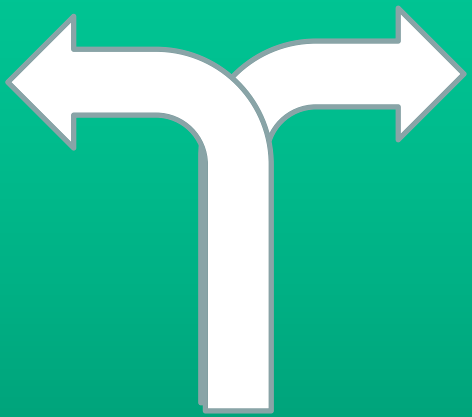


MULTIDIMENSIONAL DARK SPACES in OPEN DRIVEN SYSTEMS

Yuval Gefen Weizmann Institute
with
Raul Santos U. Cambridge
Alex Kamenev U. Minnesota
Fernando Iemini Niteroi, Brazil

see also : M. Gau, R. Egger, YG, to be published

- ☆ Lindblad dynamics
- ☆ Drive & Dissipation: dark state
- ☆ Dark space
- ☆ Fractional quantum Hall state by dissipation

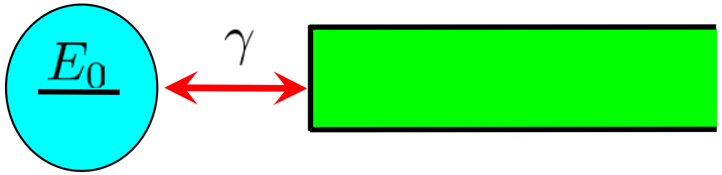
Dissipation

- Coupling to **baths** ► **Density matrix**
- **Markovian** baths ► **Lindblad** dynamics
  - **Preserves**: **Hermiticity, trace**, **positivity**

$$\partial_t \rho(t) = \hat{\mathcal{L}}\left[\rho(t)\right] = -i[H, \rho(t)] + \frac{1}{2}\sum \gamma_i \left[2L_i\rho(t)L_i^\dagger - L_i^\dagger L_i\rho(t) - \rho(t)L_i^\dagger L_i\right]$$

Hamiltonian **Quantum Jumps** **Decay, Decoherence**

- Example: **1** fermion level + fermion bath, **distribution**: $n(\omega)$



$$\hat{\mathcal{L}}^{\mathrm{out}}\rho = \frac{\gamma\left[1 - n(E_0)\right]}{2}\left(2d\rho d^\dagger - d^\dagger d\rho - \rho d^\dagger d\right)$$

$$\hat{\mathcal{L}}^{\mathrm{in}}\rho = \frac{\gamma n(E_0)}{2}\left(2d^\dagger\rho d - dd^\dagger\rho - \rho dd^\dagger\right)$$

$$H = E_0 d^\dagger d$$

  - **steady state** level population: $n(E_0)$
  - deviations from it **decay** at **rate** γ

