# Geometry and Topology in 2d Chiral Liquids

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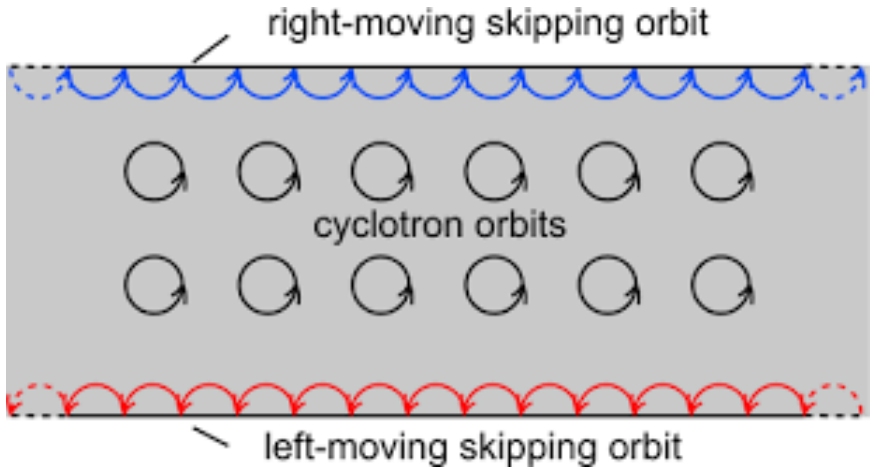
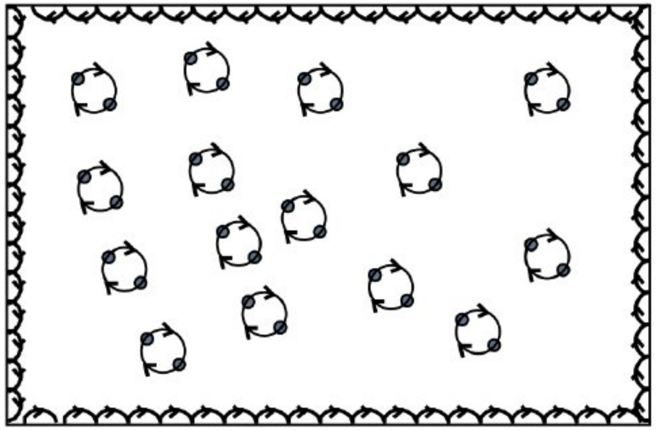
# Two two-dimensional Quantum Chiral Liquids




## Quantum Hall liquids:

Chirality due to the cyclotron
motion in the strong B field



## Chiral superconductors:

Chirality due to orbital angular
momentum of the Cooper pairs

**We shall study the response of these states to topology and geometry of the surface they live on, and in particular derive the "geo-Meißner effect", and solve a theoretical puzzle.**

# A simple QH liquid - the Laughlin state

There are many ways to describe the Laughlin states at filling fraction 1/k. One useful way is to use an effective field theory,

$$\mathscr{L}_{top} = -\frac{k}{4\pi}\varepsilon^{\mu\nu\sigma}b_\mu\partial_\nu b_\sigma - \frac{e}{2\pi}\varepsilon^{\mu\nu\sigma}A_\mu\partial_\nu b_\sigma - \frac{s}{2\pi}\omega_i\,\epsilon^{i\nu\sigma}\partial_\nu b_\sigma + b_\mu j^\mu$$

Wen & Zee, 1992

where $b$ describes the current and is referred to as a hydrodynamical field. The spin connection $\omega$ describes the geometry of the surface on which the liquid resides. This theory describes a *liquid state with quantized Hall conductivity and fractionally charged anyonic quasiparticles*:

$$\mathscr{L}_{eff}(A, j_\nu) = \underbrace{\frac{e^2}{4\pi k}}_{2\sigma_H}\epsilon^{\mu\nu\sigma}A_\mu\partial_\nu A_\sigma + \underbrace{\frac{e}{k}}_{e^\star}A_\mu j^\mu - \underbrace{\frac{\pi}{k}}_{\theta_s}j^\mu\left(\frac{1}{d}\right)_{\mu\nu}j^\nu$$

Note that Hall response implies chirality!!

