How spin currents defy our high-school intuition

Yaroslaw Bazaliy [1], Revaz Ramazashvili [2]

(1) University of South Carolina, Columbia, SC
(2) Université de Toulouse, CNRS, France

Support: National Science Foundation

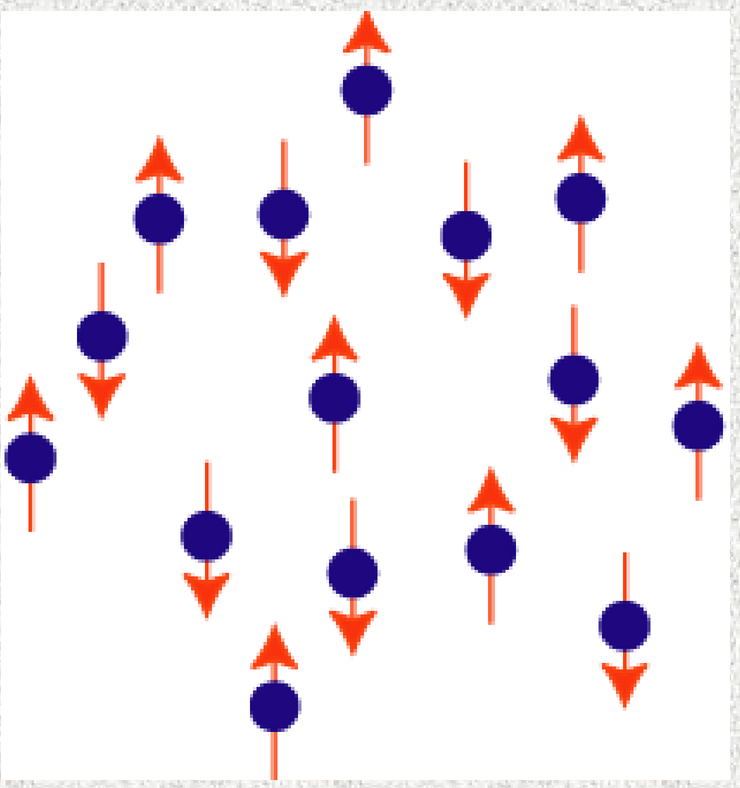
TQM Tbilisi, June 1, 2019

Outline

- Spin currents in addition to electric currents;
- Interplay between spin and electric currents;
- Resistor vs. “spin resistor”. What’s the difference?
- Spin-orbit interaction changes the game.

Spin currents in metallic conductors

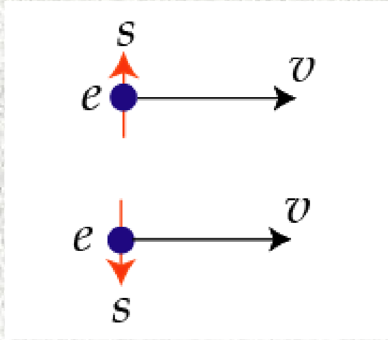
Electric and spin currents are carried by itinerant electrons



- Normal metals (Cu, etc.)
- Ferromagnets (Fe, Ni, etc.)
- Metals with spin-orbit interaction (Pt, etc)

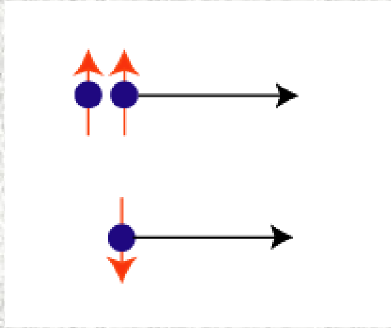
Electric and spin currents

Normal metal: pure electric current

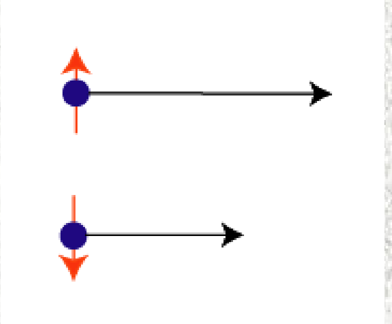


$j_i^e = e\langle v_i \rangle$	electric current
$s_\alpha = \langle \sigma_\alpha \rangle$	average spin
$j_{\alpha,i}^s = \langle \sigma_\alpha v_i \rangle$	spin current

Ferromagnetic metal: electric and spin currents

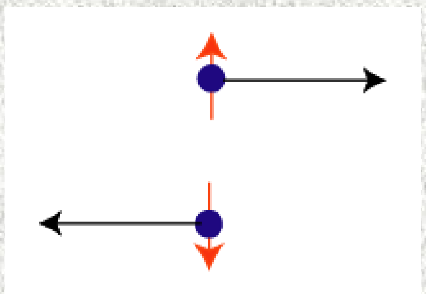


OR

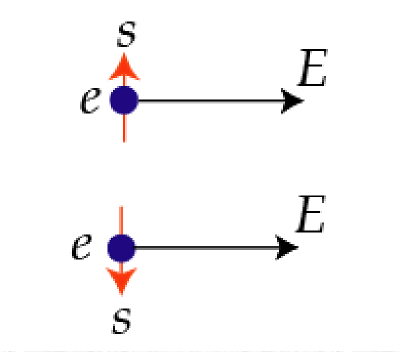


(or both)

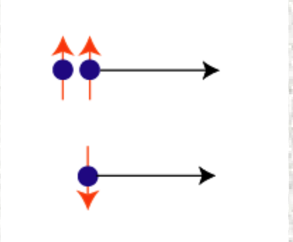
Pure spin current

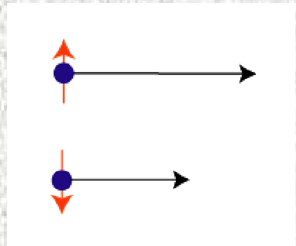


Driving currents with E field



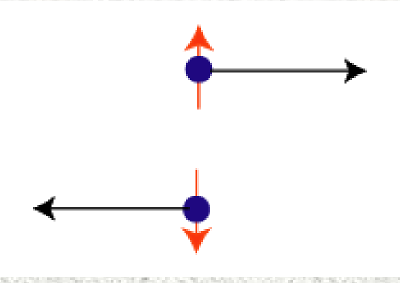
Normal metal: electric current, no spin current





Ferromagnet: both electric and spin current

"Current spin polarization" $p = \dfrac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$



How can one ever produce a pure spin current?
Or at least some spin current in a normal metal?

Diffusive model

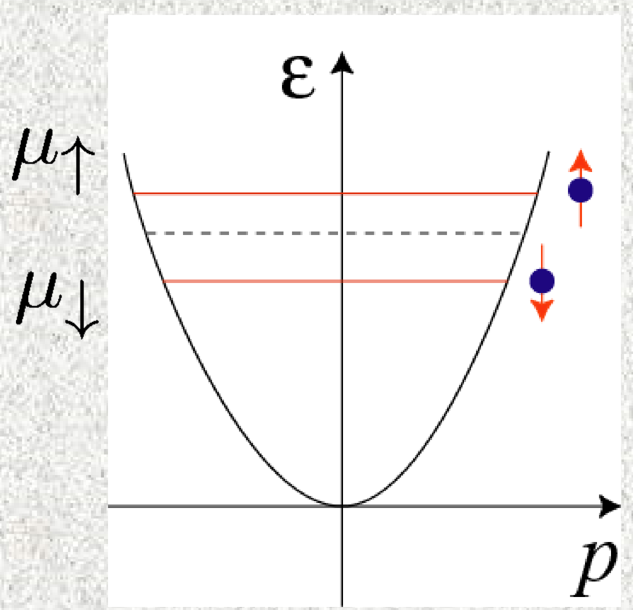
Campbell, Fert, and Pomeroy, Philos.Mag. **15**, 977 (1967)
Valet and Fert, Phys. Rev. B **48**, 7099 (1993).

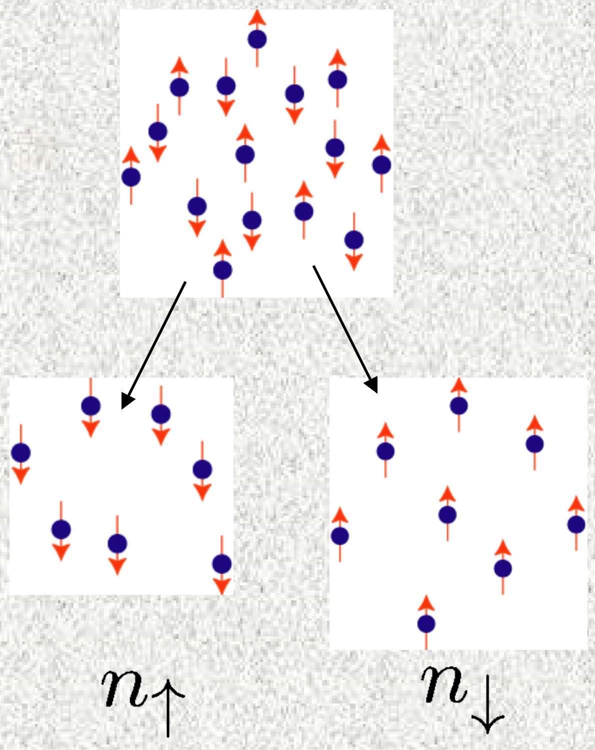
<u>Assumptions:</u>

a) Short mean free path of electrons => diffusive regime
b) Spin flip time is much longer than momentum relaxation time

$\uparrow$ and $\downarrow$ electrons form separate Fermi gases with slow equilibration between the two

Metallic regime: $k_B T << \mu$

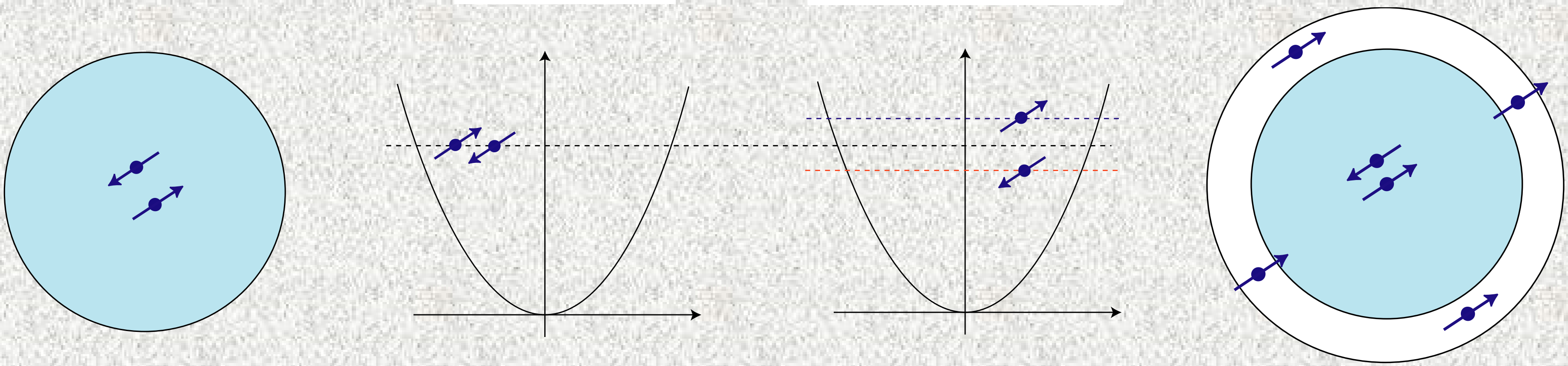




$$n_\uparrow + n_\uparrow = n = const$$

Diffusive regime

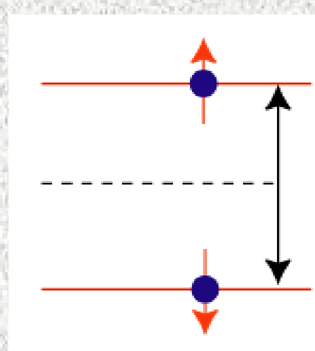
$$\tau_p \ll \tau_s \qquad l_p \ll \lambda_s \sim L$$



