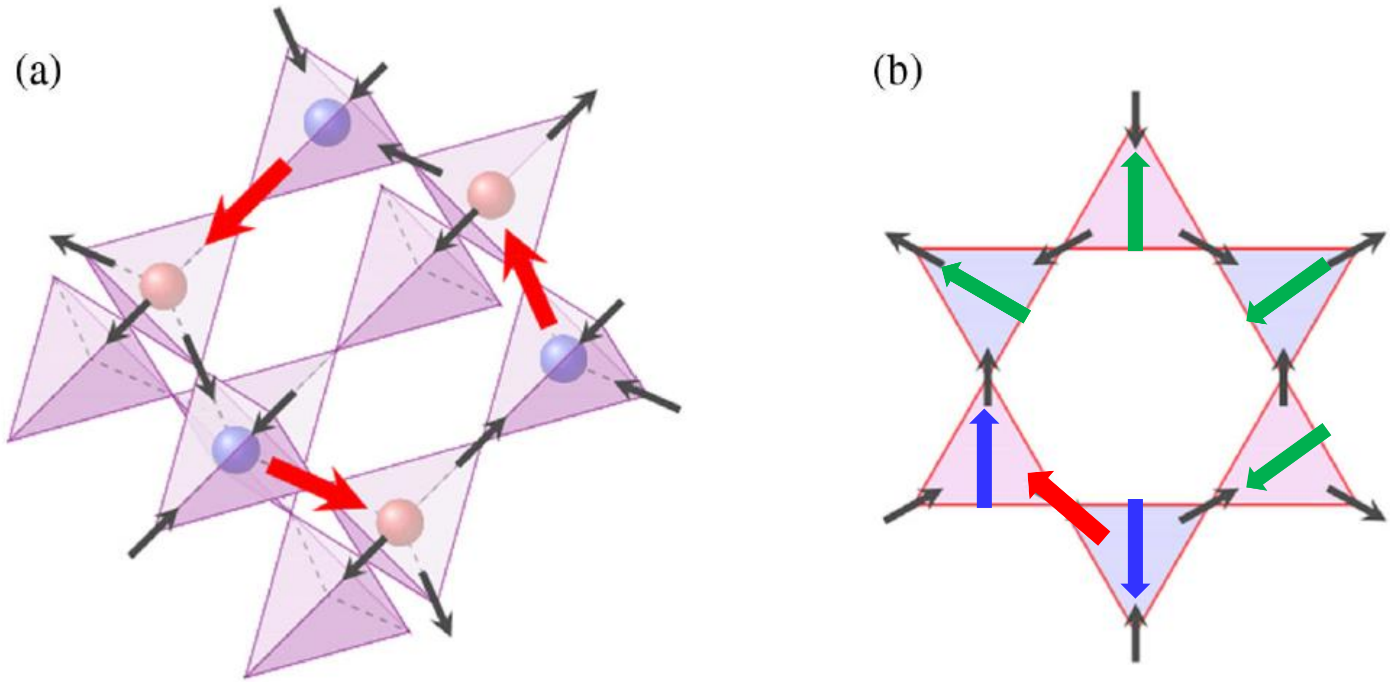
Magnetic charge injection in spin ice: a new way to fragmentation

E. Lefrançois,[1,2,*] V. Cathelin,[2] E. Lhotel,[2] J. Robert,[2] P. Lejay,[2] C. V. Colin,[2] B. Canals,[2] F. Damay,[3] J. Ollivier,[1] B. Fåk,[1] L. C. Chapon,[1] R. Ballou,[2] and V. Simonet[2]

Dipole activity of spin fragmentation materials?



- dipole moments.

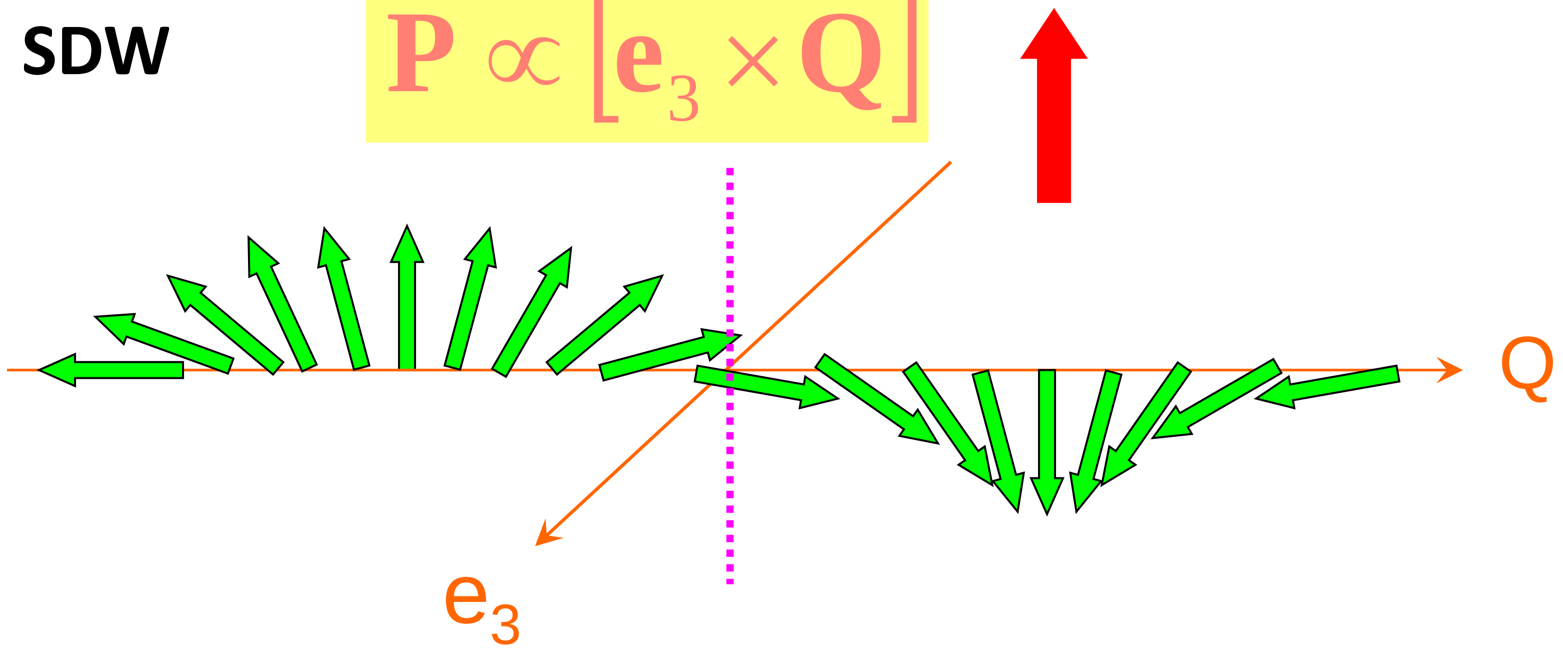


(a)
(b)