The essence of the geometric response is described by,

$$\mathscr{L}_{eff}(A,\omega) = -\frac{1}{4\pi k}\epsilon^{\mu\nu\sigma}(eA_\mu + s\omega_\mu)\partial_\nu(eA_\sigma + s\omega_\sigma) + \dots$$

That $\omega$ comes in the combination $(eA_\sigma + s\omega_\sigma)$ will be important below!

# QH liquids — U(1) charge insulators

* Conserved U(1) electric current
* Gap to charged excitations

## Charge response:

$$W[A_\mu] = W_{CS}[A_\mu] = \frac{\nu e^2}{4\pi}\int d^3x\, \epsilon^{\mu\nu\sigma} A_\mu \partial_\nu A_\sigma \qquad \nu = p/q$$

- **Hall conductance is quantized:** $J^i = \frac{\delta W}{\delta A_i} = \frac{\nu e^2}{2\pi}\epsilon^{ij}E_i$

$$\rho = \frac{\delta W}{\delta A_0} = \frac{\nu e^2}{2\pi}\epsilon^{ij}\partial_i A_j = \frac{\nu e}{2\pi} eB$$ implies that

- **Flux is proportional to charge:** $N_Q = \nu N_\Phi$

## Effect of geometry:

$$W_{WZ}[A_\mu, \omega_\mu] = \frac{se}{2\pi}\int d^3x\, \epsilon^{\mu\nu\sigma}\omega_\mu \partial_\nu A_\sigma \quad , \quad \epsilon^{ij}\partial_i \omega_j = \sqrt{g}\, K$$

***K* is the Gaussian curvature of the surface**

**and *s* is the orbital spin of the electron**

Note: $\omega$ is to $K$
as $A$ is to $B$.

This gives a correction to the density:

$$\rho = \frac{\delta W}{\delta A_0} = \frac{\nu e^2}{2\pi}\epsilon^{ij}\partial_i A_j + \frac{\nu e}{2\pi}\epsilon^{ij} s \partial_i \omega_j = \frac{\nu e}{2\pi}(eB + sK)$$ ,which implies that

- **Curvature acts like flux:** $N_Q = \nu\,(N_\Phi + s\,\chi)$

**"Shift"**

where $$\chi = \frac{1}{2\pi}\int d^2x\,\sqrt{g}\,K$$

On a closed surface, this is the Euler characteristics, which is a topological number depending on the genus, $g$, of the surface, $\chi = 2(1-g)$ , so *e.g.* $\chi_{sp} = 2$ .

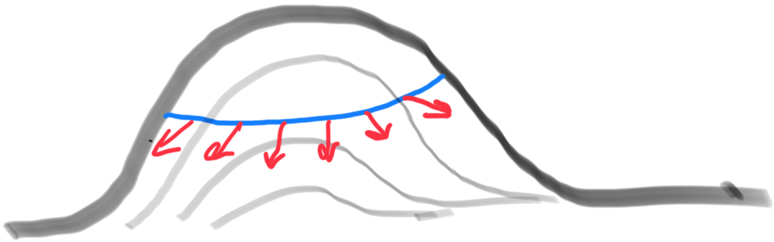
- **What is the orbital spin?**
- **What does it have to do with curvature?**
- **What is its physical significance?**

# What is the orbital spin?

- For the integer QH liquids, it is just the cyclotron motion of the electrons.
- For more complicated states, it also depends on interactions.
- In general the shift on the sphere is *2 x average orbital spin of the electrons*.

# What does it have to do with curvature?

A spin tracing out a loop on a curved surface is the example of a Berry phase. This Berry phase is added to the AB phase due to the magnetic field.



# What is its physical significance?

The shift on higher genus surfaces seems like rather exoteric stuff. What would the orbital spin spin mean in a real sample?

The orbital spin is related to the Hall viscosity, which is a non-dissipative $T$-symmetry breaking transport coefficient.