Without spin accumulation

With spin accumulation

$$\frac{\mu_\uparrow + \mu_\downarrow}{2} = \mu$$

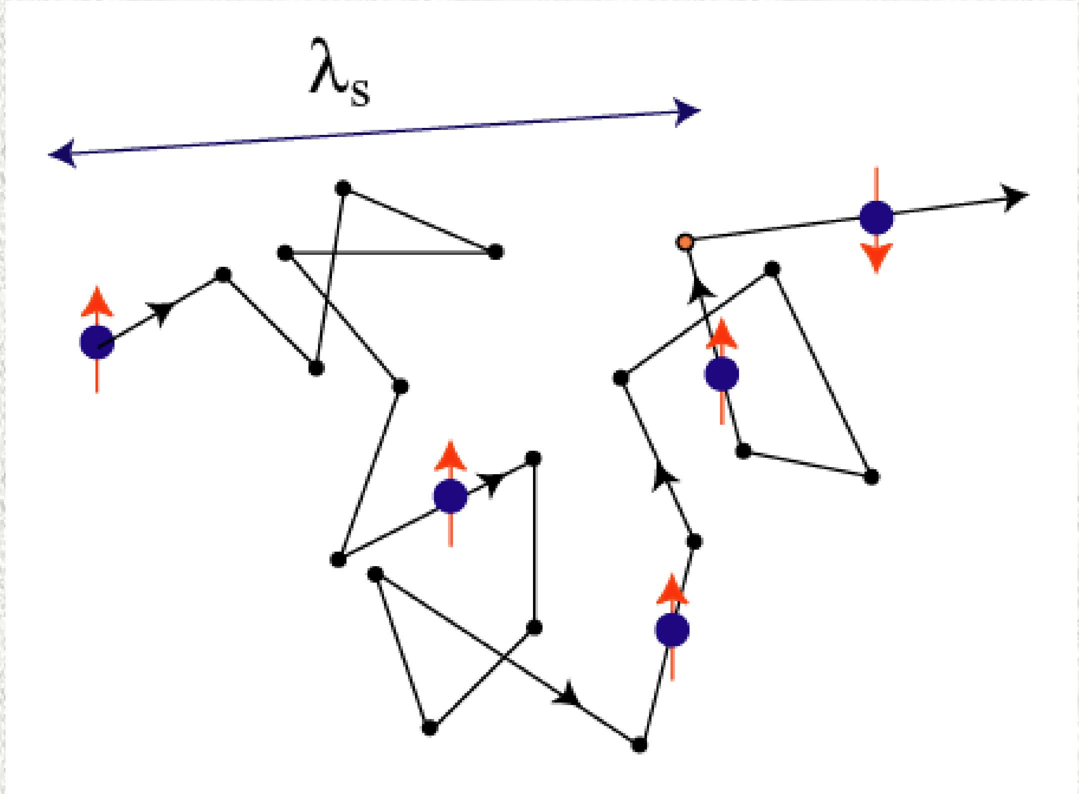


$$\mu_s = \mu_\uparrow - \mu_\downarrow$$

voltage

non-equilibrium spin accumulation

Spin diffusion length



Coupled diffusion equations

$$\begin{cases} j &= -\dfrac{\sigma}{e^2}\left[\nabla\mu + p\left(\dfrac{\nabla\mu_s}{2}\right)\right] \\ j_s &= -\dfrac{\sigma}{e^2}\left[\left(\dfrac{\nabla\mu_s}{2}\right) + p\nabla\mu\right] \end{cases}$$

$$\boxed{p = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}}$$

Time-dependent flow

$$\begin{cases} \dot{n} + \mathrm{div} j = 0 \\ \dot{s} + \mathrm{div} j_s = -\dfrac{s}{\tau_s} \end{cases}$$

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Spin relaxation

Coupled diffusion equations

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$$\boxed{p = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}}$$

Time-dependent flow

$$\begin{cases} \dot{n} + \mathrm{div} j = 0 \\ \dot{s} + \mathrm{div} j_s = -\dfrac{s}{\tau_s} \end{cases} \Rightarrow$$

Stationary flow

$$\begin{cases} \mathrm{div} j = 0 \\ \mathrm{div} j_s = -\dfrac{s}{\tau_s} \end{cases}$$

Spin relaxation

Coupled diffusion equations

$$\begin{cases} j &= -\dfrac{\sigma}{e^2}\left[\nabla\mu + p\left(\dfrac{\nabla\mu_s}{2}\right)\right] \\ j_s &= -\dfrac{\sigma}{e^2}\left[\left(\dfrac{\nabla\mu_s}{2}\right) + p\nabla\mu\right] \end{cases}$$

$$\boxed{p = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}}$$

$$\Rightarrow \begin{cases} \Delta\mu = 0 \\ \Delta\mu_s = \dfrac{\mu_s}{\lambda_s^2} \end{cases}$$

Time-dependent flow

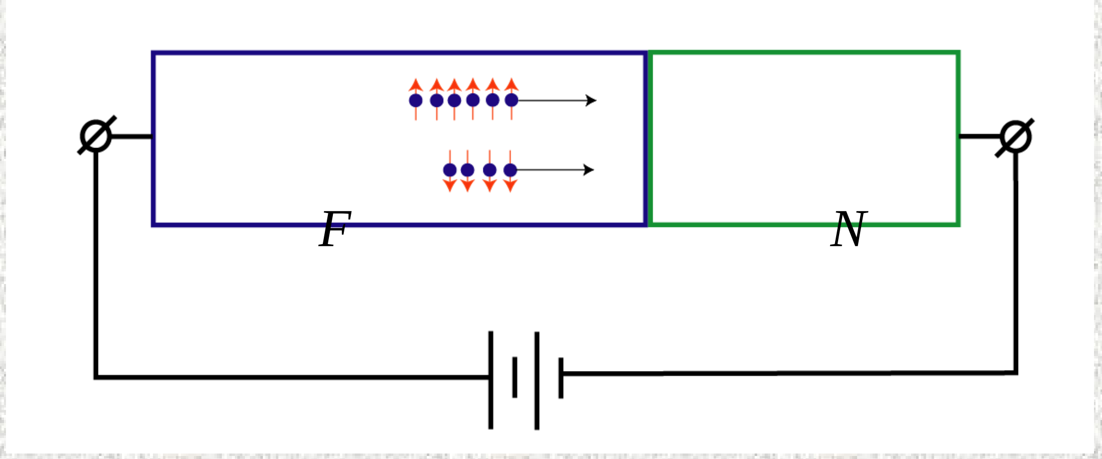
$$\begin{cases} \dot{n} + \mathrm{div} j = 0 \\ \dot{s} + \mathrm{div} j_s = -\dfrac{s}{\tau_s} \end{cases}$$

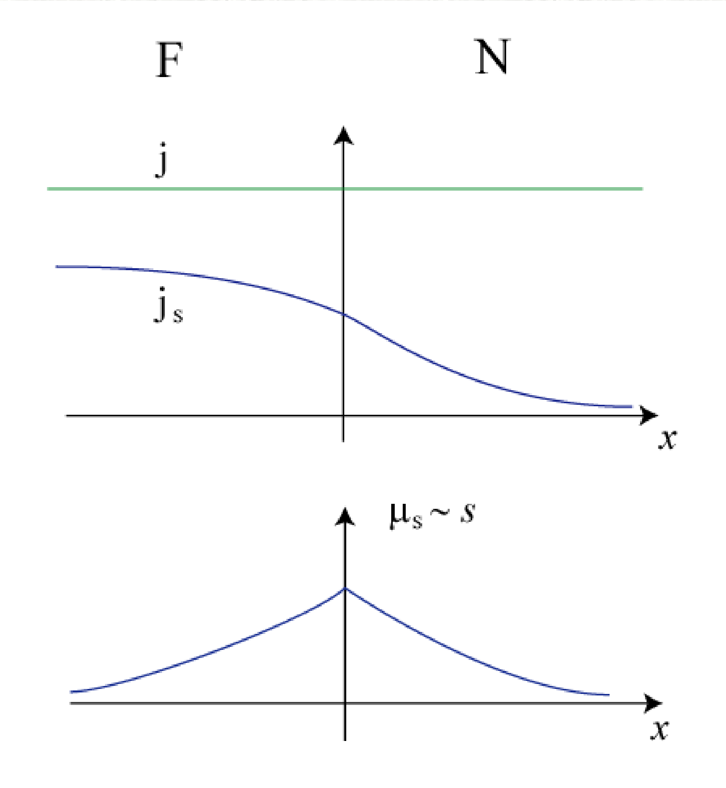
Stationary flow

$$\Rightarrow \begin{cases} \mathrm{div} j = 0 \\ \mathrm{div} j_s = -\dfrac{s}{\tau_s} \end{cases}$$