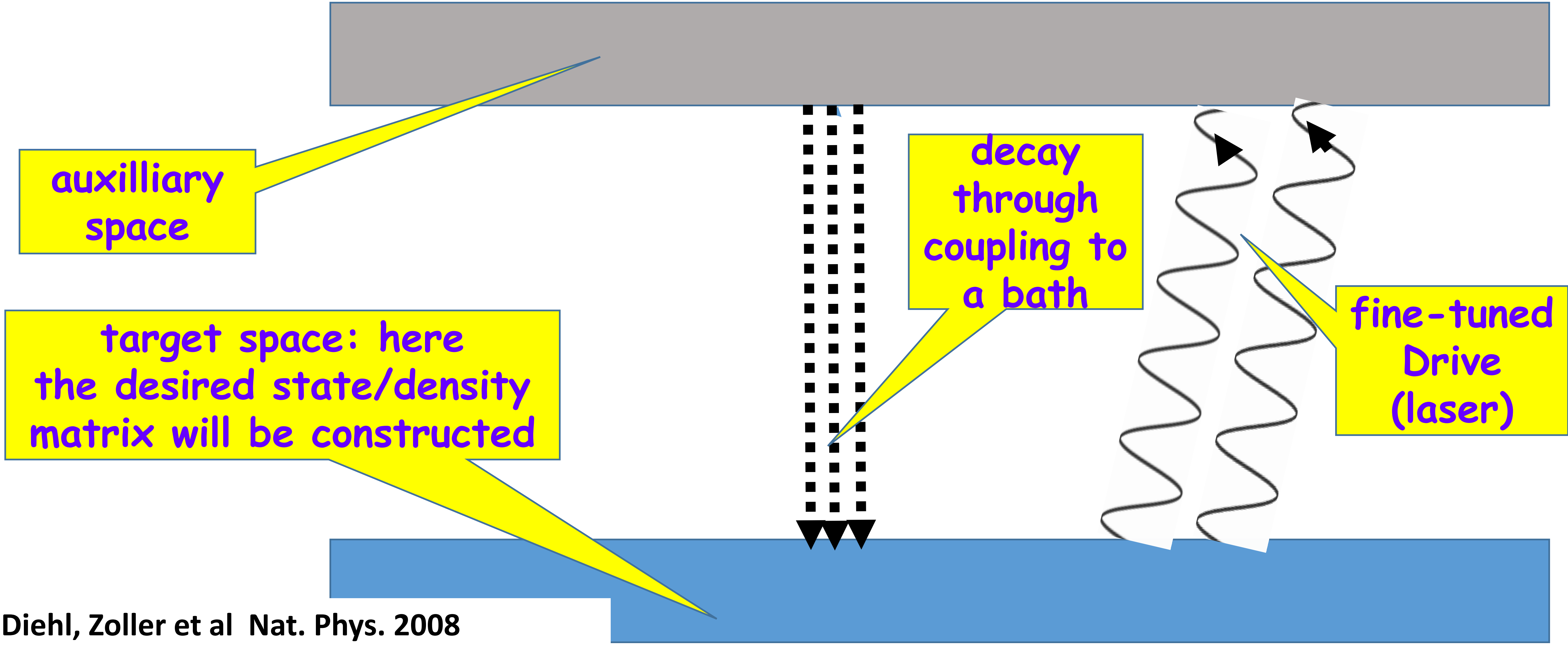
M. Goldstein, private communication

Diehl, Zoller et al Nat. Phys. 2008
Verstraete, Wolf, Cirac, Nat. Phys. 2009
Diehl, Rico, Baranov, Zoller, Nat. Phys. 2011

See also:
Legthas, Girvin, Devoret et al. PRA, 2013
Liu, Shankar, Schoelkopf, Devoret et al., Phys. Rev. X (2016)

- ☆ Lindblad dynamics
- ☆ Drive & Dissipation: dark state
- ☆ Dark space
- ☆ Fractional quantum Hall state by dissipation

TOPOLOGY BY DISSIPATION: a caricature



Diehl, Zoller et al Nat. Phys. 2008
Verstraete, Wolf, Cirac, Nat. Phys. 2009
Diehl, Rico, Baranov, Zoller, Nat. Phys. 2011