Read's formula: $$\eta^H = \frac{\hbar}{2}\rho\bar{s} = \frac{\hbar}{4}\rho S$$

Checked numerically for several states, including the 2/5 hierarchical Jain state

# What is a chiral superconductor?

**Cooper pairing:**

**spin singlet, s-wave:** $\bar{\Delta}\,\epsilon^{\alpha\beta}c_\alpha(\vec{p})c_\beta(\vec{p}) + h.c.$ In general, $l$ even

**spin triplet:** $\bar{\Delta}(\vec{p})^{\alpha\beta}c_\alpha(\vec{p})c_\beta(\vec{p}) + h.c.$ In general, $l$ odd

**Chiral order parameters:**

$$\text{Odd pairing: } p_x \pm ip_y,\ f_x \pm if_y\ \ etc.$$

$$\text{Even pairing: } d_x \pm id_y,\ etc.$$

**Even pairing *candidates*: Doped graphene, SrPtAs, $Na_xCoO\ .\ yH_2O$**

**Odd pairing *candidates*: $UPt_3$, $Li_2Pt_3B$, $Sr_2RuO_4$**

**Many of these materials are *layered*, and an effective 2d description should be relevant for films thicker than $\lambda_L$.**

# Superconductors = Flux insulators

**2+1 d Maxwell eq:** $$\partial_\mu j_F^\mu = \frac{1}{2}\partial_\mu \epsilon^{\mu\nu\omega} F_{\mu\nu} = 0$$

$$j_F^0 = \epsilon^{ij}\partial_i A_j = B \qquad\qquad Q_F = \int d^3x\, B = \Phi$$

**Couple the conserved flux current to an auxiliary gauge field b:**

$$\int d^3x\, b_\mu j_F^\mu = \int d^3x\, A_\mu \epsilon^{\mu\nu\sigma}\partial_\mu b_\sigma = \int d^3x\, A_\mu j_{ch}^\mu$$

**so magnetic response can be calculated as**

$$\frac{\delta W[b_\mu]}{\delta b^0} = B$$

**but what is** $W[b_\mu]$ **??**

- **Ordinary SCs have chiral symmetry and are trivial flux insulators**
- **χSCs could be topologically nontrivial**

**What are the possible terms consistent with symmetry?**

**In principle there could be a flux Hall effect** $\sim \int d^3x\, \epsilon^{\mu\nu\sigma} b_\mu \partial_\nu b_\sigma$ **, but ….**

**so we instead concentrate on the flux Wen-Zee term:**

$$W_{WZ}[b_\mu, \omega_\mu] = -\frac{\kappa_\phi \tilde{\phi}_0}{2\pi} \int d^3x\, \epsilon^{\mu\nu\sigma} \omega_\mu \partial_\nu b_\sigma \quad , \quad \tilde{\phi}_0 = \frac{\pi}{e}$$

$$\boxed{N_\Phi = -\kappa_\phi \chi}$$

This relation can be derived from Ginzburg Landau theory:

$$\boxed{\left(\lambda_L^2 \triangle - 1\right) B = \frac{\Phi_0}{4\pi} K}$$

**Thus curvature generates flux: This is the geo-Meißner effect**

# Why do we expect a non-trivial geometric response?

Consider a very thin film with small curvature $K\xi^2 \ll 1$
where $\xi$ is the size of the Cooper pair, so that

the orbital spin of the pair (i.e. the angular momentum due to the orbital motion, is well defined and perpendicular to the surface.

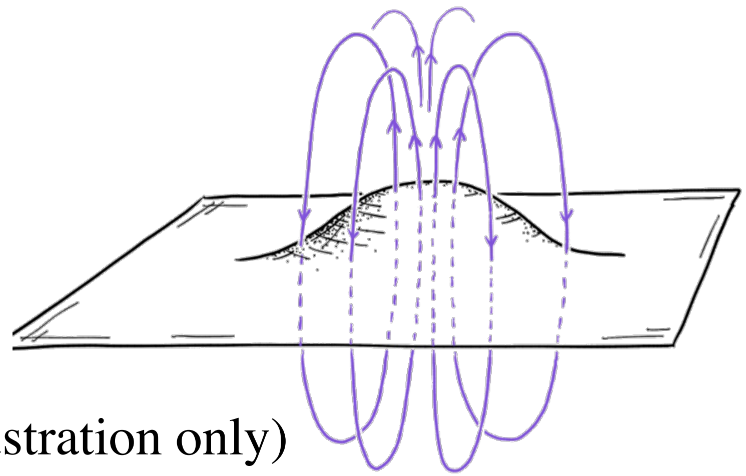
I further all the pairs have the same chirality, the pair will respond to curvature in the same way as to a magnetic field, so that:

in addition to the AB phase due to the charge 2e, there will be a Berry phase $2\pi\chi l$ , where *l* is the orbital spin of the pair, so $\kappa_\phi = \ell$

and the Meißner effect will amount to expelling the combination $B + lK\Phi_0/4\pi$ rather than the magnetic field itself!

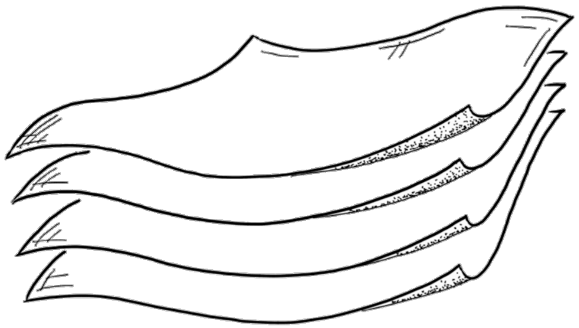
# The geo-Meissner effect

**Curvature induces flux:**



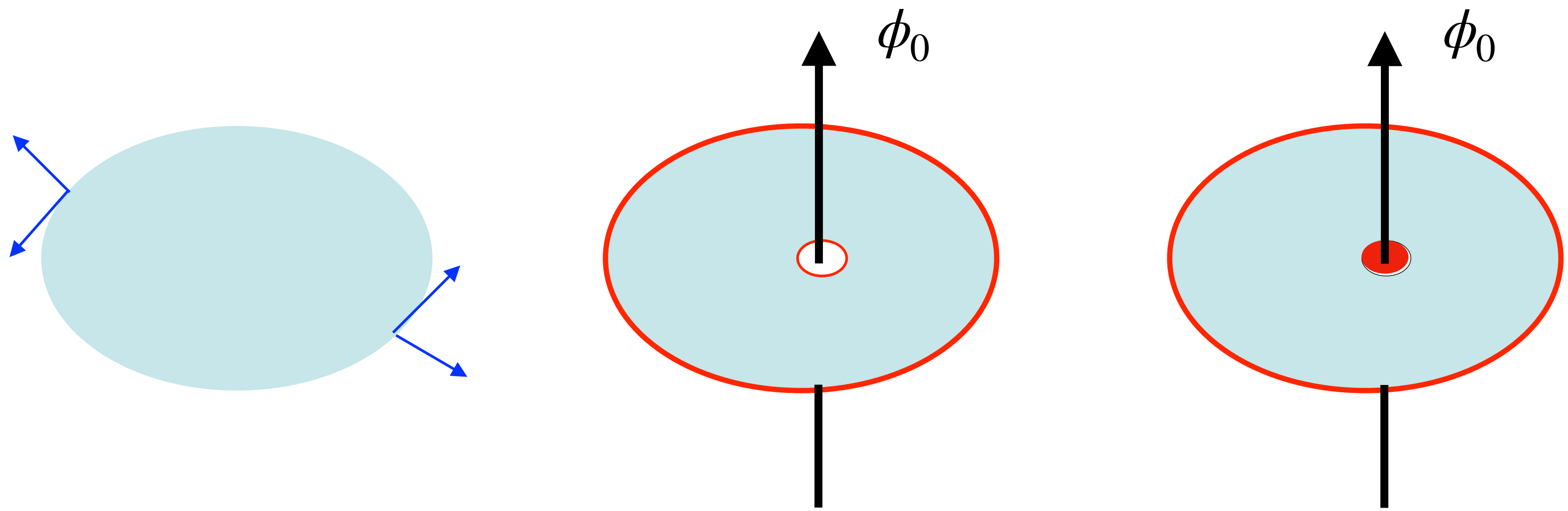
(Arrows for illustration only)