Dipoles on spirals

Cycloidal SDW

$$\bar{\mathbf{P}} \propto [\mathbf{e}_3 \times \mathbf{Q}]$$



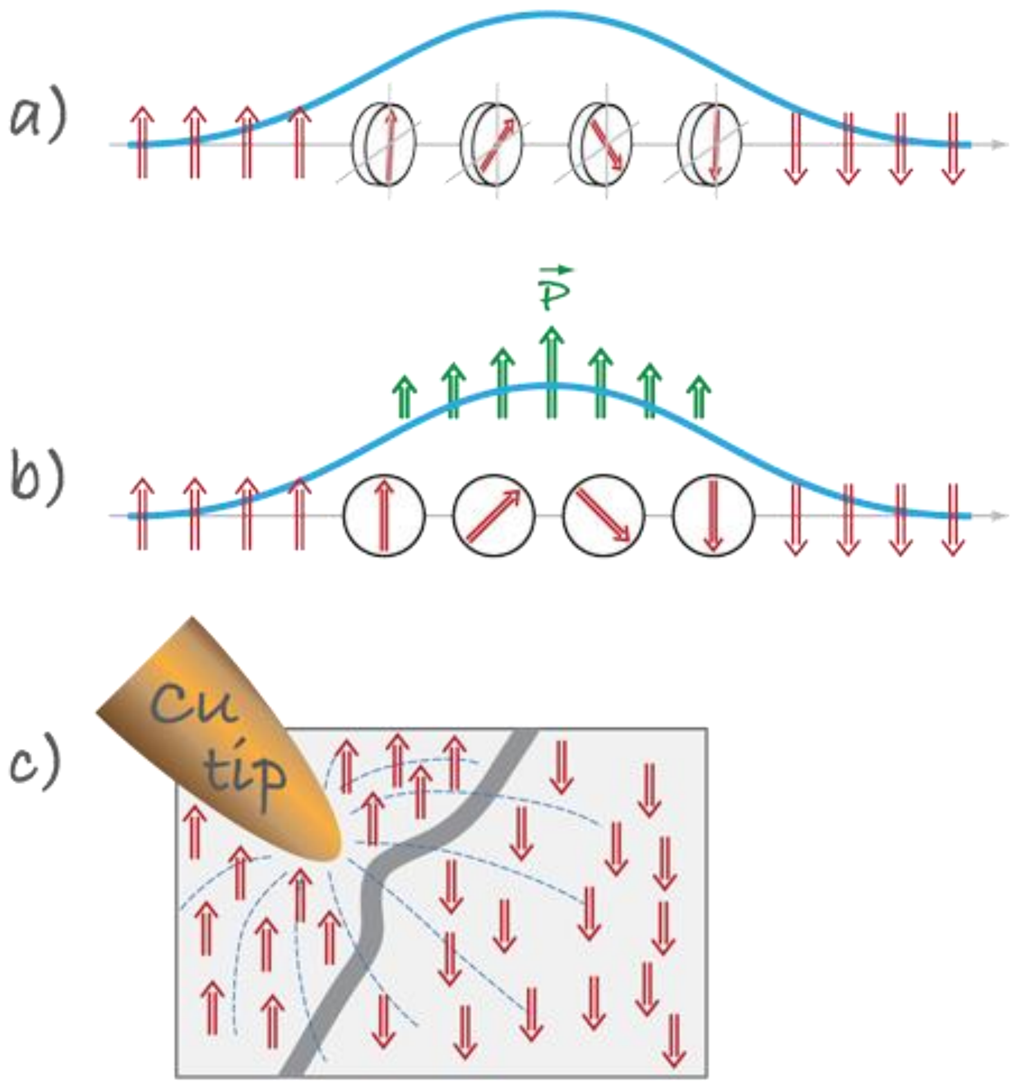
Katsura, Nagaosa and Balatsky, 2005

Mostovoy 2006

-- gives dipoles/polarization on the Neel domain walls

Electric dipoles at domain walls

Was already observed for Neel domain walls in ferromagnets (cf. spiral multiferroics):



Logginov, Pyatakov et al. (Moscow State Univ.)

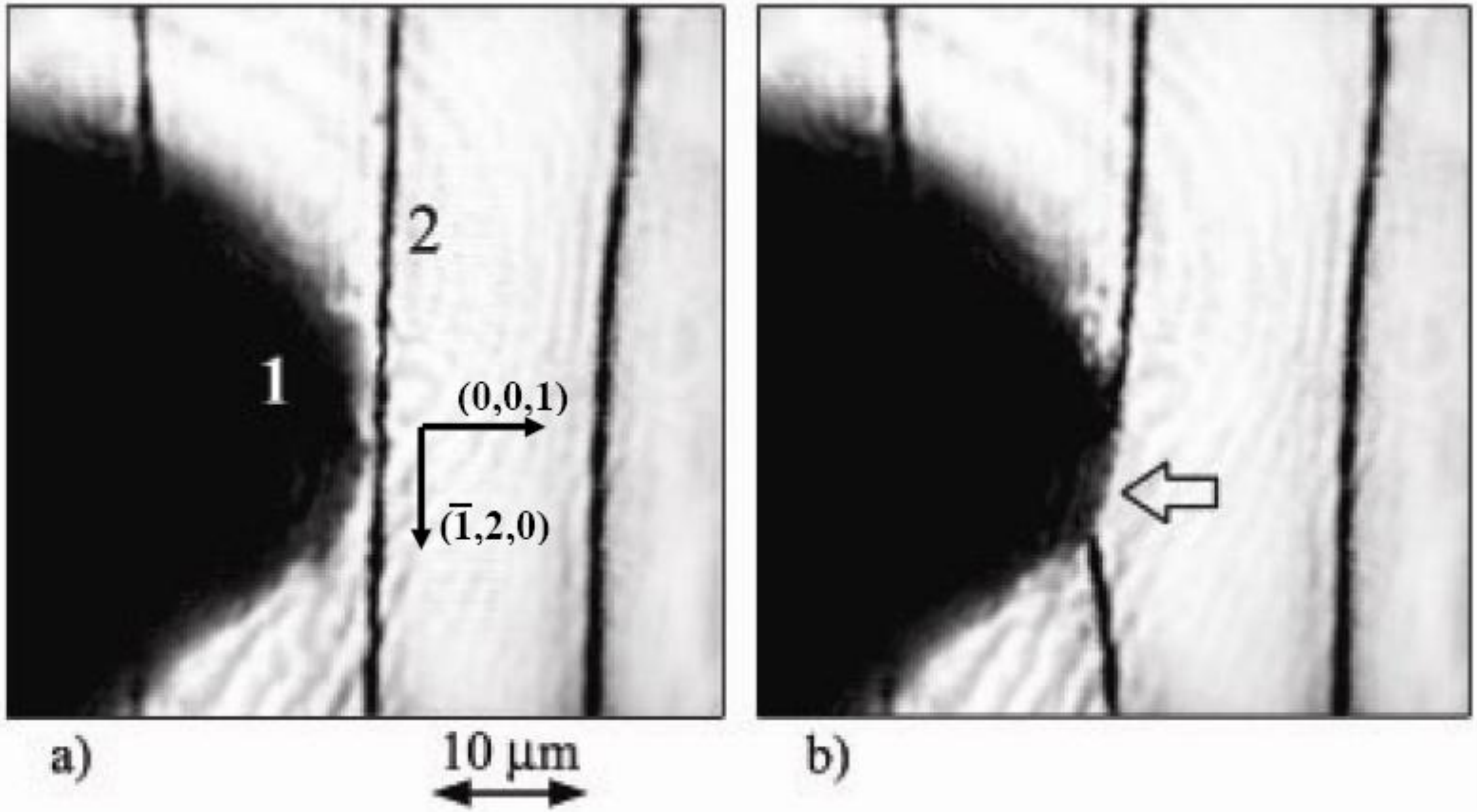
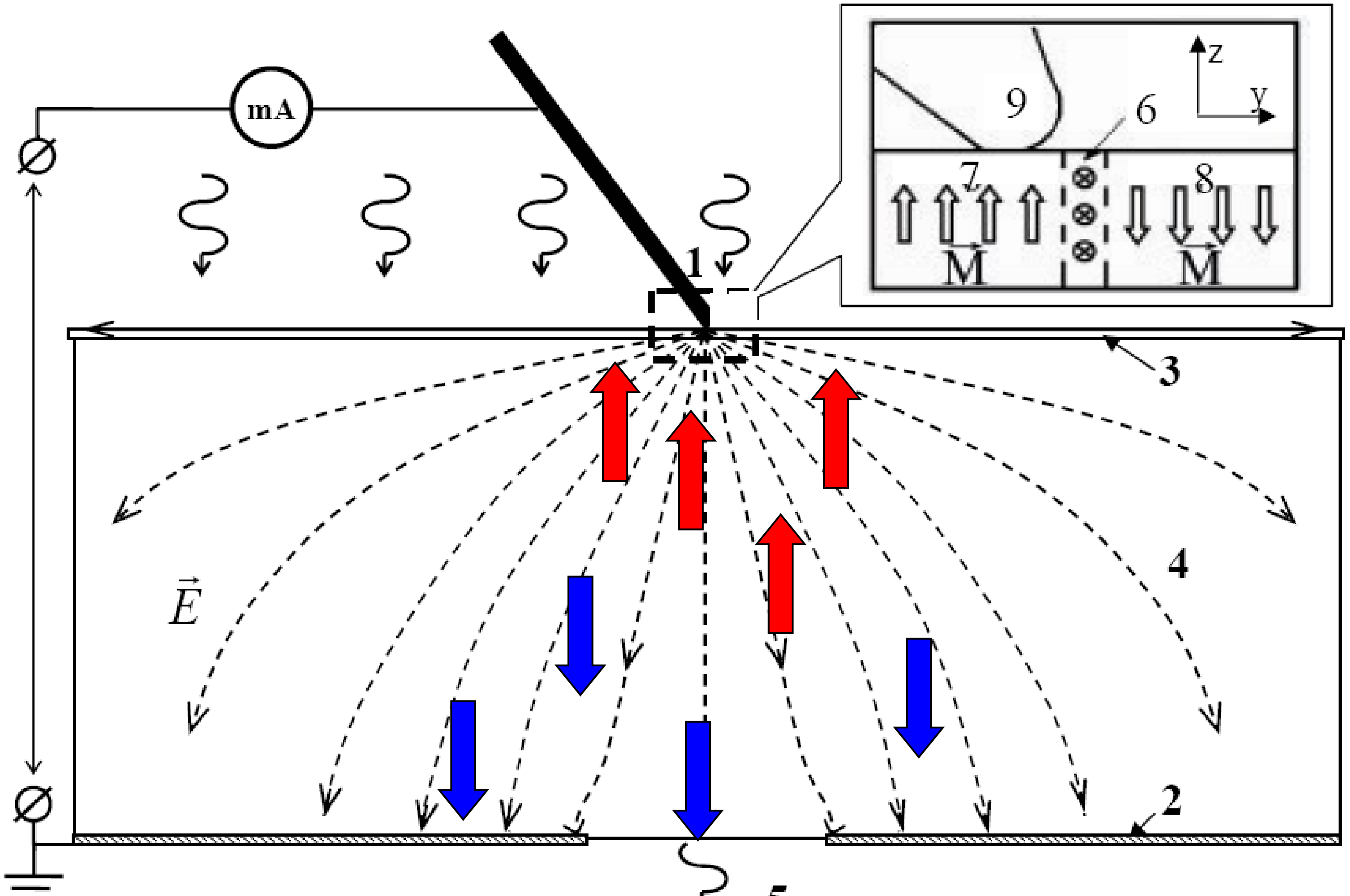