Spin relaxation

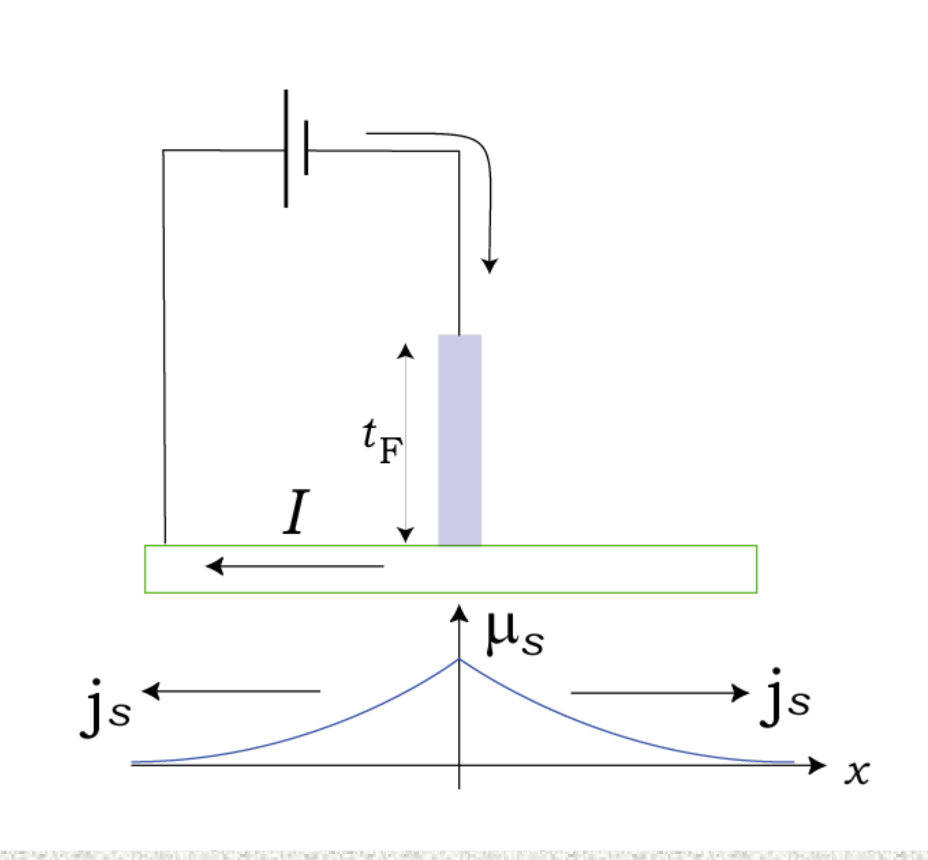
Injection from a ferromagnet





Electric and spin currents in diffusion regime

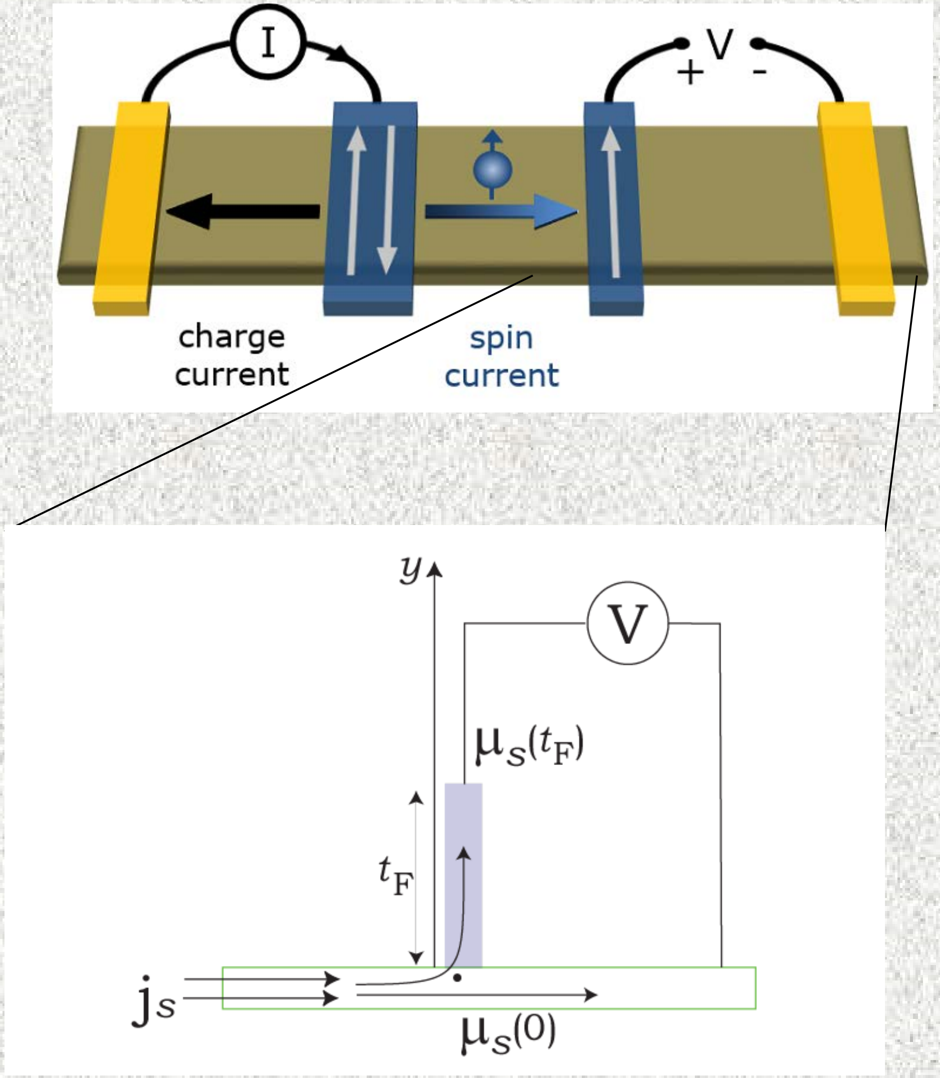
Point injection produces pure spin current



<u>Electric current</u> completes the circuit; no I in dangling part

<u>Spin current</u> diffuses symmetrically in both directions

Non-local spin injection



"Non-local resistance"

M. Johnson and R. H. Silsbee, PRL **55**, 1790 (1985) .

- Diffusive regime
- Slow spin relaxation (Valet-Fert)
- collinear magnetization

$$\mu_s(t_F) \to 0$$

$$\boxed{V = \frac{p\mu_s(0)}{2e}}$$

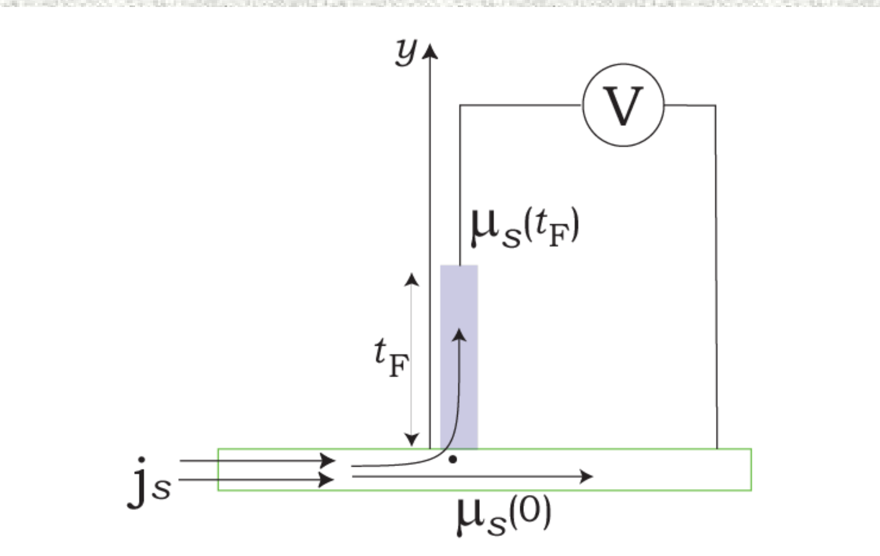
Johnson-Silsbee formula

Johnson-Silsbee formula from diffusive equations

$$\begin{cases} j = -\dfrac{\sigma}{e^2}\left[\nabla\mu + p\left(\dfrac{\nabla\mu_s}{2}\right)\right] \\ j_s = -\dfrac{\sigma}{e^2}\left[\left(\dfrac{\nabla\mu_s}{2}\right) + p\nabla\mu\right] \end{cases}$$

effective EMF developed in a ferromagnet

$$V = \frac{\mu(t_F) - \mu(0)}{e} = \frac{1}{e}\int_0^{t_F} \nabla\mu \, dr$$

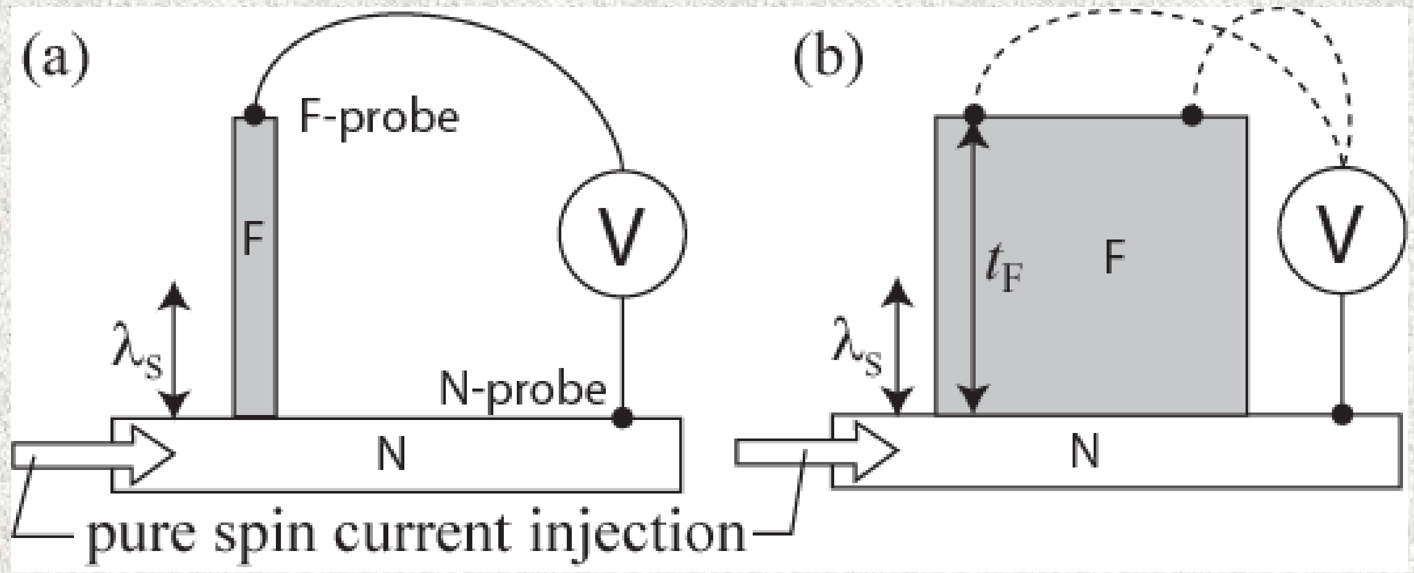


$$j = 0 \;\Rightarrow\; V = p\left(\frac{\mu_s(0)}{2e} - \frac{\mu_s(t_F)}{2e}\right)$$

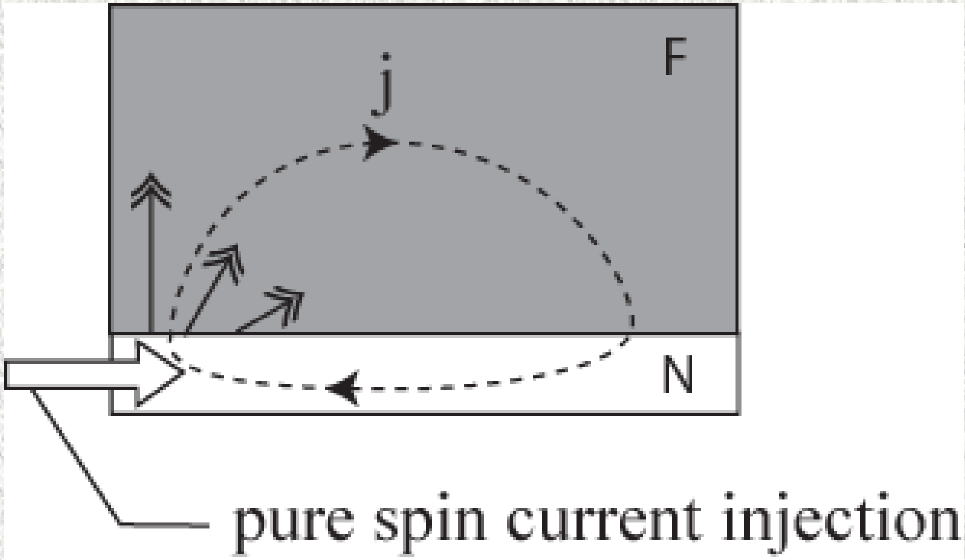
$$\mu_s(t_F) = 0 \;\Rightarrow\; V = \boxed{\frac{p\mu_s(0)}{2e}}$$

Problem of a wide measuring contact

When μ_s is variable, which μ_s should be substituted into $V = \dfrac{p\mu_s(0)}{2e}$?