- **Assume 1% bond-length stretching.**
- **Curved region large compared to $\lambda_L \lesssim 1$ μm corresponding to B $\lesssim$ μT.**
- **Easily detected by a SQUID**
- **Gives clear signature of chiral layered S.C.**
- **Thin films are more complicated**



# The geo-Meissner effect at work:

**Majorinos in spin less 2d $p_x \pm ip_y$ chiral superconductors:**



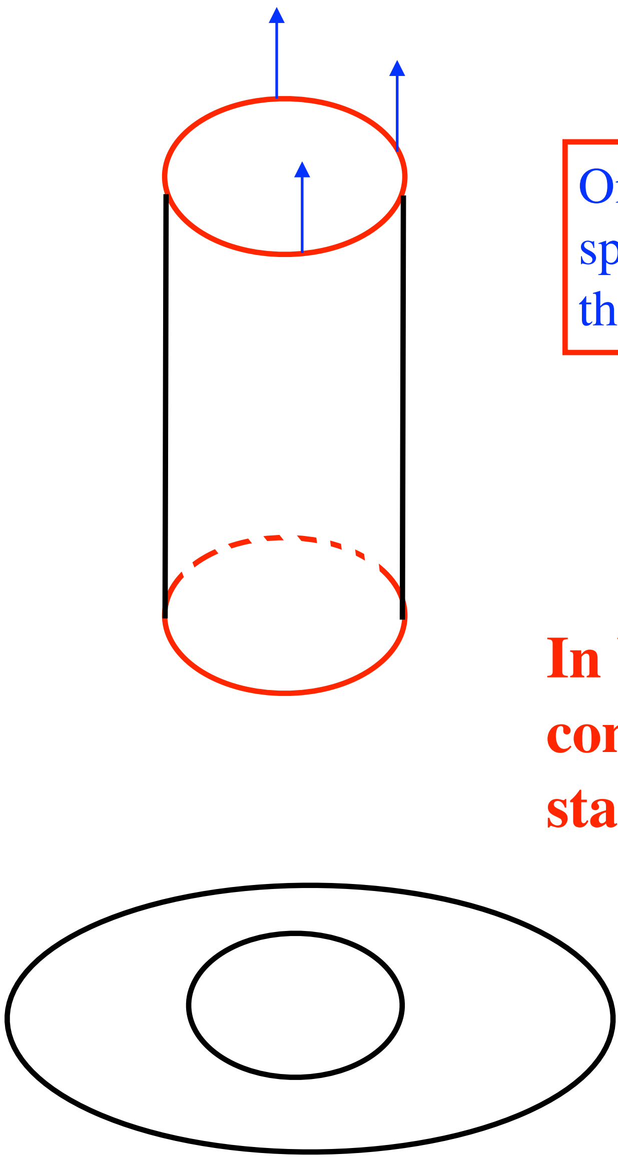

**The effective mean field theory has two Nambu components and the edge is described by an effective Majorana (i.e. a real Dirac) equation.**