Fig. 2 The effect of electric field in the vicinity of electrode (1) on magnetic domain wall (2) in the films of ferrite garnets: a) initial state b) at the voltage of +1500 V applied

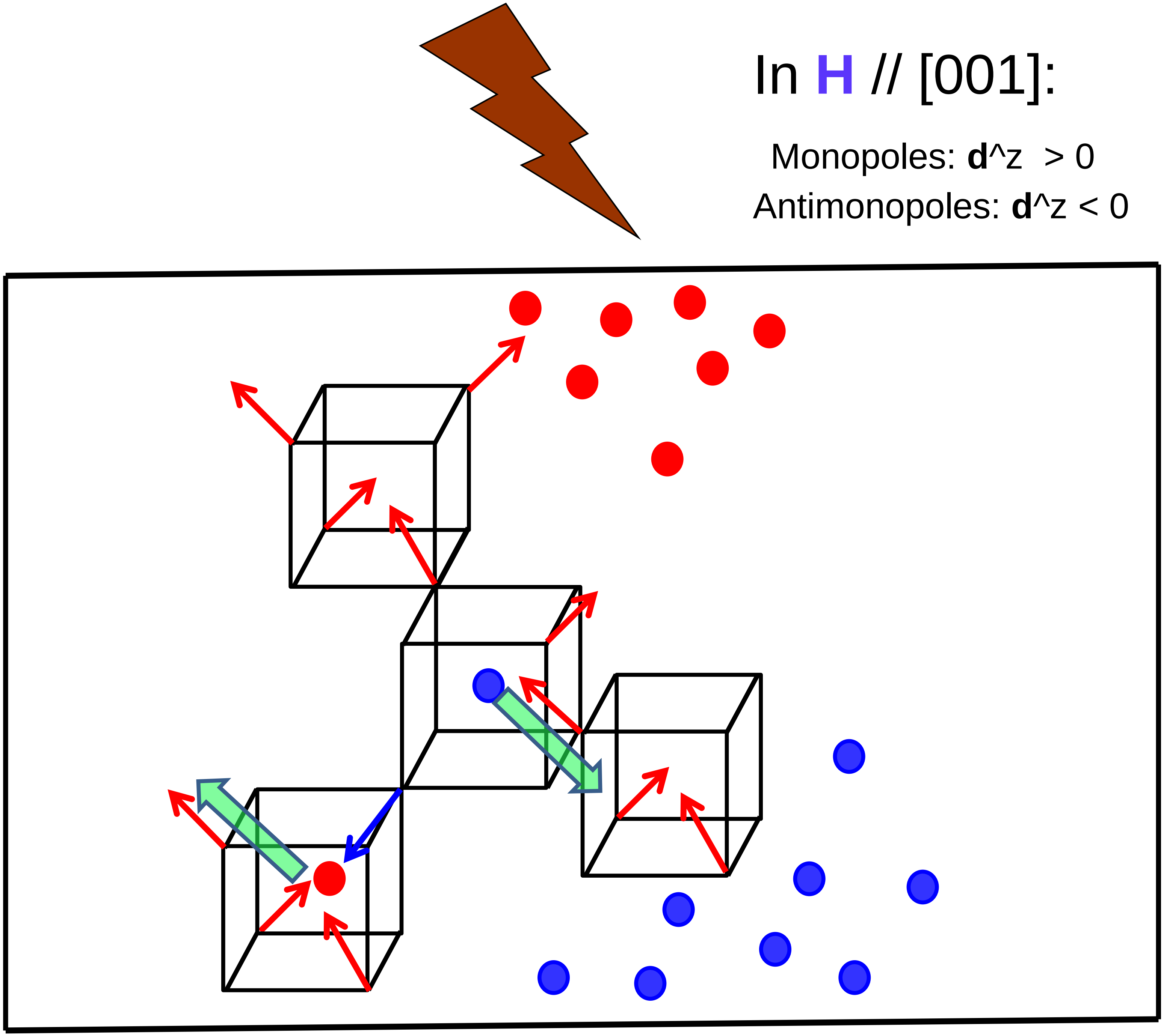
Inhomogeneous electric field in spin ice



In **H** // [001]:

Monopoles: **d**^z > 0

Antimonopoles: **d**^z < 0



nature Vol 447|10 May 2007|doi:10.1038/nature05802

LETTERS

Spiral structures in metal monolayers

Chiral magnetic order at surfaces driven by inversion asymmetry

M. Bode[1]†, M. Heide[2], K. von Bergmann[1], P. Ferriani[1], S. Heinze[1], G. Bihlmayer[2], A. Kubetzka[1], O. Pietzsch[1], S. Blügel[2] & R. Wiesendanger[1]

PRL 101, 027201 (2008) PHYSICAL REVIEW LETTERS week ending 11 JULY 2008

Atomic-Scale Spin Spiral with a Unique Rotational Sense: Mn Monolayer on W(001)

P. Ferriani,[1,*] K. von Bergmann,[1] E. Y. Vedmedenko,[1] S. Heinze,[1] M. Bode,[1,†] M. Heide,[2] G. Bihlmayer,[2] S. Blügel,[2] and R. Wiesendanger[1]