Generation of loop electric currents



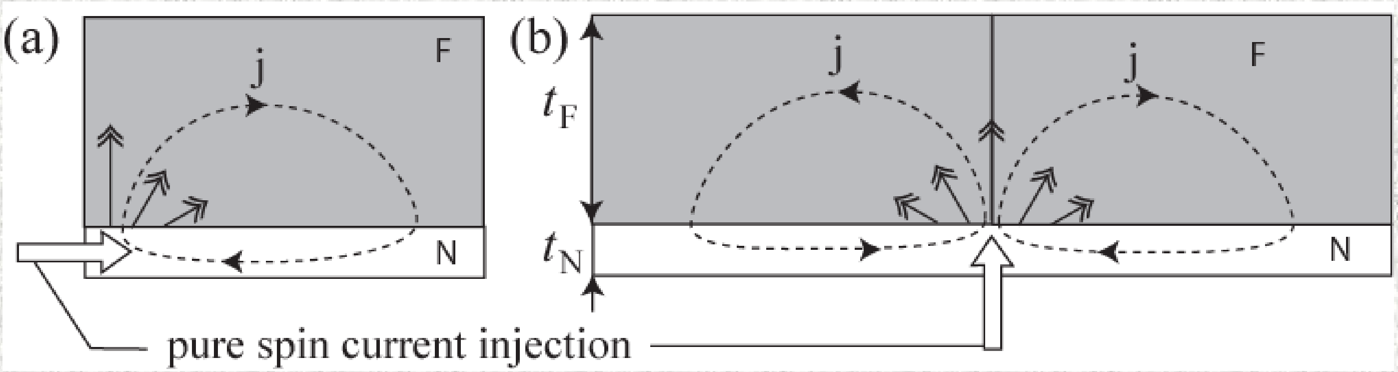
$$\mathrm{curl}\left(\frac{\mathbf{j}}{\sigma}\right) = \frac{1}{2}\nabla p \times \nabla \mu^s \ .$$

Theorem:
For $\nabla p \times \nabla \mu^s \neq 0$ circular electric current is generated in the F/N structure

Physics: EMF is concentrated near the spin injection point. The circuit is completed through the bulk.

Consequence #1

Voltage depends on the thickness of the F-layer, and on the measurement point



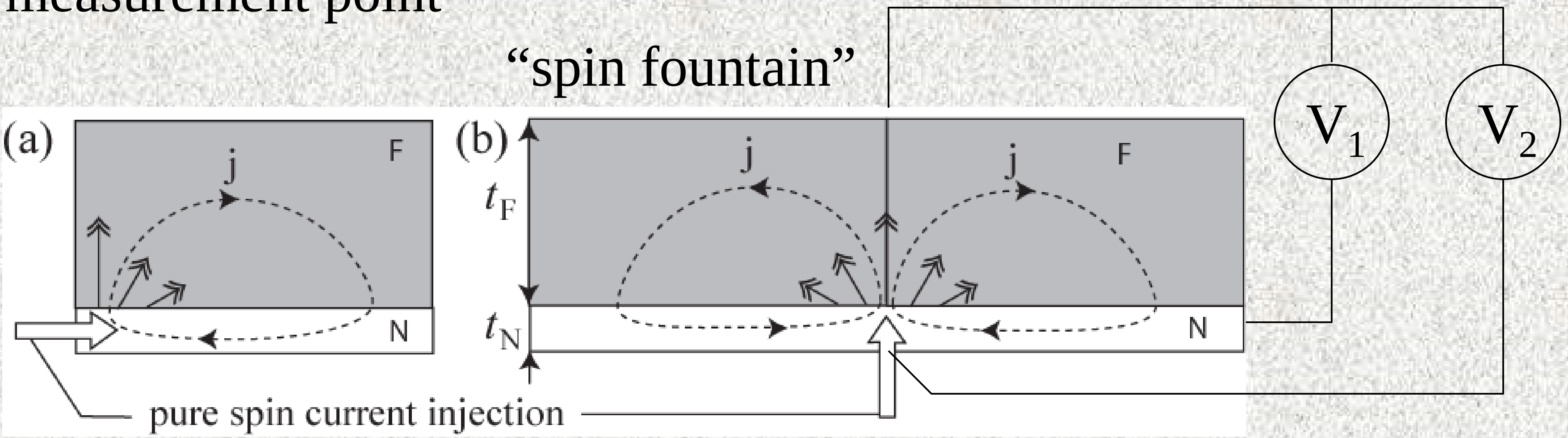
"spin fountain"

Bazaliy, Ramazashvili, APL **110**, 092405 (2017)

Consequence #1

Voltage depends on the thickness of the F-layer, and on the measurement point

“spin fountain”



Consequence #1

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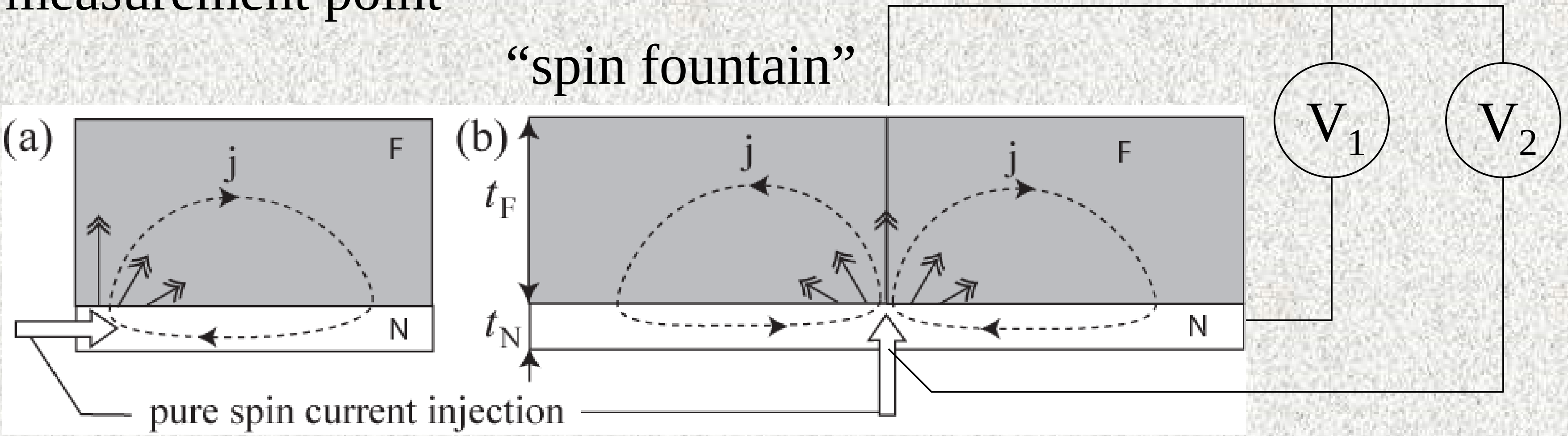
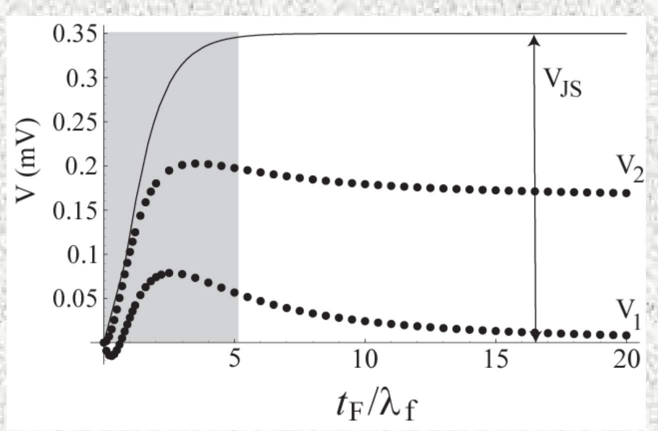
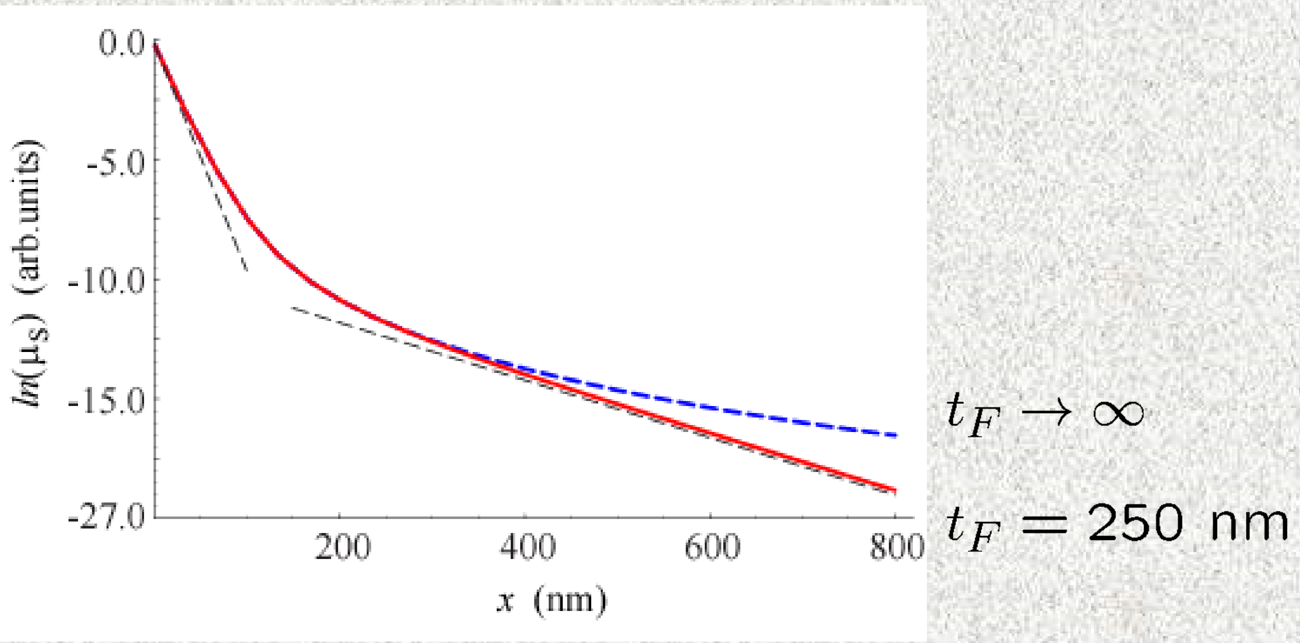




Consequence #2

How far do the spins propagate along the F/N interface?