**In the presence of a flux, there are zero energy modes at vortices and edges.**

**These Majorinos always come in pairs.**

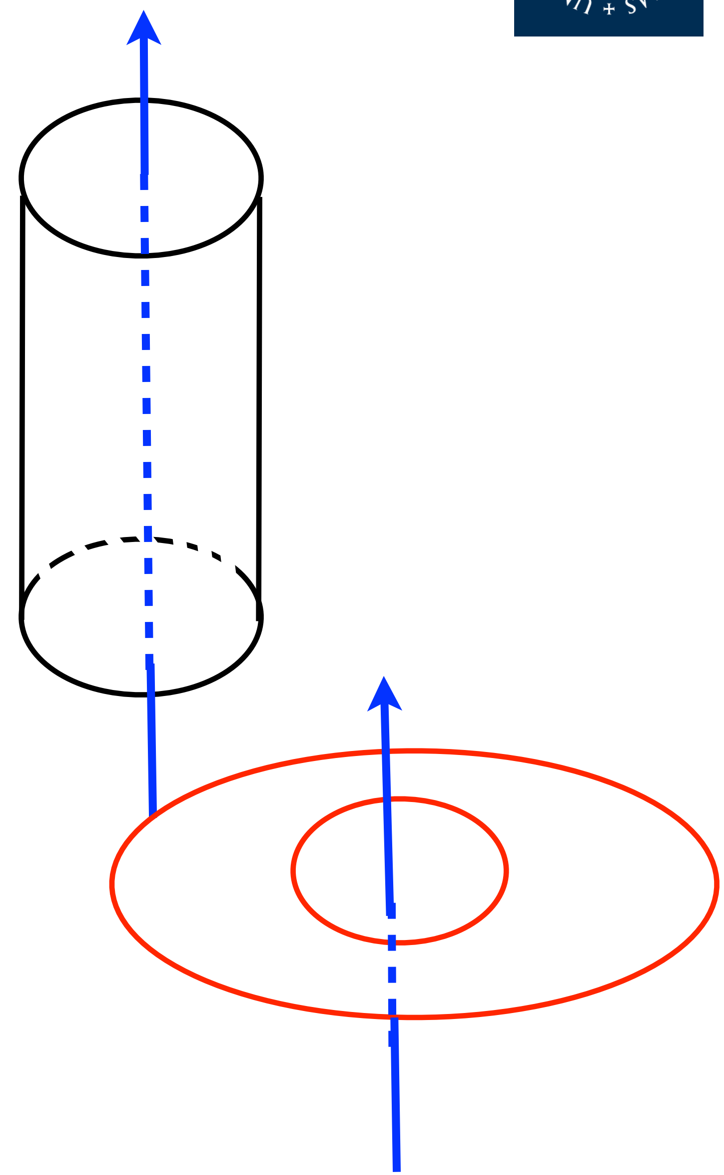
**The absence of a Majorino for the edge of the disc is because the rotation of the Nambu spinor gives an extra minus sign.