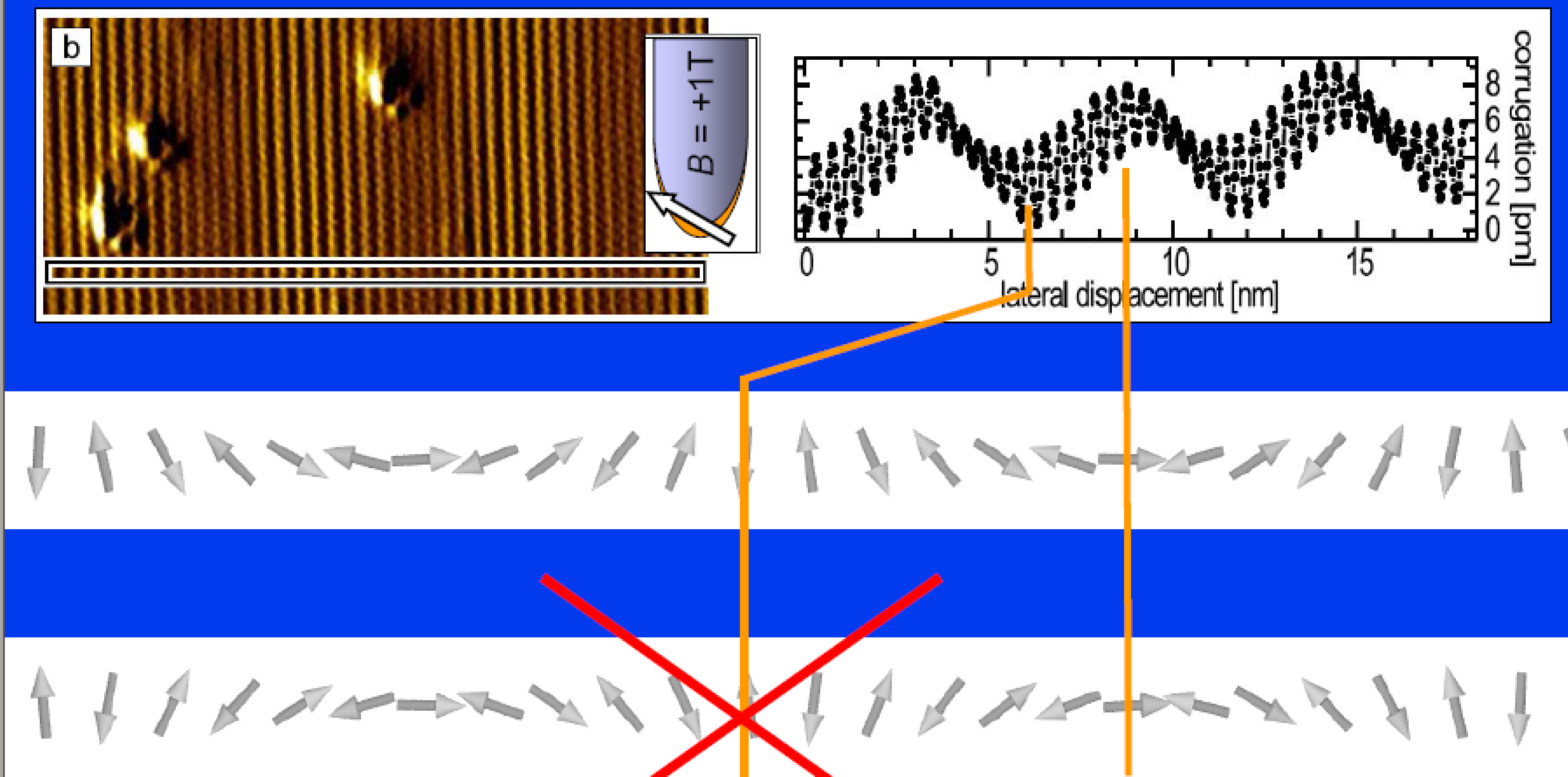
[1]*Institute of Applied Physics, University of Hamburg, Jungiusstrasse 11, 20355 Hamburg, Germany*

[2]*Institut für Festkörperforschung, Forschungszentrum Jülich, 52425 Jülich, Germany*

(Received 15 April 2008; published 7 July 2008)

Using spin-polarized scanning tunneling microscopy we show that the magnetic order of 1 monolayer Mn on W(001) is a spin spiral propagating along ⟨110⟩ crystallographic directions. The spiral arises on the atomic scale with a period of about 2.2 nm, equivalent to only 10 atomic rows. *Ab initio* calculations identify the spin spiral as a left-handed cycloid stabilized by the Dzyaloshinskii-Moriya interaction, imposed by spin-orbit coupling, in the presence of softened ferromagnetic exchange coupling. Monte Carlo simulations explain the formation of a nanoscale labyrinth pattern, originating from the coexistence of the two possible rotational domains, that is intrinsic to the system.

rotation direction of spin spiral



6 out of 6 independent islands: same rotational sense
→ DM-driven spin spiral

K. von Bergmann
University of Hamburg

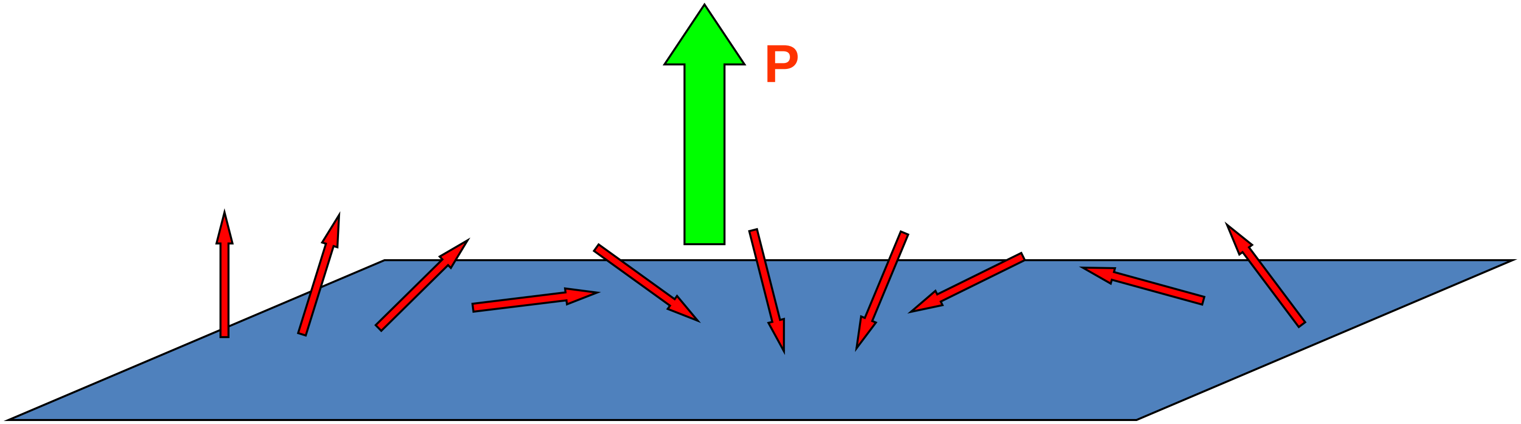
Simple explanation: at the surface there is a drop of a potential (work function, double layer)

I.e. there is an electric field **E**, or polarization **P** perpendicular to the surface

By the relation

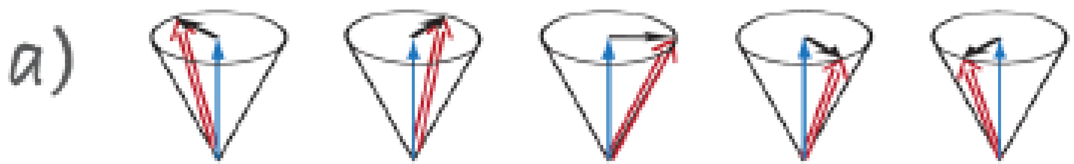
$$\overline{\mathbf{P}} \propto \left[\mathbf{e}_3 \times \mathbf{Q}\right]$$

there will appear magnetic spiral with certain sense of rotation, determined by **P**