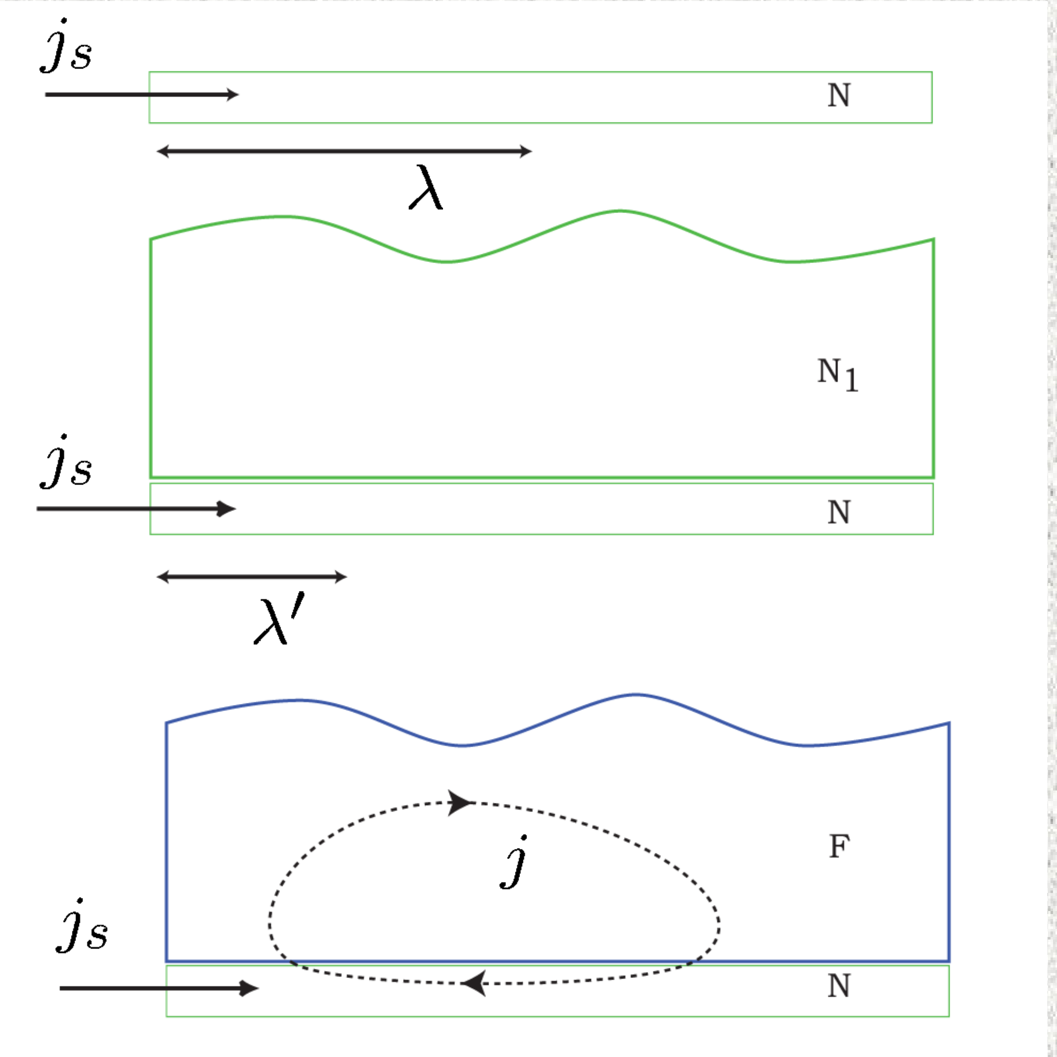


Bazaliy, Ramazashvili, APL **110**, 092405 (2017)

Long-range propagation in finite-thickness overlayers

$$\lambda_c = \frac{t_F}{\pi}\left(1 + \frac{R t_N}{t_F} + \dots\right)$$

Long range spin propagation



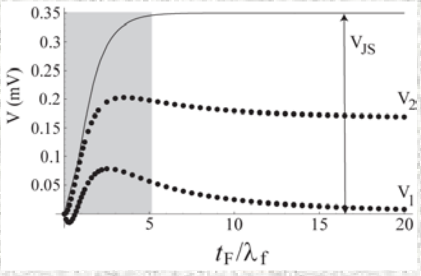
$\mu_s(x) \sim \exp(-x/\lambda)\ (x \to \infty)$

$\mu_s(x) \sim \exp(-x/\lambda')\ (x \to \infty)$

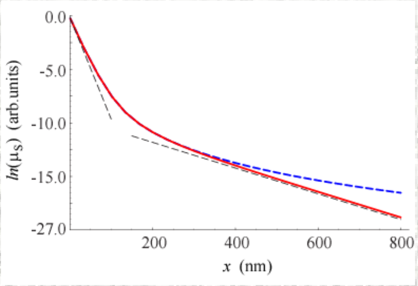
$\mu_s(x) \sim x^{-4}\ (x \to \infty)$

Conclusions-I

1. Pure spin currents injected into electrically "floating" regions produce loop electric currents



2. These currents lead to long-range spin propagation along the F/N interface



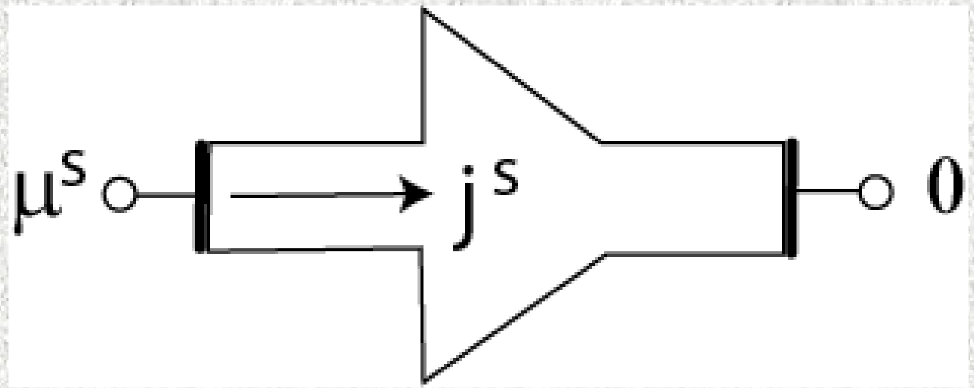
Bazaliy, Ramazashvili, APL **110**, 092405 (2017)

Amplification of pure spin current?

R. M. Abdullah *et al.*, “Spin-current signal amplification by geometrical ratchet”,

J. Phys. D: Appl. Phys. **47**, 482001 (2014);

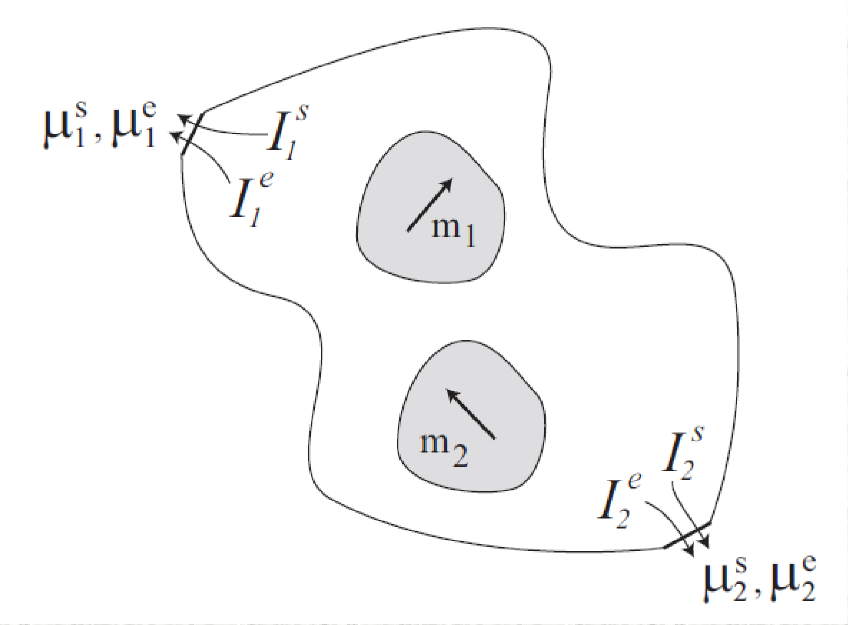
R. M. Abdullah *et al.*, “Optimisation of geometrical ratchets for spin-current amplification”,
J. Appl. Phys. **117**, 17C737 (2015).



Spins are injected into a normal metal element and propagate to the other terminal.

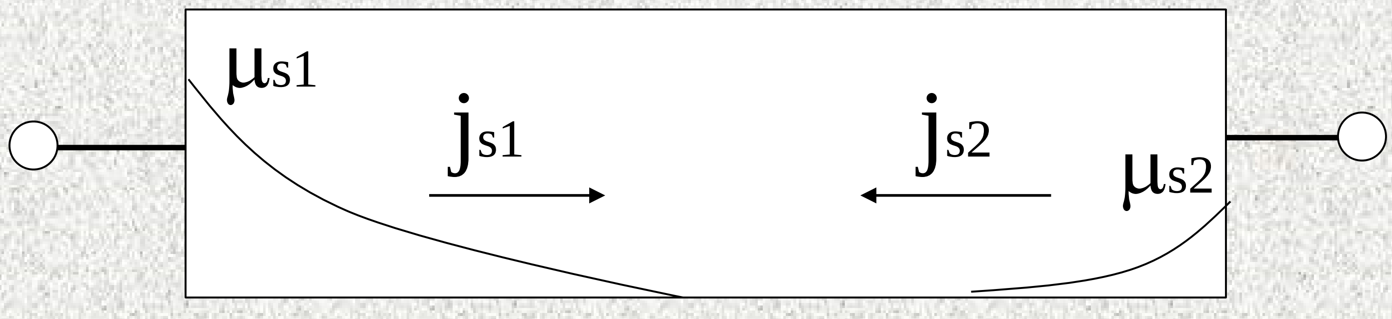
<u>Claim</u>: Geometrical directionality of the conductor’s shape enhances spin current flowing along the arrow

General lumped element



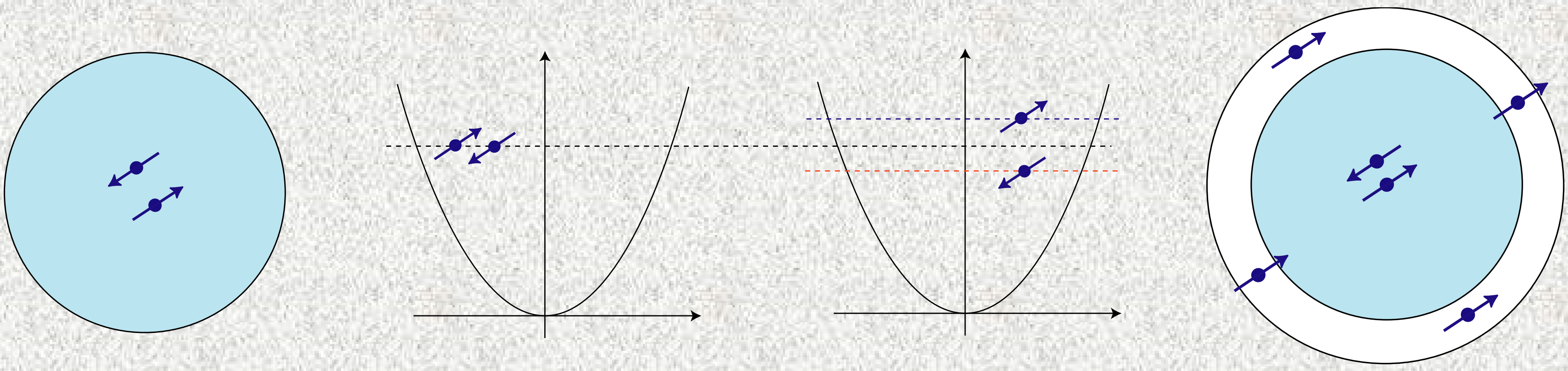
- Spin currents at two terminals are not the same