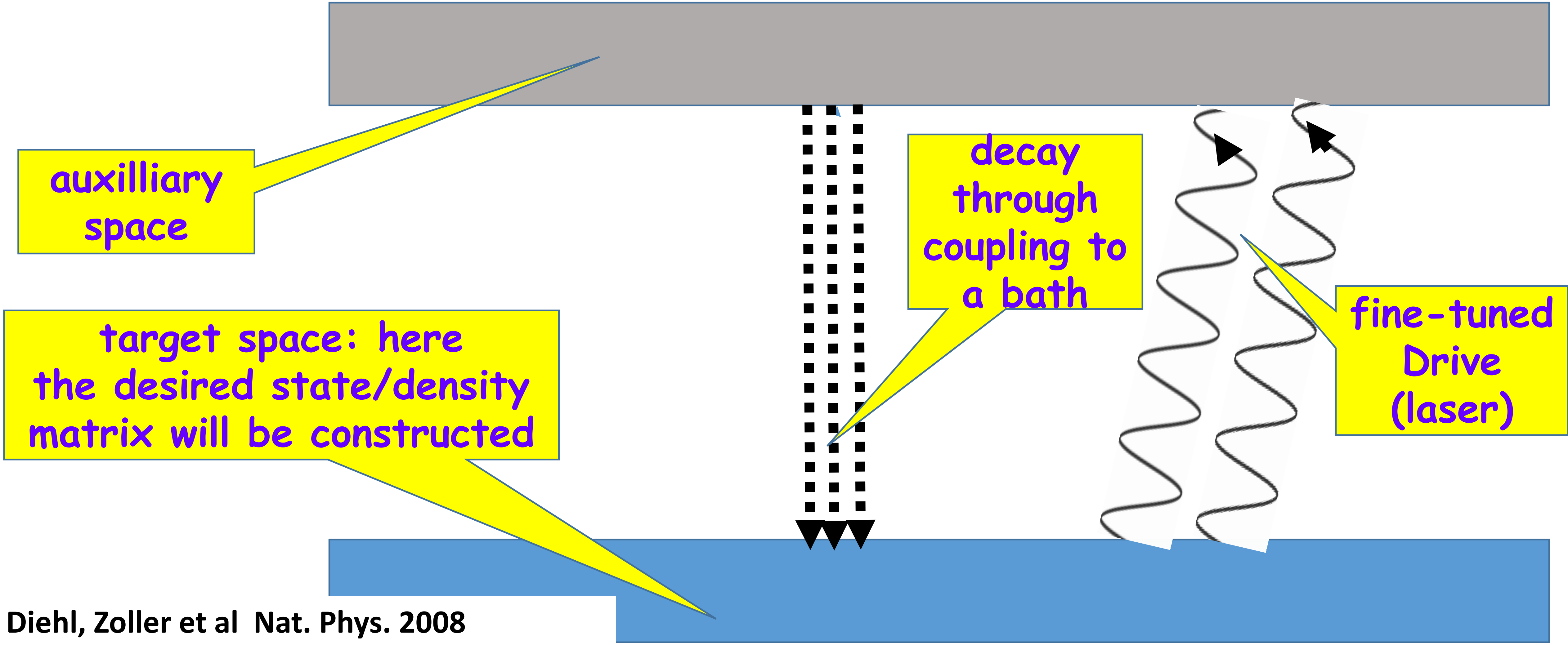
A DARK STATE

$$\partial_t \rho(t) = \hat{\mathcal{L}}\left[\rho(t)\right] = -i[H, \rho(t)] + \frac{1}{2}\sum \gamma_i \left[2L_i\rho(t)L_i^\dagger - L_i^\dagger L_i \rho(t) - \rho(t)L_i^\dagger L_i\right]$$

$$L_i \left|\psi\right\rangle_{DARK} = 0 \quad \text{for } all \quad i$$

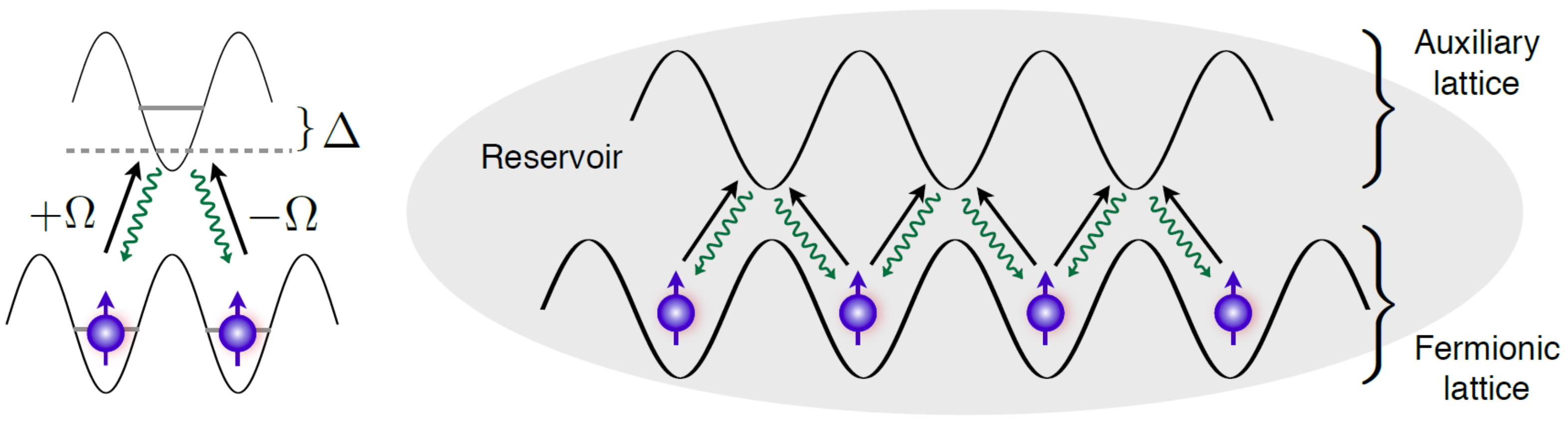
example: drive (orthogonal to target dark state)→ relaxation
continuous leakage to the target space

TOPOLOGY BY DISSIPATION: a caricature



Diehl, Zoller et al Nat. Phys. 2008
Verstraete, Wolf, Cirac, Nat. Phys. 2009
Diehl, Rico, Baranov, Zoller, Nat. Phys. 2011

EXAMPLE: KITAEV CHAIN with MAJORANA END-POINTS



c

a

$b \rightarrow \text{bath}$

drive: $c_i^\dagger (a_i - a_{i+1})$

dissipative relaxation: $b^\dagger (a_i^\dagger + a_{i+1}^\dagger) c_i$

tracing out auxiliary+bath

$$L_i = C_i^\dagger A_i$$

$$C_i^\dagger = a_i^\dagger + a_{i+1}^\dagger$$

$$A_i = a_i - a_{i+1}$$

Lindbladian quartic in a
→ Lindbladian quadratic in a
→ Conserves parity

Bardyn, Baranov, Kraus, Rico, Imamoglu, Zoller, Diehl, 2013

Kitaev chain: two E=0 states: $|0\rangle, |1\rangle$

given parity → a single state

→ a single dark state →

PURE STATE

cf. Iemini, Rossini, Fazio, Dihel, Mazza
Phys. Rev. B 2016

See also:

Bardyn, Zoller, Diehl et al., NJP (2013) classification according to symmetries Gaussian states

Albert and Jiang, PRA (2014) classification but not how to generate the states note...

eigenvalue of L =0 ➔ target (steady state) density matrix (dark state)

next eigenvalues ➔ rate of approaching the target density matrix

cf. Iemini, Rossini, Fazio, Dihel, Mazza
Phys. Rev. B 2016

- Lindblad dynamics
- Drive & Dissipation: dark state
- Dark space
- Fractional quantum Hall state by dissipation

See also:

Bardyn, Zoller, Diehl et al., NJP (2013) classification according to symmetries Gaussian states

Albert and Jiang, PRA (2014) classification but not how to generate the states

- Lindblad dynamics
- Drive & Dissipation: dark state
- Dark space
- Fractional quantum Hall state by dissipation

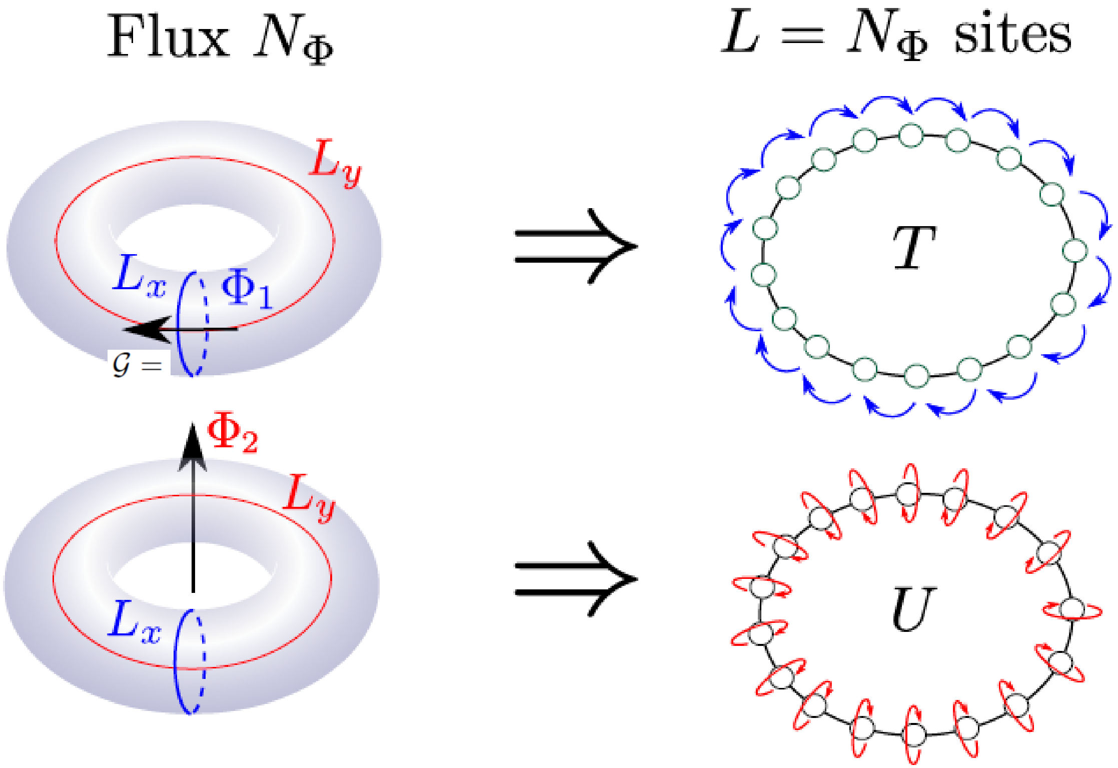
EXAMPLE: FQHE $\nu = 1/3$

on a torus :

T, U fluxon insertion

$\mathcal{G} = Z_3 \times Z_3$

$U^3 = T^3 = \mathbf{1}$



degeneracy of dark space $= p = 3$

MAPPING LAUGHLIN STATES TO 1D

Laughlin state, filling $= 1/m$

exact ground-state of Landau+ $V(\mathbf{r}) = V_0 \nabla^{m-1} \delta(\mathbf{r})$

Trugman, Kivelson 1985
Seidel, Fu, Lee, Leinaas,Moore 2005
Ortiz, Nussinov, Dukelsky, Seidel 2013