Electric dipole carried by the usual spin wave

D.Khomskii, Physics (Trends) **2**, 20 (2009)



Physics (Trends) **2**, 20 (2009)

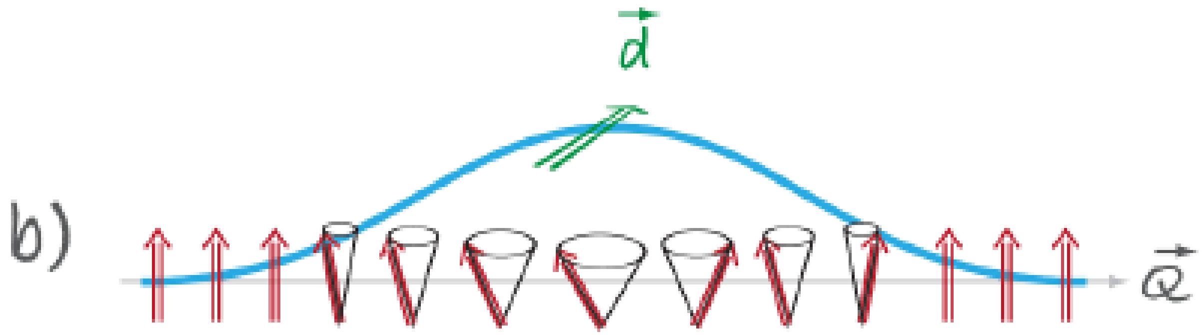
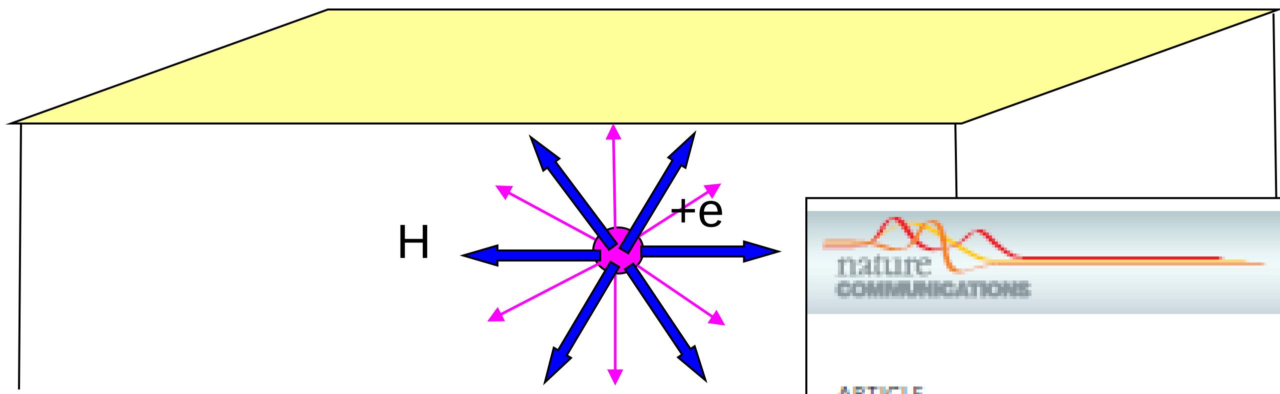


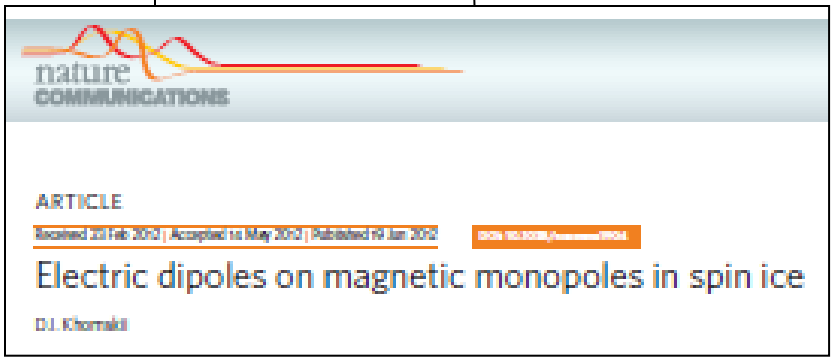
FIG. 5: How polarization emerges in a spin wave (magnon). (a) The classical picture of a spin wave in a ferromagnet: the spin (red arrow) precesses about a fixed axis (blue). The deviation is measured by the black arrows. (b) According to Eq. (1), as a spin-wave packet propagates along Q, it will also carry an electric dipole moment. (Illustration: Alan Stonebraker)

Magnetic monopoles in magnetoelectrics ?

Charge inside of of ME material: $\mathbf{M_i = \alpha_{ij} E_j}$, , $\mathbf{H=4\pi M}$

Let $\boldsymbol{\alpha_{ij} = \alpha\delta_{ij}}$, diagonal: magnetic field outside of the charge looks like a field of a magnetic monopole $\boldsymbol{\mu = 4\pi\alpha e}$




nature
COMMUNICATIONS

ARTICLE

Electric dipoles on magnetic monopoles in spin ice

D.I. Khomskii

Moving electron → moving monopole.

Electron in a magnetic field: force $\mathbf{F} = \boldsymbol{\mu}\mathbf{H} = 4\pi\alpha e\mathbf{H}$

(But one can also consider it as an action of the electric field created in magnetoelectric material on the electric charge: $\mathbf{E} = 4\pi\alpha\mathbf{H}$, $\mathbf{F} = \mathbf{E}e = 4\pi\alpha e\mathbf{H}$)

Other possible effects ? (how to find, to measure such monopoles)

"Electric Hall effect": if electric charge e moving in **H** gives a Hall effect, a monopole moving in electric field will do the same

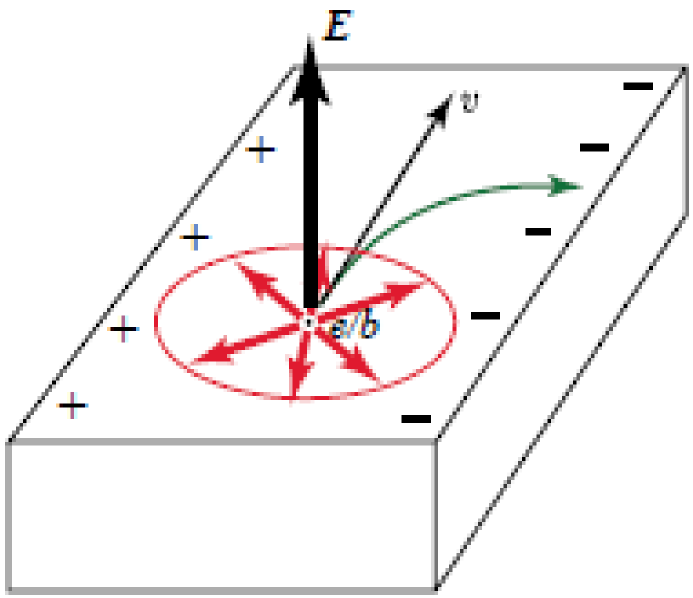
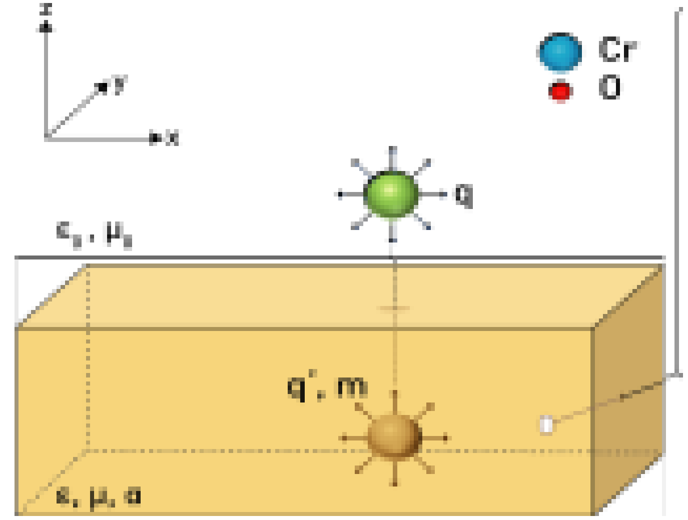


Figure 2: Motion of a magnetic monopole b, accompanying electric charge e, in a perpendicular electric field. The force $b[\boldsymbol{v} \times \boldsymbol{E}]$ will bend the trajectory of this object, leading to an "electric Hall effect".