- Spin potentials do not enter current expressions as differences



$$J_s = J_s(\mu_{s1}, \mu_{s2}, V_1 - V_2)$$

Spin potential is a vector

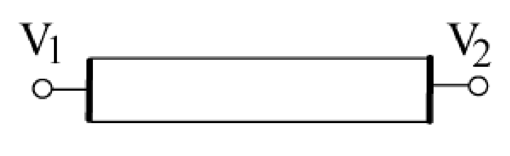


Spin potential has information about energy difference AND the direction of unpaired spins

$$\vec{\mu}_s = (\mu^{sx}, \mu^{sy}, \mu^{sz})$$

Lumped element description for non-conserved currents

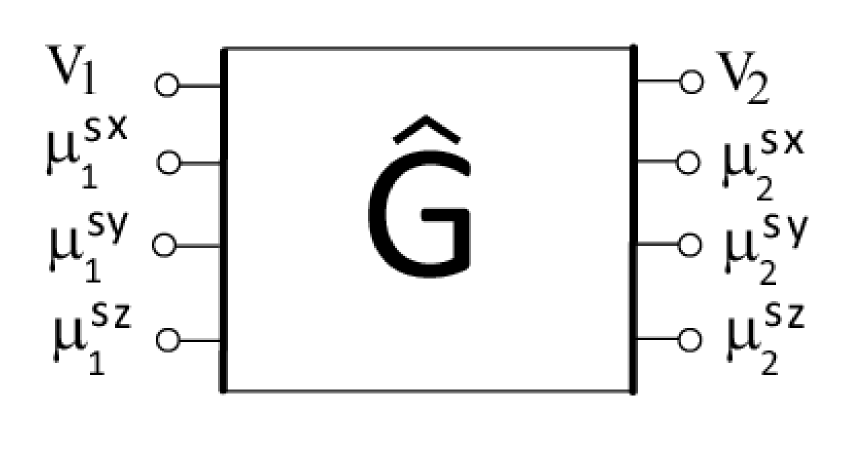
Electronics element



$$R, \ G = 1/R$$

$$I = G(V_1 - V_2)$$

Spintronics element

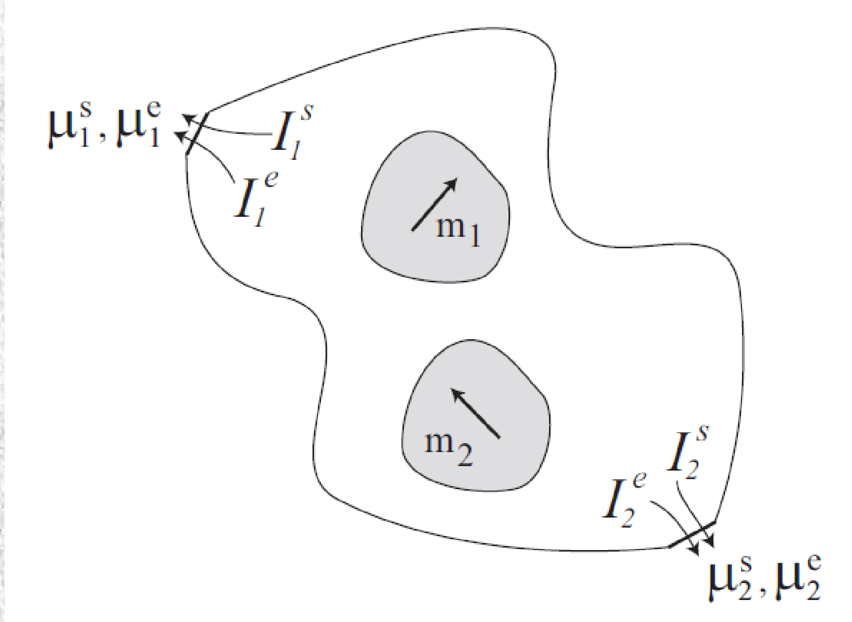


7 potentials $\{(V_1 - V_2), \mu_1^{sx}, \mu_2^{sx}, \mu_1^{sy}, \mu_2^{sy}, \mu_1^{sz}, \mu_2^{sz}\}$

7 x 7 matrix $I^a = G^{ab} \ \mu^b$

7 currents $\{I, I_1^{sx}, I_2^{sx}, I_1^{sy}, I_2^{sy}, I_1^{sz}, I_2^{sz}\}$

Diffusive elements



Bulk diffusive equations

$$j_i^e = -\frac{\sigma}{e^2}\left(\nabla_i \mu^e + \frac{1}{2} p \nabla_i \mu^s\right),$$

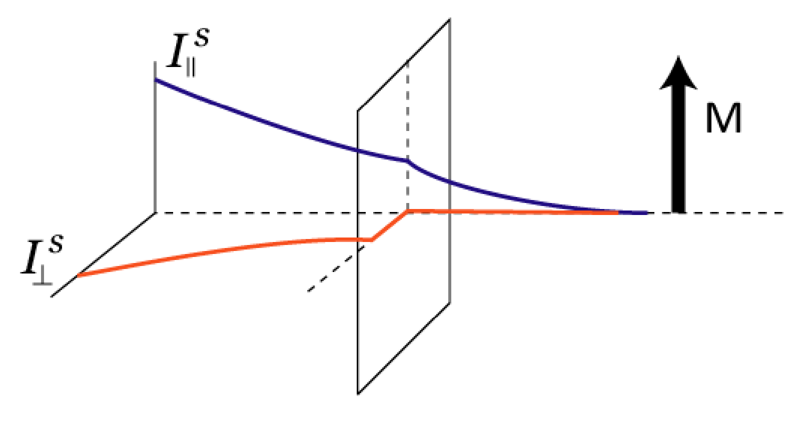
$$j_i^{s\alpha} = -\frac{m^\alpha \sigma}{e^2}\left(\frac{1}{2}\nabla_i \mu^s + p \nabla_i \mu^e\right)$$

Boundary conditions with strong ferromagnets

$$\mu^a(N)|_S = \mu^a(F)|_S$$

$$j_i^e(N) n_i\big|_S = j_i^e(F) n_i\big|_S,$$

$$m^\alpha j_i^{s\alpha}(N) n_i\big|_S = m^\alpha j_i^{s\alpha}(F) n_i\big|_S,$$



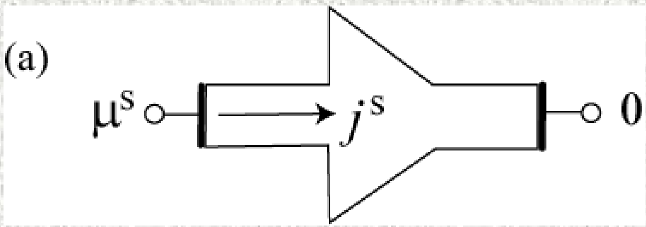
Reciprocity theorem for diffusive elements

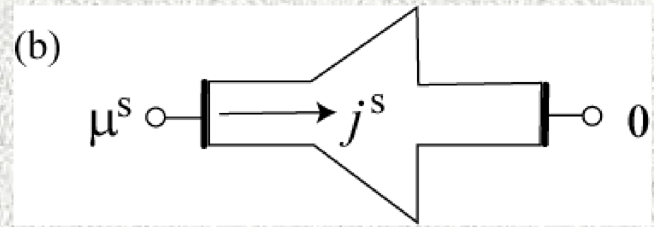
$$\begin{pmatrix} I \\ \mathbf{I}^s \end{pmatrix} = \begin{pmatrix} G & \underset{(1\text{x}6)}{C} \\ \underset{(6\text{x}1)}{C^T} & \underset{(6\text{x}6)}{S = S^T} \end{pmatrix} \begin{pmatrix} (V_1 - V_2) \\ \boldsymbol{\mu}^s \end{pmatrix}$$

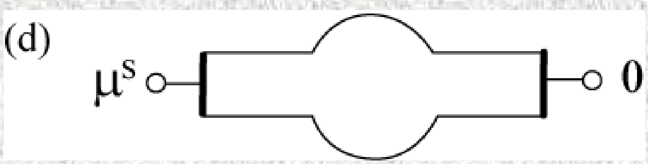
Spin-electric conductance matrix is symmetric

(28 independent parameters)

No geometric spin ratchets

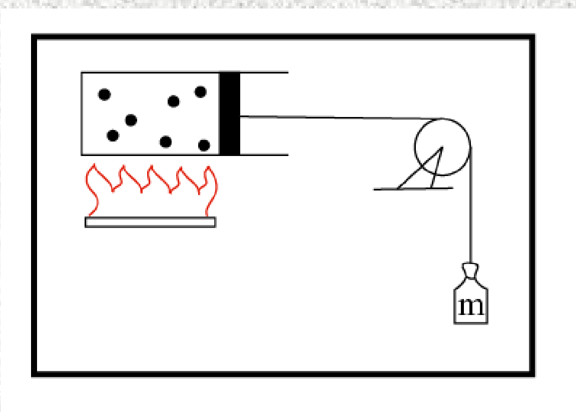






- Abdullah *et al.* were comparing (a) and (c).
- Yet (a) and (b) have identical spin conductance. The arrow of opposite direction “enhances” spin conductance by the same measure.
- Case (d) may have even larger spin conductance.

Onsager reciprocity



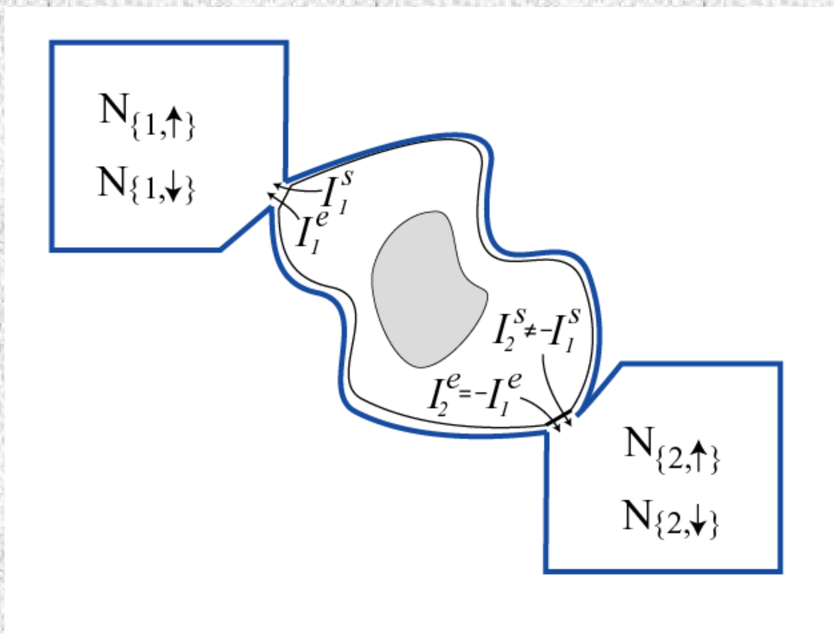
x_i — thermodynamic variables

$$S = S_0 - \frac{1}{2} A_{ij} x_i x_j.$$

$Y_i = A_{ij} x_j$ — thermodynamic forces

$$\dot{x}_i = -\Gamma_{ij} x_j = -\gamma_{ij} Y_j$$

Time-reversal invariance $\Rightarrow \ \gamma_{ji} = \pm \gamma_{ij}$



$$S = S_0 - A_{t\sigma;t'\sigma'} \, \delta N_{t\sigma} \, \delta N_{t'\sigma'}$$

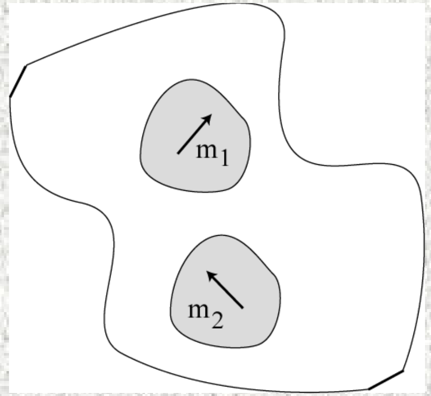
Can prove that forces are chemical potentials

$$\delta \dot{N}_{t\sigma} = -\gamma_{t\sigma;t'\sigma'} \ \mu_{t'\sigma'}$$

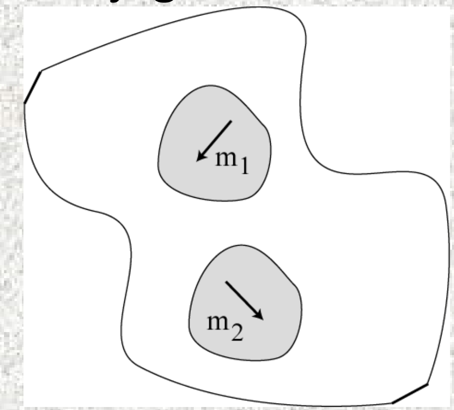
$$\delta \dot{N}_{t\sigma} = I_{t\sigma}$$