Fpro**projecting H onto LLL**

$$\mathcal{H} = \sum_n Q^\dagger_{0,n} Q_{0,n} + Q^\dagger_{1,n} Q_{1,n}$$

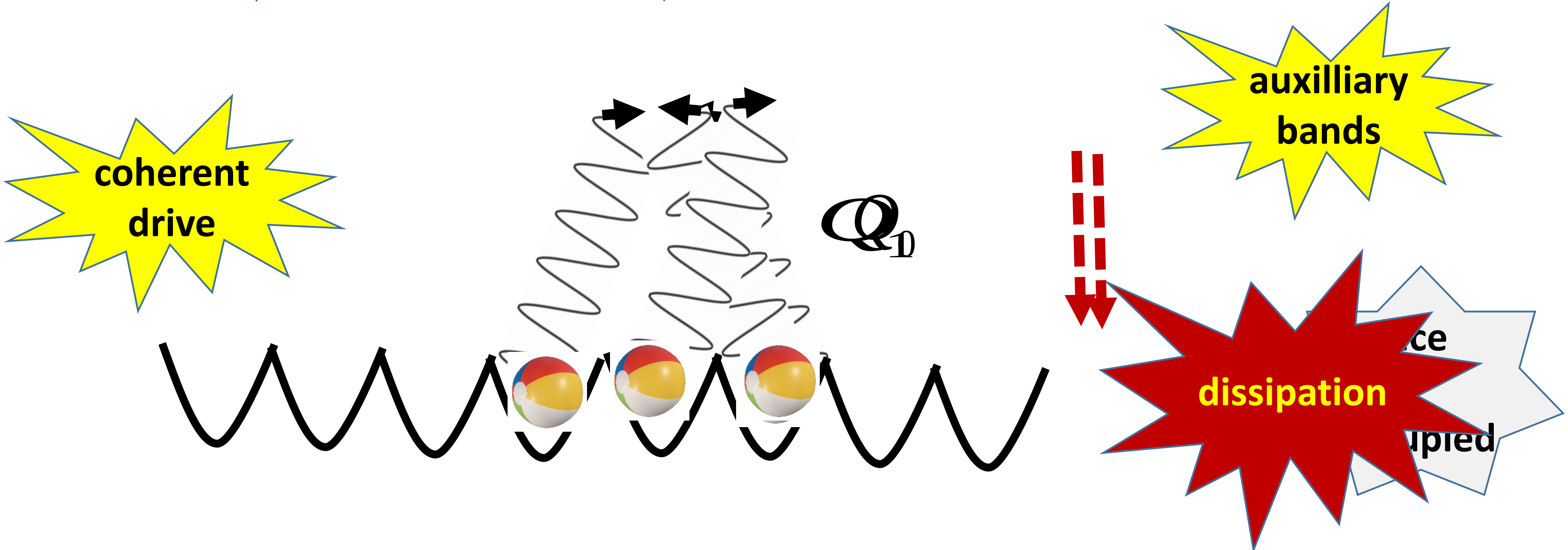
$$Q_{s,n} = \sum_{l \geq 0} \eta \left(l + \frac{s}{2} \right) c_{n-l-s} c_{n+l}.$$

$$\eta(x) = \sum_{r \in \mathbb{Z}} (x + r N_\Phi) e^{-\kappa^2 (x + r N_\Phi)^2}$$

$$\kappa = L_x / L_y \xrightarrow[\text{thin torus}]{} \infty$$

$$Q_{s,n} = \sum_{l \geq 0} \eta \left(l + \frac{s}{2} \right) c_{n-l-s} c_{n+l} \xrightarrow{\text{thin torus}}$$

$$Q_{0,i} = c_i c_{i+2} \quad \text{and} \quad Q_{1,i} = c_i c_{i+1} + \beta c_{i-1} c_{i+2}$$