**

# Majorinos on disc and cylinder - a puzzle



On the cylinder, the Nambu spinor does not rotate, and there is a pair of Majorinos.

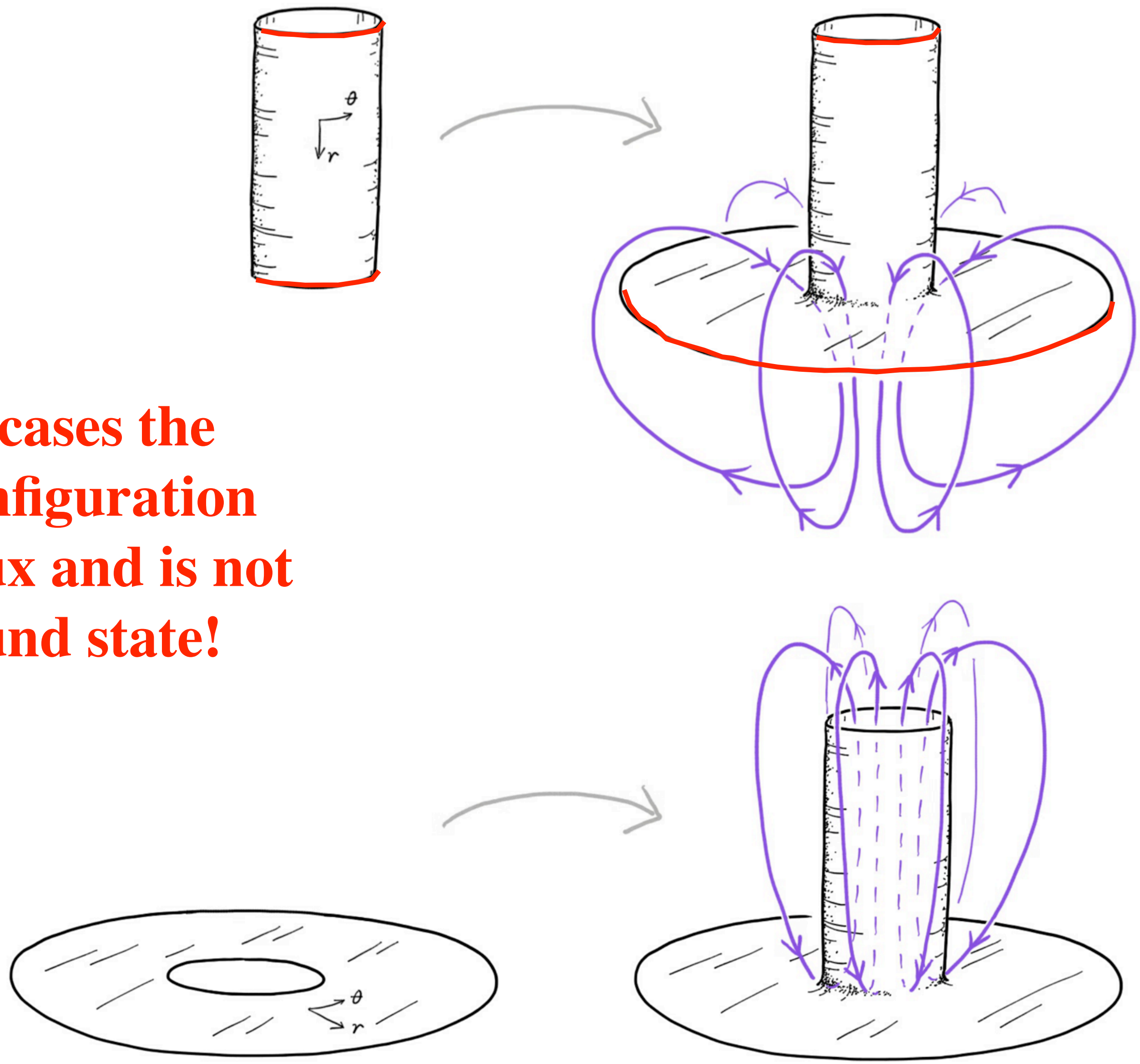
**In both cases the flux free configuration is the ground state**