Diffusive reciprocity vs. Onsager reciprocity

Element



Conjugated element



$$\hat{G}(M_i) = \begin{pmatrix} G & \mathbf{C} \\ \mathbf{D}^T & \hat{S} \end{pmatrix} \quad \xleftrightarrow{\text{Onsager}} \quad \hat{G}(-M_i) = \begin{pmatrix} G & -\mathbf{D} \\ -\mathbf{C}^T & \hat{S}^T \end{pmatrix}$$

Diffusive reciprocity

$$\hat{G}(M_i) = \begin{pmatrix} G & \mathbf{D} \\ \mathbf{C}^T & \hat{S}^T \end{pmatrix}$$

For $M = 0$

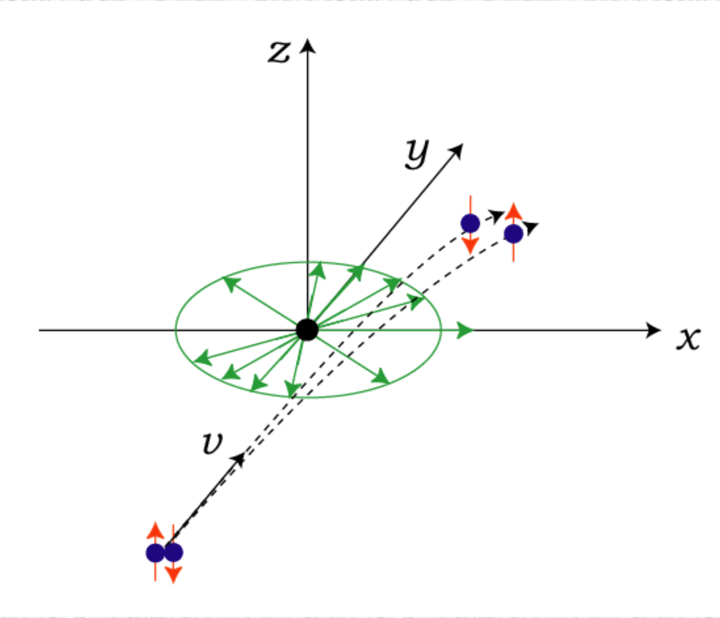
$$S_{ij} = S_{ji}$$

$$\mathbf{C} = \mathbf{D}^T \quad \xleftrightarrow{???} \quad \mathbf{C} = -\mathbf{D}^T$$

Resolution of the apparent paradox

Normal metal: $\mathbf{C} = \mathbf{D} = 0$ both diffusive and Onsager reciprocity relations are valid.

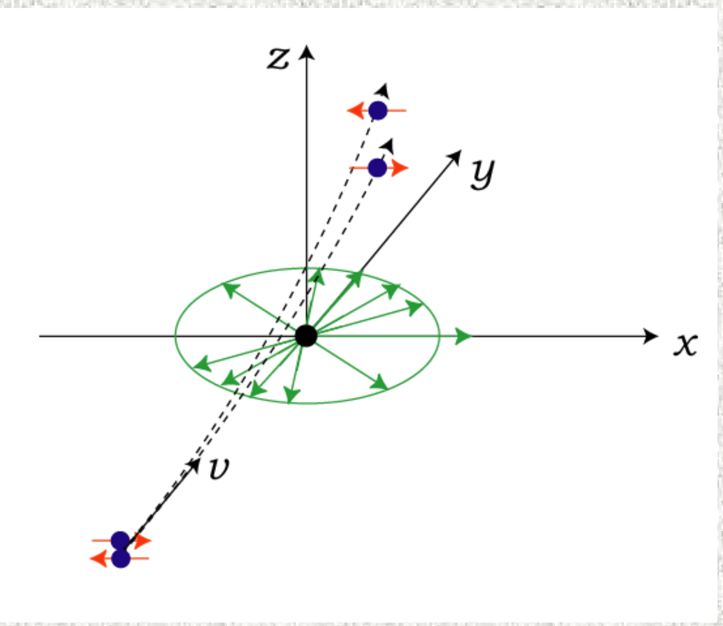
Limitations of diffusive reciprocity: Metal with spin-orbit interaction



$$H_{SO} = \frac{e\hbar}{4m^2c^2}\mathbf{s} \cdot [\mathbf{E} \times \mathbf{p}]$$

$$\mathbf{F} = -\nabla \langle H_{SO} \rangle$$

Particle current along *y* produces spin current along *x.*

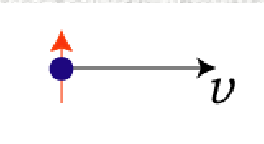


But you can look at it differently, and then the spin current flows along *z!*

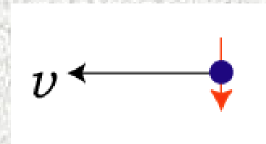
So where does the spin current flow?!

Spin current is a tensor

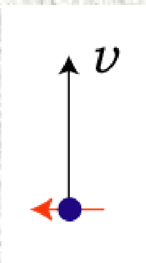
$$j_{\alpha i} = \langle s_\alpha v_i \rangle = \begin{pmatrix} s_x v_x & s_x v_y \\ s_y v_x & s_y v_y \end{pmatrix}$$



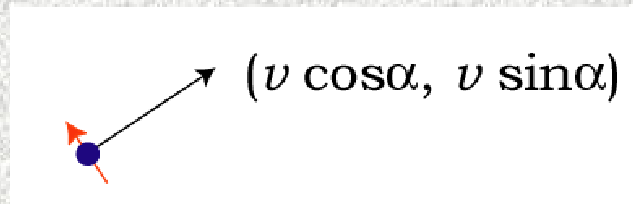
$$\delta j_{\alpha i} = v \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$



$$\delta j_{\alpha i} = v \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$



$$\delta j_{\alpha i} = v \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}$$



$$\delta j_{\alpha i} = v \begin{pmatrix} \sin\alpha\cos\alpha & -\sin^2\alpha \\ \cos^2\alpha & -\sin\alpha\cos\alpha \end{pmatrix}$$

After averaging over all possible incoming spin directions

$$\boxed{j_{\alpha i} = \begin{pmatrix} 0 & -J \\ J & 0 \end{pmatrix}}$$

Diffusion equations in Pt

$$j_i^e = -\frac{\sigma}{e^2}\left(\nabla_i\mu^e + \alpha\epsilon_{ij\alpha}\nabla_j\mu^{s\alpha}\right)$$

$$j_i^{s\alpha} = -\frac{\sigma}{e^2}\left(\nabla_i\mu^{s\alpha} \underbrace{- \alpha\epsilon_{i\alpha j}\nabla_j\mu^e}\right)$$

Equations are now coupled by spin-orbit interaction

As a result of equation coupling, the proof of diffusive reciprocity breaks down. Yet starting from new equations it is possible to prove the Onsager reciprocity.

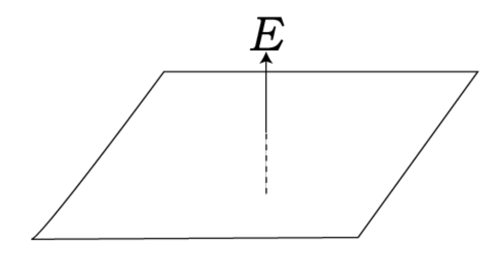
$$\hat{G}(M_i) = \begin{pmatrix} G & \mathbf{C} \\ \mathbf{D}^T & \hat{S} \end{pmatrix}$$

For $M = 0$

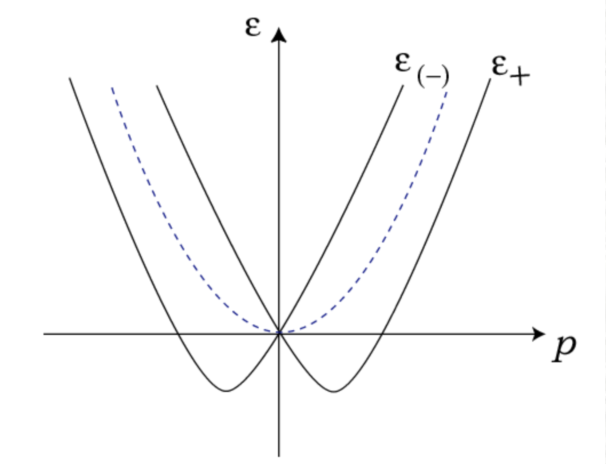
$$\mathbf{C} = -\mathbf{D}^T$$

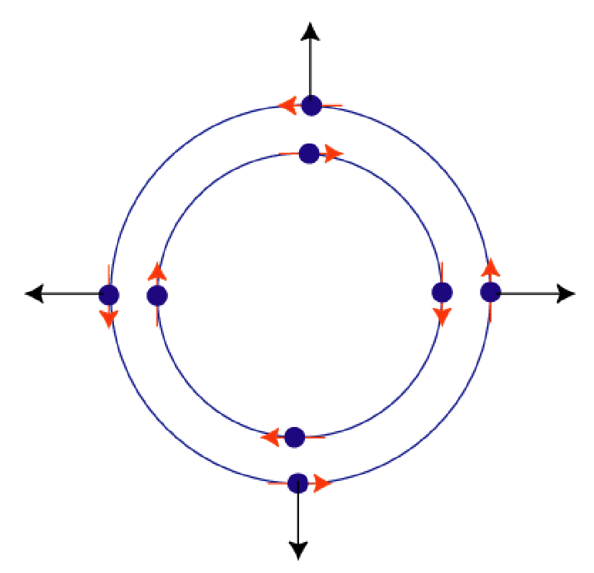
$S_{ij} = S_{ji}$ $\Rightarrow$ "Spin ratchets" are still impossible in Pt wires

Spin current in Rashba 2DEG



$$H_{SO} = \alpha[\mathbf{s} \times \mathbf{p}] \cdot \hat{z}$$





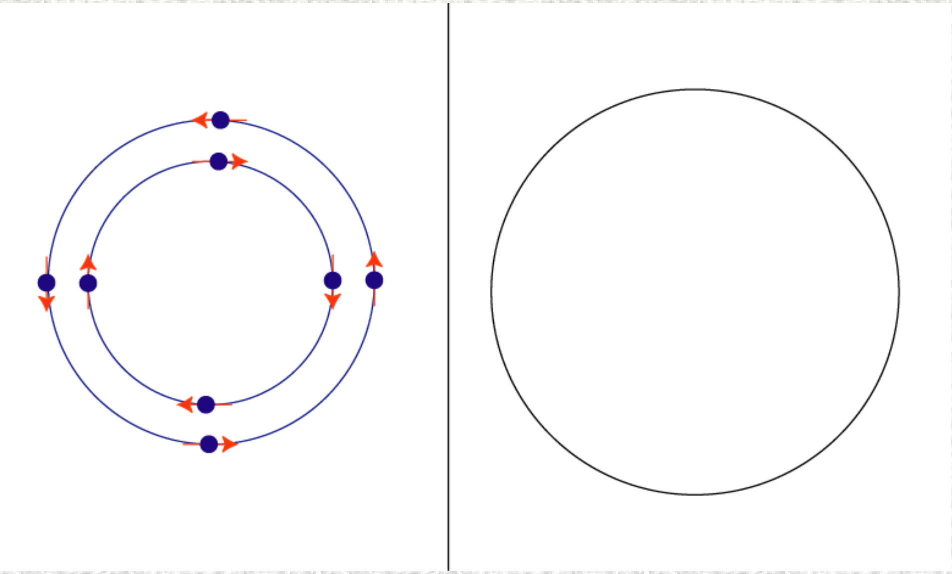
$$j_+ = \begin{pmatrix} 0 & -J_+ \\ J_+ & 0 \end{pmatrix}$$

$$j_{(-)} = \begin{pmatrix} 0 & -J_{(-)} \\ J_{(-)} & 0 \end{pmatrix}$$

$$j_{tot} = \begin{pmatrix} 0 & -J \\ J & 0 \end{pmatrix} \neq 0$$

In 2DEG with Rashba spin-orbit interaction there is a spin current in equilibrium. What can it mean? Is the definition of this current even meaningful?