PRESERVING SYMMETRIES MAY BE BAD

an initial may not decay into the target dark space

decay through dissipation should help us

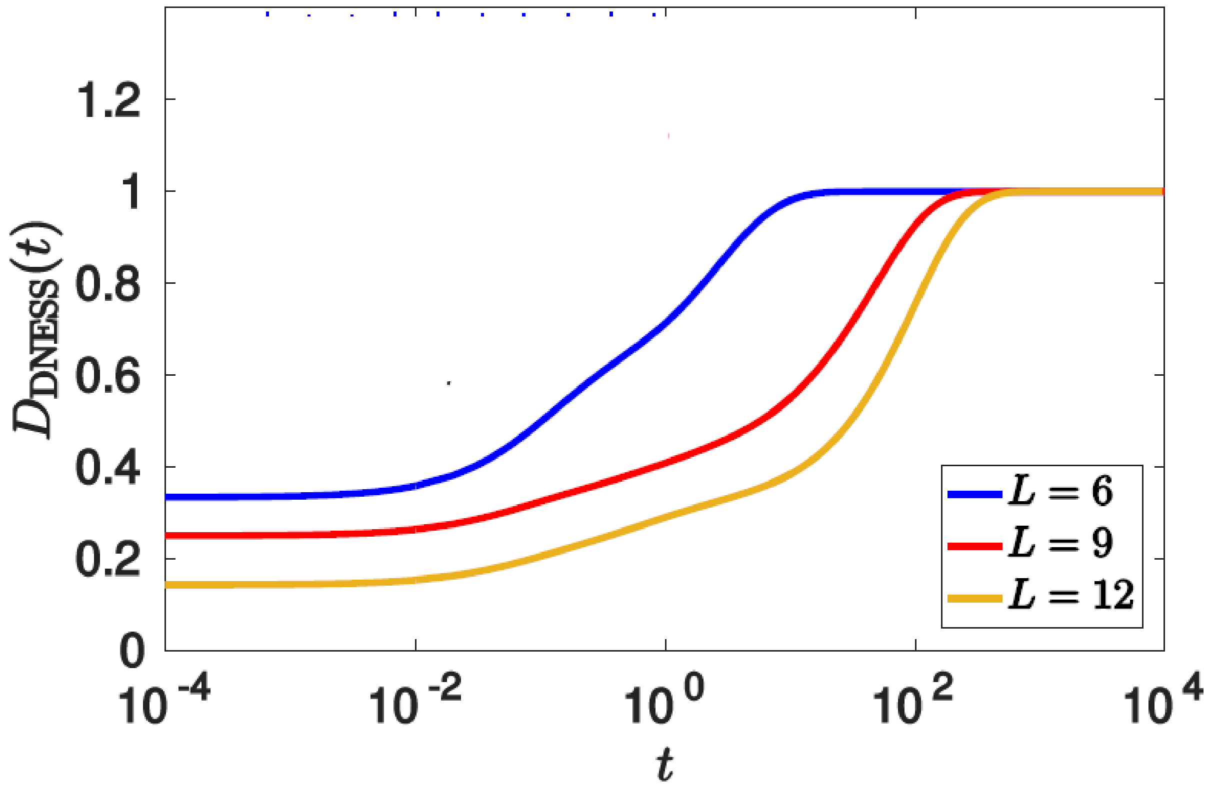
$$\ell_{s,i} = R_i^\dagger Q_{s,i}$$

$$R_i^\dagger = c_i^\dagger c_{i+1}^\dagger$$

$$Q_{0,i} = c_i c_{i+2} \quad \text{and} \quad Q_{1,i} = c_i c_{i+1} + \beta c_{i-1} c_{i+2}$$