But one can also explain this effect as the usual Hall effect in an effective magnetic field **B** ~ α**E**

Search for the magnetic monopole at a magnetoelectric surface
Q. N. Meier, M. Fechner, T. Nozaki, M. Sahashi, Z. Salman, T. Proschka, A. Suter, P. Schoenherr, M. Lilienblum, P. Borisov, I. E. Dzyaloshinskii, M. Fiebig, H. Luetkens, N. A. Spaldin Phys. Rev. X 9, 011011 (2019)



Qi, X. L., Li, R., Zang, J. & Zhang, S. C. Inducing a magnetic monopole with topological surface states. Science 323, 1184–1187 (2009).

Zang, J. & Nagaosa, N. Monopole current and unconventional Hall response on topological insulators. Phys. Rev. B81, 245125

Magnetic vortices as magnetoelectrics

PRL 102, 157203 (2009) PHYSICAL REVIEW LETTERS week ending 17 APRIL 2009

Superexchange-Driven Magnetoelectricity in Magnetic Vortices

Kris T. Delaney,[1] Maxim Mostovoy,[2] and Nicola A. Spaldin[1]

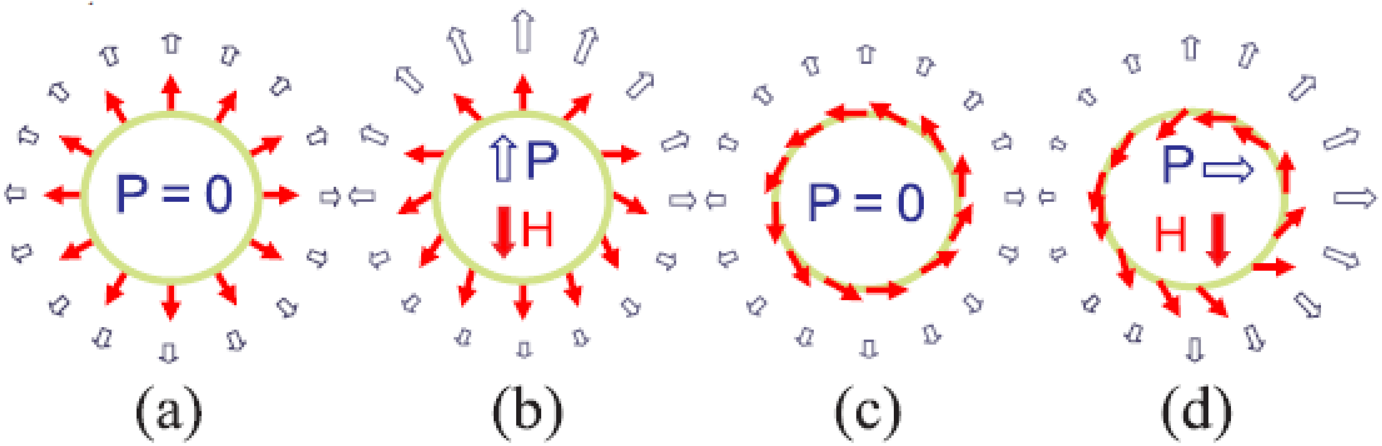
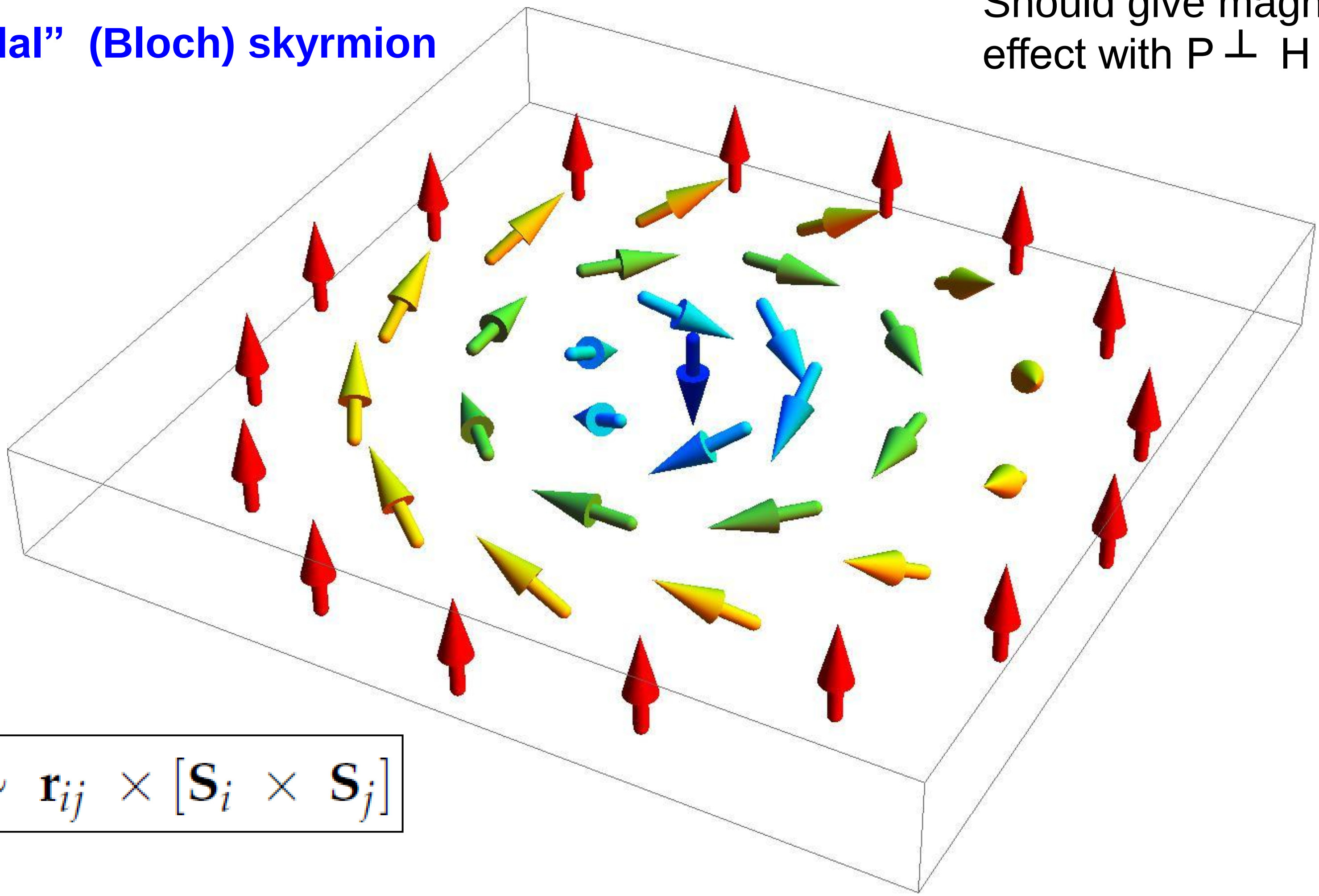


FIG. 1 (color online). (a) A magnetic vortex carrying a monopole moment. The thin solid arrows indicate the spin orientation, while the thick open arrows show the local polarization vector. (b) A magnetic field applied to the vortex shown in (a) induces a net polarization along the field direction. (c) A magnetic vortex with a toroidal moment. (d) A magnetic field applied to (c) induces an electric polarization perpendicular to the field.