Can equilibrium spin current be used in a "perpetual spin source"?



Rashba SO metal | **Normal metal**

$$J^s_{\alpha i} \neq 0 \qquad J^s_{\alpha i} = 0$$

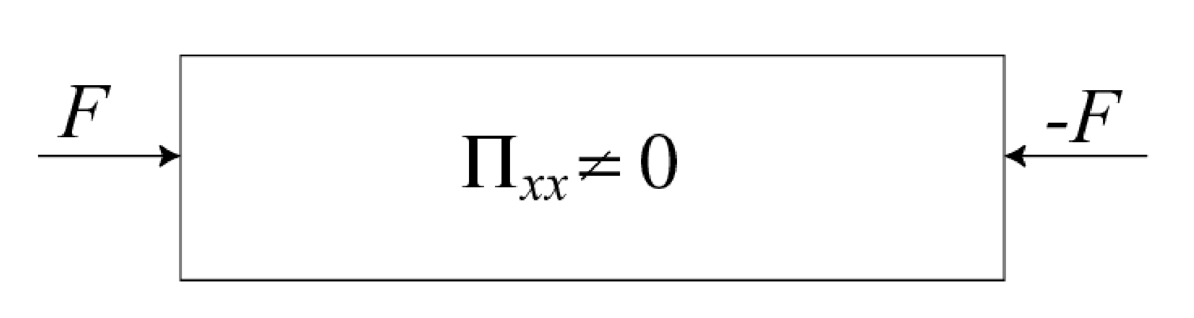
In the presence of spin-orbit interactions, spins can rotate upon reflection. Spin current does not have to be continuous on the boundary.

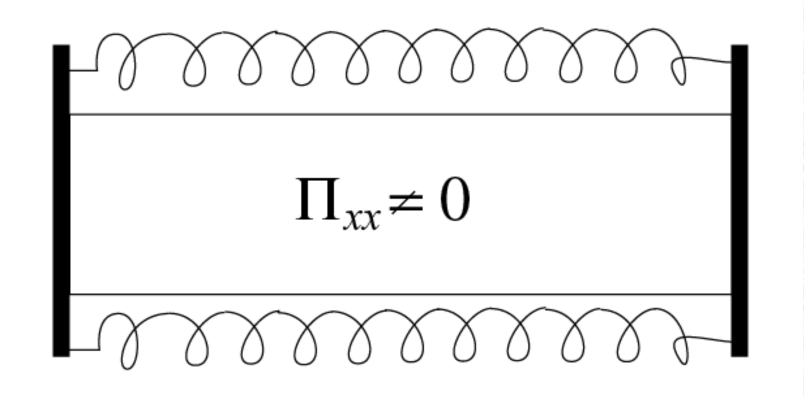
$$\Downarrow$$

Rashba SO mechanically twists the film

Meaning of equilibrium spin currents

In mechanics, compressed rod in equilibrium carries a momentum flux





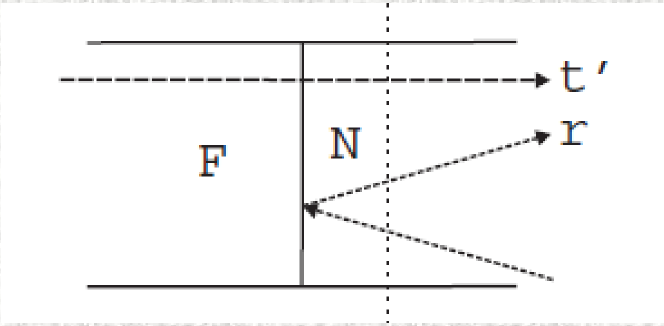
A hypothetical system above develops equilibrium momentum flux if the equilibrium length of the springs can be changed and made smaller than the length of the rod. Spring length here is the analogy of SO interaction.

Diffusive reciprocity and circuit theory

Eur. Phys. J. B **22**, 99–110 (2001)

Spin-transport in multi-terminal normal metal-ferromagnet systems with non-collinear magnetizations

A. Brataas[1,a], Y.V. Nazarov[2], and G.E.W. Bauer[2]



Landauer-Buttiker formula for conductance

$$G^{\uparrow} = \frac{e^2}{h}\left[M - \sum_{nm} |r_{\uparrow}^{nm}|^2\right] = \frac{e^2}{h}\sum_{nm} |t_{\uparrow}^{nm}|^2 ,$$

$$G^{\downarrow} = \frac{e^2}{h}\left[M - \sum_{nm} |r_{\downarrow}^{nm}|^2\right] = \frac{e^2}{h}\sum_{nm} |t_{\downarrow}^{nm}|^2$$

Partial conductances for up/down spins

$$G^{\uparrow\downarrow} = \frac{e^2}{h}\left[M - \sum_{nm} r_{\uparrow}^{nm} (r_{\downarrow}^{nm})^*\right] .$$

“mixing conductance”

$$j_i = \langle \psi_s^{\dagger} \hat{v}_i \psi_s \rangle$$

$$j_i^{s\alpha} = \langle \psi_s^{\dagger} \hat{v}_i \hat{\sigma}_{ss'}^{\alpha} \psi_{s'} \rangle$$

Diffusive

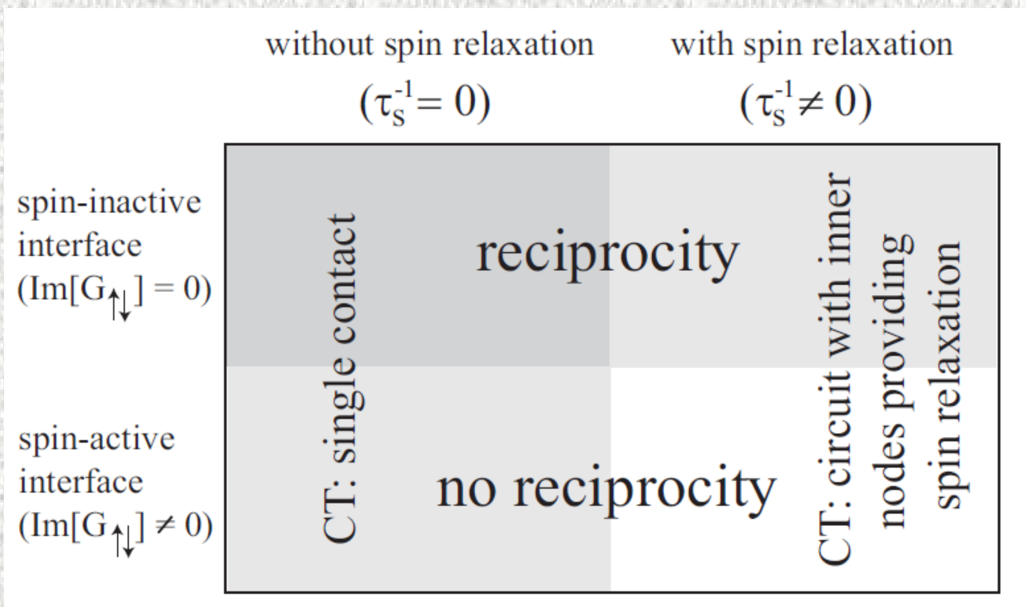
$$G^{ab}_{NN} = \begin{vmatrix} -G & C^x_N & C^y_N & C^z_N \\ C^x_N & S^x_N & S^{xy}_N & S^{xz}_N \\ C^y_N & S^{xy}_N & S^y_N & S^{yz}_N \\ C^z_N & S^{xz}_N & S^{yz}_N & S^z_N \end{vmatrix}$$

10 parameters

Circuit theory

$$\hat{\mathcal{G}} = -\begin{vmatrix} G_\uparrow + G_\downarrow & 0 & 0 & G_\uparrow - G_\downarrow \\ 0 & 2Re[G_{\uparrow\downarrow}] & 2Im[G_{\uparrow\downarrow}] & 0 \\ 0 & -2Im[G_{\uparrow\downarrow}] & 2Re[G_{\uparrow\downarrow}] & 0 \\ G_\uparrow - G_\downarrow & 0 & 0 & G_\uparrow + G_\downarrow \end{vmatrix}$$

4 parameters



Diffusive

$$G^{ab}_{NN} = \begin{vmatrix} -G & C^x_N & C^y_N & C^z_N \\ C^x_N & S^x_N & S^{xy}_N & S^{xz}_N \\ C^y_N & S^{xy}_N & S^y_N & S^{yz}_N \\ C^z_N & S^{xz}_N & S^{yz}_N & S^z_N \end{vmatrix}$$

Circuit theory

$$\hat{\mathcal{G}} = -\begin{vmatrix} G_\uparrow + G_\downarrow & 0 & 0 & G_\uparrow - G_\downarrow \\ 0 & 2Re[G_{\uparrow\downarrow}] & 2Im[G_{\uparrow\downarrow}] & 0 \\ 0 & -2Im[G_{\uparrow\downarrow}] & 2Re[G_{\uparrow\downarrow}] & 0 \\ G_\uparrow - G_\downarrow & 0 & 0 & G_\uparrow + G_\downarrow \end{vmatrix}$$

Question: Is Onsager reciprocity violated in a coherent contact with non-zero Im(Gm)?

Diffusive

$$G^{ab}_{NN} = \begin{vmatrix} -G & C^x_N & C^y_N & C^z_N \\ C^x_N & S^x_N & S^{xy}_N & S^{xz}_N \\ C^y_N & S^{xy}_N & S^y_N & S^{yz}_N \\ C^z_N & S^{xz}_N & S^{yz}_N & S^z_N \end{vmatrix}$$

Circuit theory

$$\hat{\mathcal{G}} = - \begin{vmatrix} G_\uparrow + G_\downarrow & 0 & 0 & G_\uparrow - G_\downarrow \\ 0 & 2Re[G_{\uparrow\downarrow}] & 2Im[G_{\uparrow\downarrow}] & 0 \\ 0 & -2Im[G_{\uparrow\downarrow}] & 2Re[G_{\uparrow\downarrow}] & 0 \\ G_\uparrow - G_\downarrow & 0 & 0 & G_\uparrow + G_\downarrow \end{vmatrix}$$

Question: Is Onsager reciprocity violated in a coherent contact with non-zero Im(Gm)?

Answer: Since ferromagnets are involved, Onsager relations connect different system (M and –M).

Conclusions-II

- Conductance matrix of diffusive elements is symmetric;
- This property is in addition to other symmetries of conductance matrix;
- “Directional geometry” does not influence spin transport;
- Bazaliy+Ramazashvili, PRB **99**, 184443 (2019)