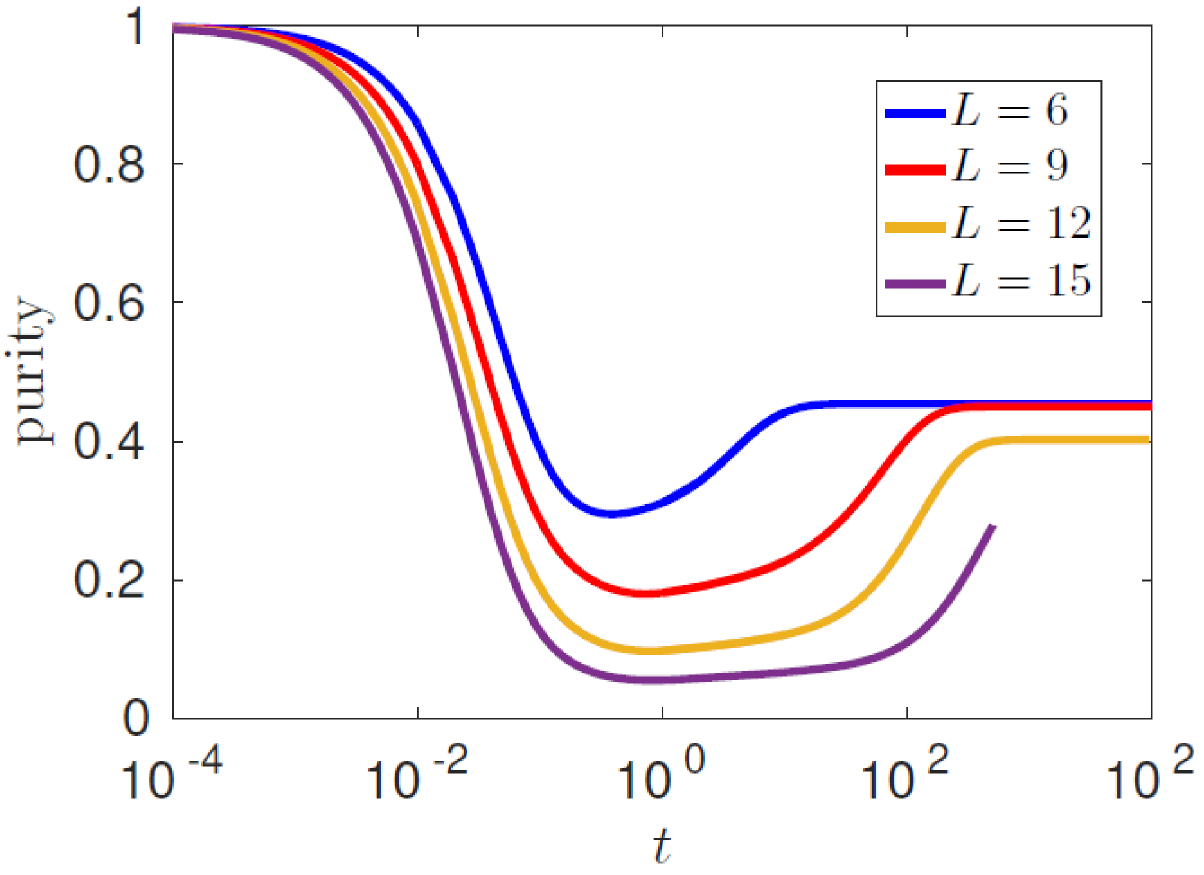
good news

approach towards the dark space

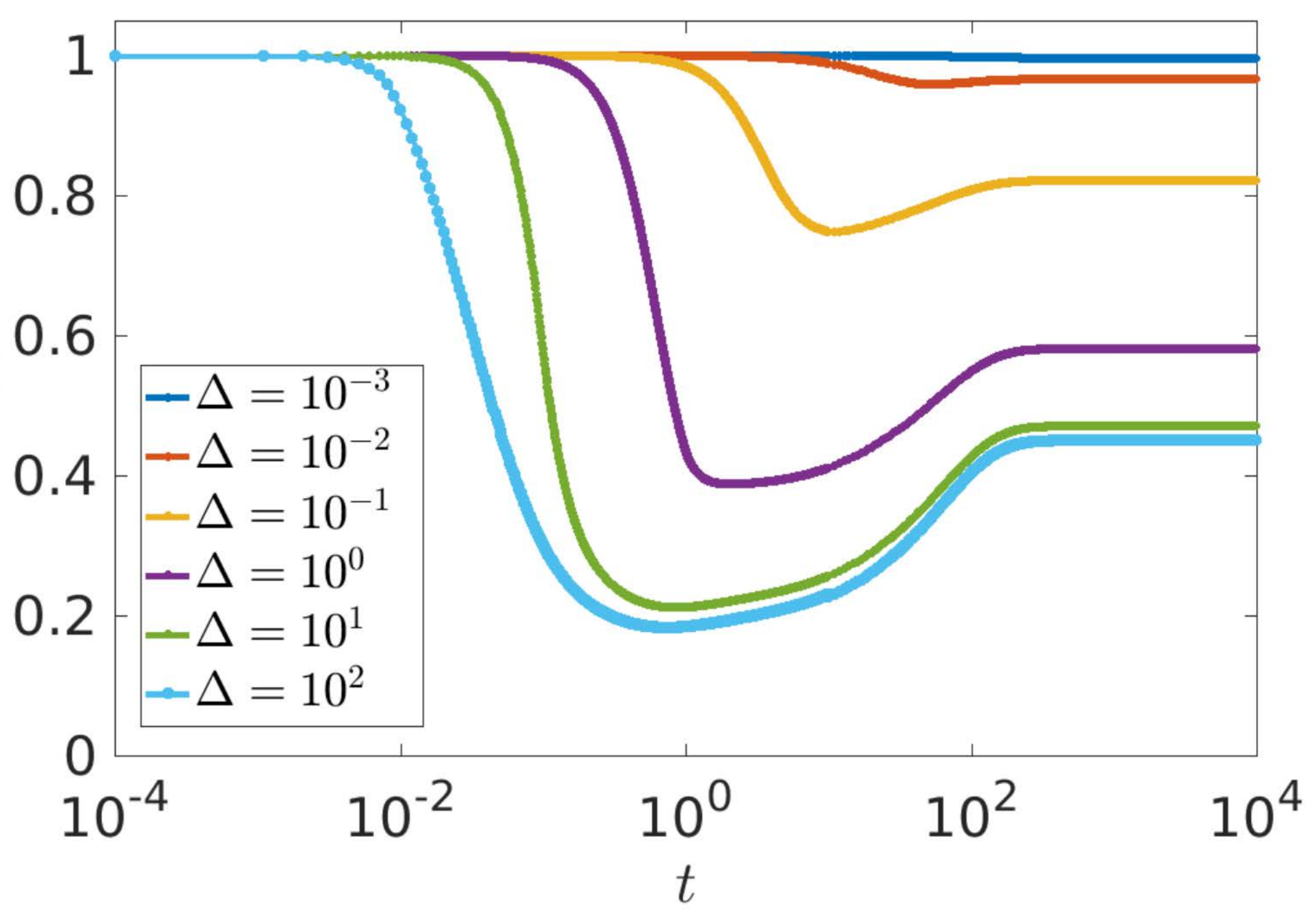


bad news

purity of the state



how to correct
adiabatic switching of Lindbladian

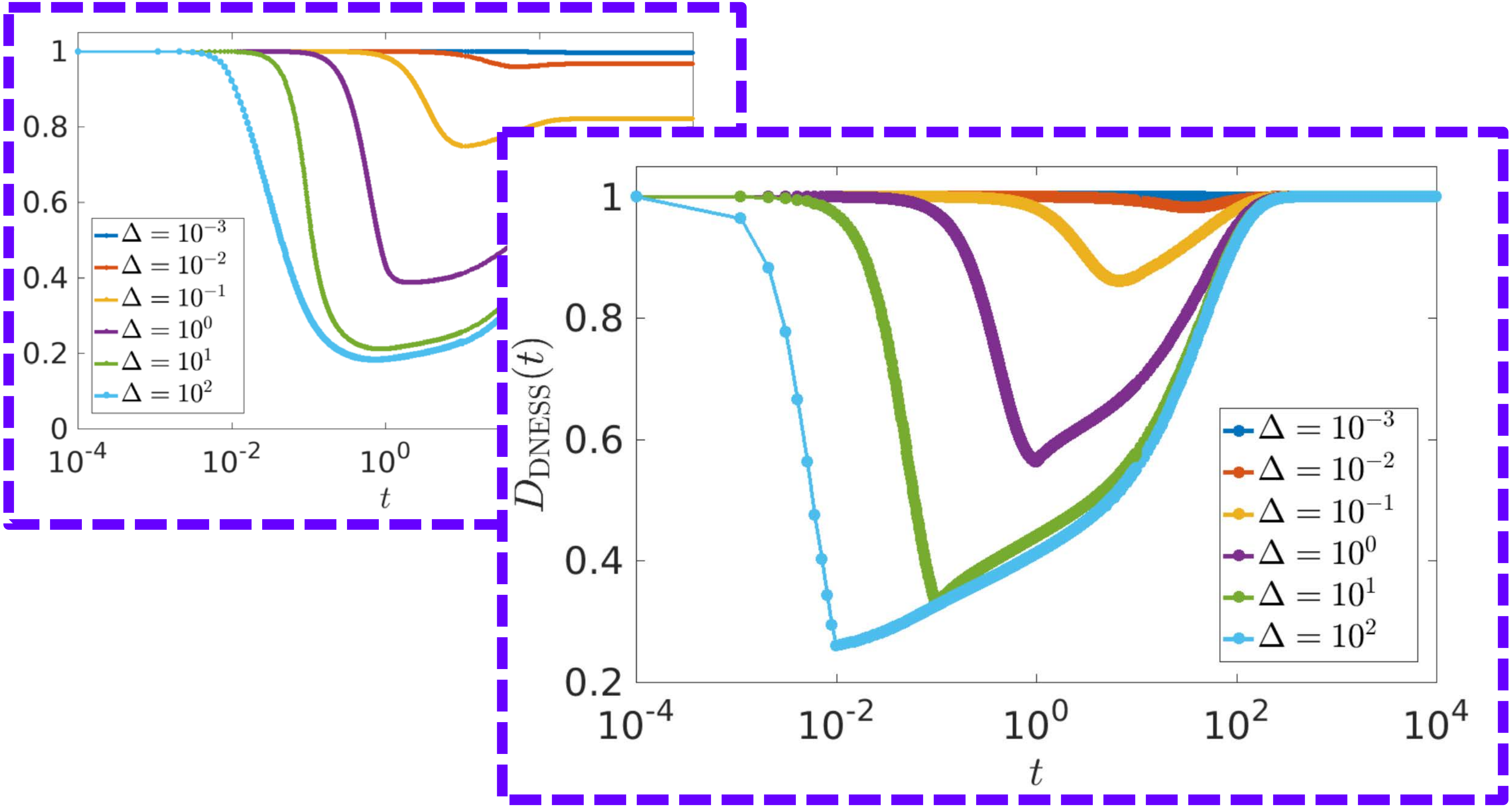


$$\beta(t) = \begin{cases} \Delta t, & \text{for} \quad 0 < t \leq 1/\Delta, \\ 1, & \text{for} \quad t > 1/\Delta. \end{cases}$$

controlling the thin torus limit

$$Q_{0,i} = c_i c_{i+2} \quad \text{and} \quad Q_{1,i} = c_i c_{i+1} + \beta c_{i-1} c_{i+2}$$

how to correct
adiabatic switching of Lindbladian



WHAT'S NEXT?

- controlling efficiently the mixed vs. pure state
  (e.g., in the 3x3 target space)
- adiabatic variations of the target state
  (e.g. braiding):
  what is "adiabatic"? keeping purity intact
- error correction

THANK YOU