Skyrmions in magnetic crystals

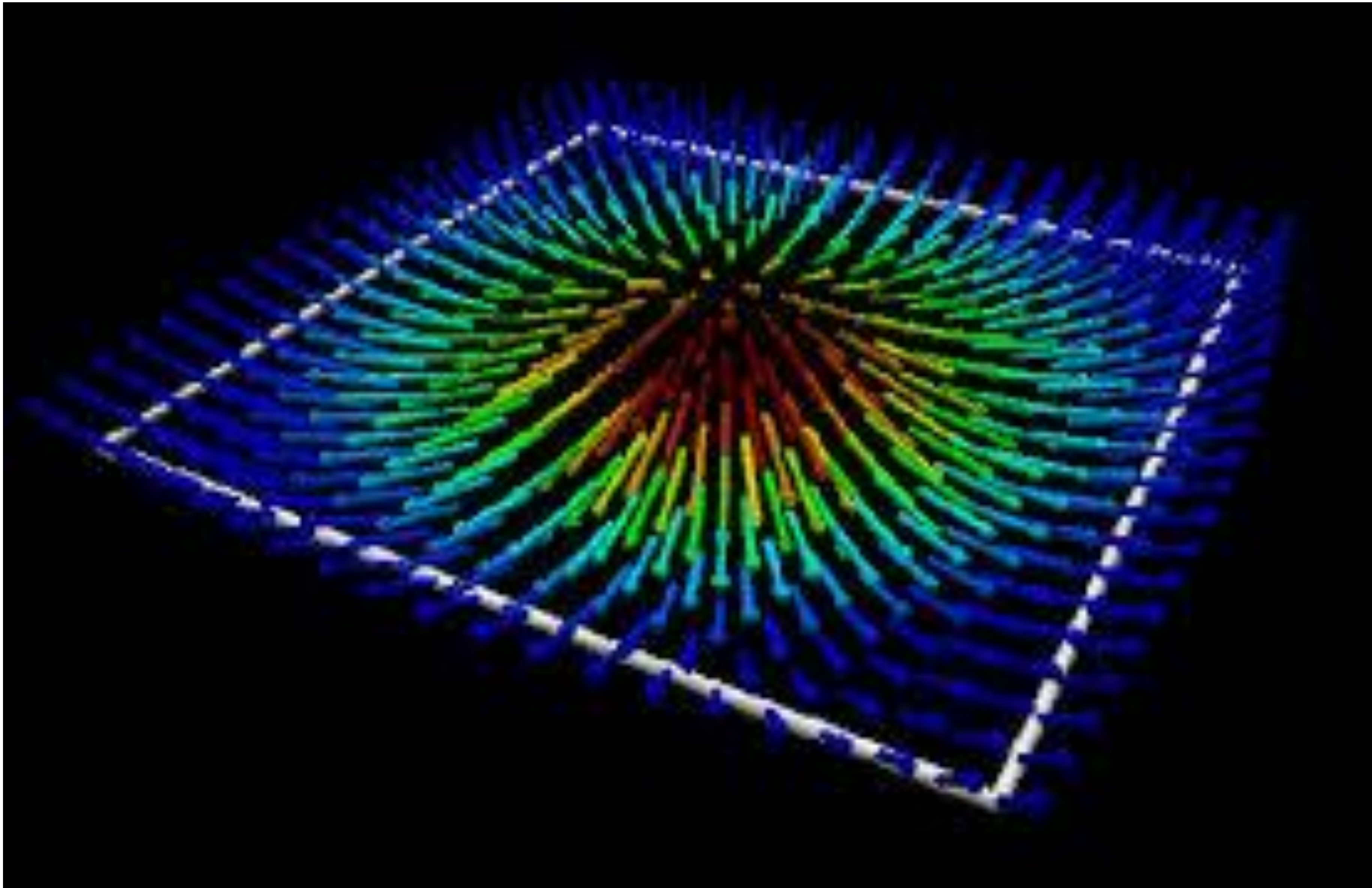
“Toroidal” (Bloch) skyrmion

Should give magnetoelectric effect with P ⊥ H



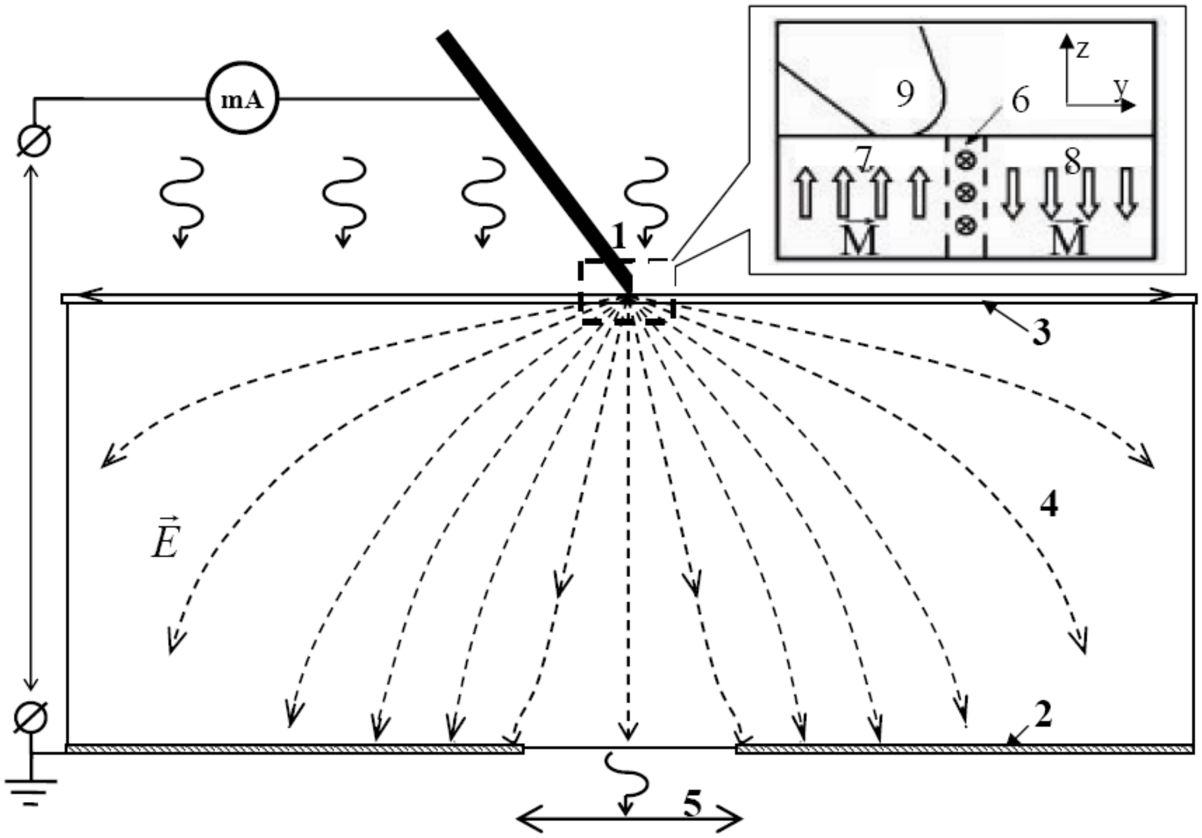
$$\mathbf{P} \sim \mathbf{r}_{ij} \times [\mathbf{S}_i \times \mathbf{S}_j]$$

“Radial” (Neel) skyrmion

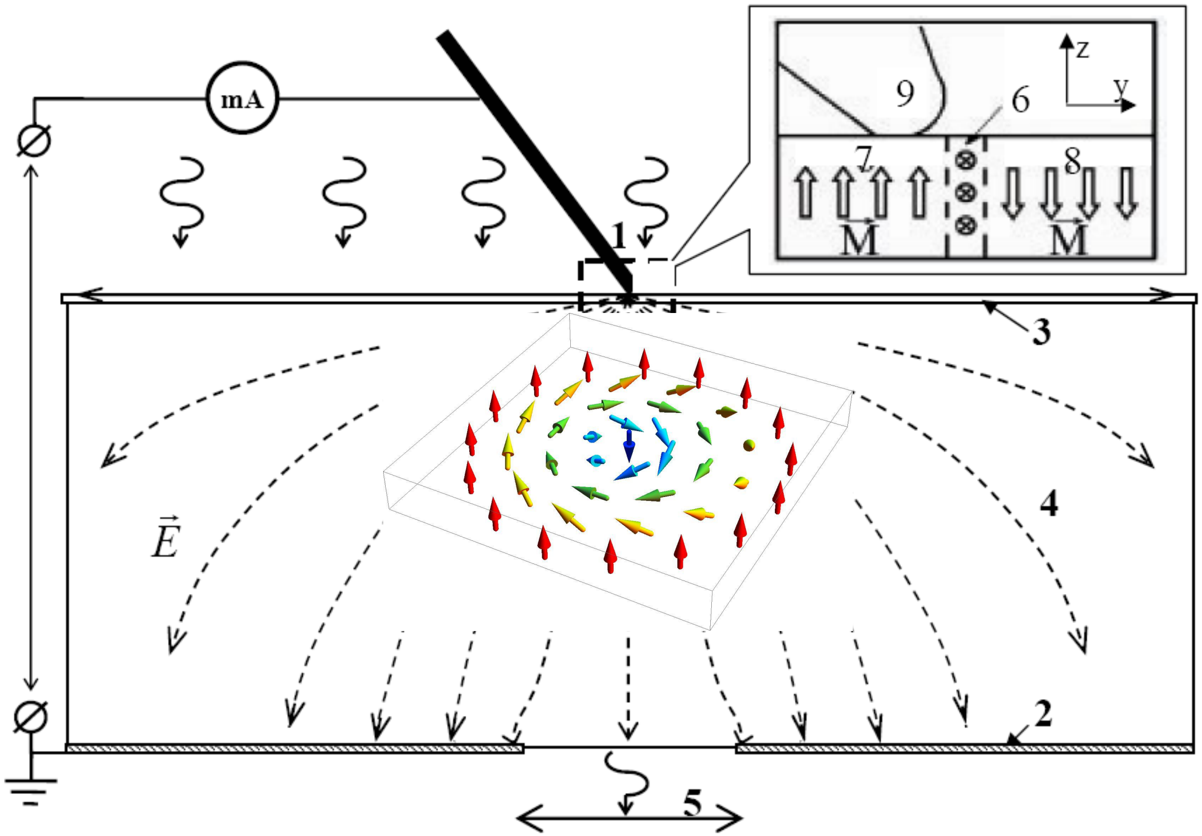


Should give magnetoelectric effect with P ‖ H

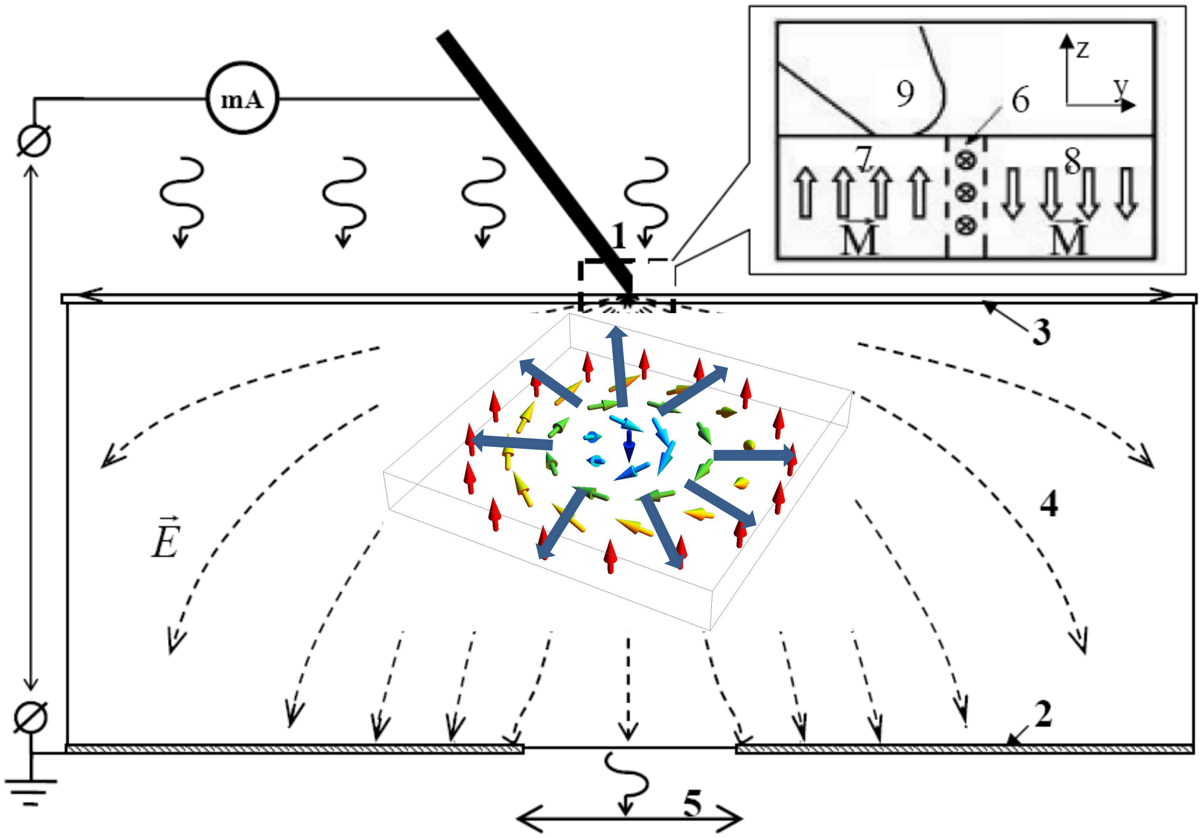
Skyrmions in inhomogeneous electric field?



Skyrmions in inhomogeneous electric field?



Skyrmions in inhomogeneous electric field?



nature nanotechnology

LETTERS

PUBLISHED ONLINE: 7 NOVEMBER 2016 | DOI: 10.1038/NNANO.2016.234

Electric-field-driven switching of individual magnetic skyrmions

Pin-Jui Hsu, André Kubetzka, Aurore Finco, Niklas Romming, Kirsten von Bergmann*
and Roland Wiesendanger

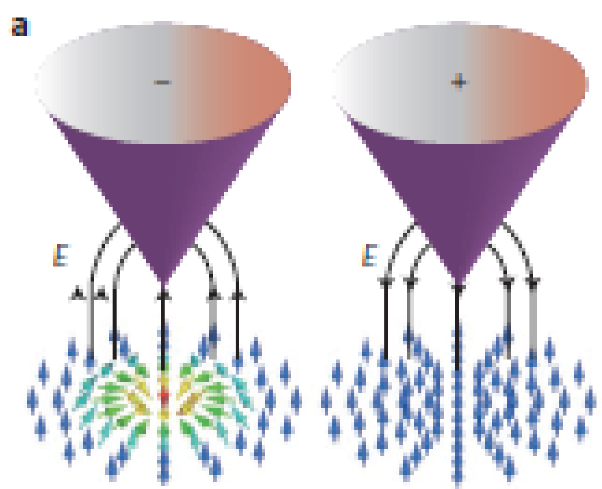
- generation of skyrmions on a surface by the STM tip with the electric voltage. Only for one polarity.

SPINTRONICS

Electric control of skyrmions

The creation and destruction of magnetic whirls by the application of an electric field may be the basis for novel memory technologies.

Achim Rosch



CONCLUSIONS

Electric activity of many different magnetic textures, and even of spin waves.