**Ground states**

**Excited states**

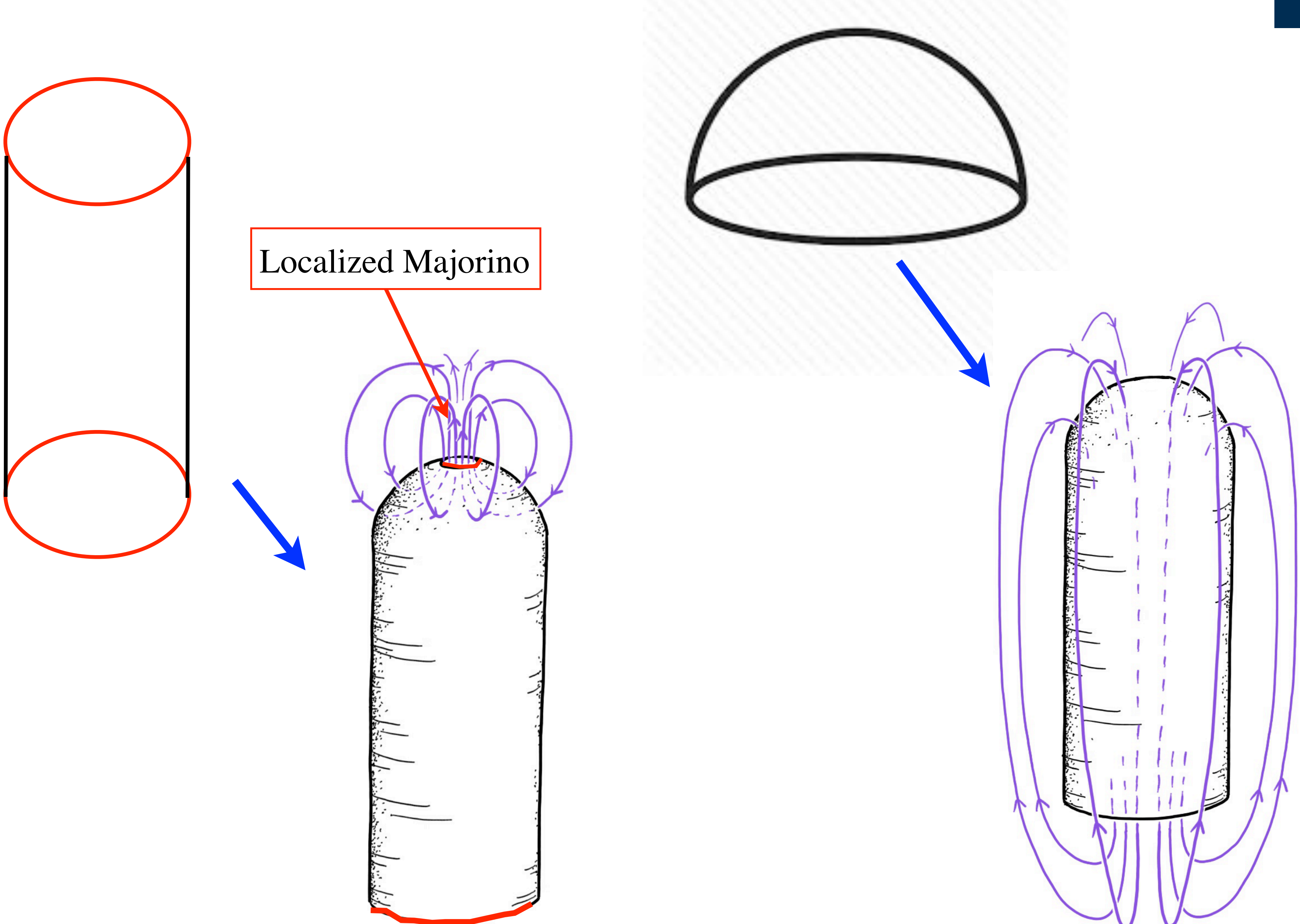
# Majorana - Quo Vadis?  II




**In both cases the final configuration has a flux and is not the ground state!**

Figures by S. Holst

# Edge modes and vortices




Figures by S. Holst

# Chiral SC's on closed surfaces

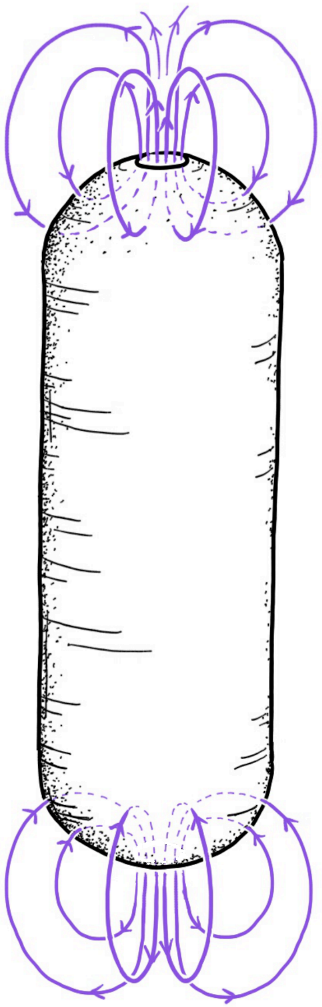


Figure by S. Holst

**Compare with the geo-Meißner relation:**

$$\boxed{N_\Phi = \kappa_\phi \chi}$$

**The Gauss-Bonnet theorem for a sphere:** $\chi = 2$

**p-wave pairing:** $\kappa_\phi = 1$

**So there cannot be a homogeneous flux distribution on a sphere - two vortices are spontaneously created, and the rotational invariance is spontaneously broken.**

# From Ginzburg-Landau to geo-Meissner

**Ginzburg-Landau free energy:**

$$F = \hbar^2 \int \sqrt{g}\, d^2x \left( \frac{g^{ij}}{2m} (D_i\varphi)^* D_j\varphi + \frac{B^2}{2\mu_0} \right) + V(\varphi)$$

**a p-wave order parameter:**

$$\varphi = \sqrt{\rho_+} e^{i\theta_+} (\hat{e}_1 + i\hat{e}_2) + \sqrt{\rho_-} e^{i\theta_-} (\hat{e}_1 - i\hat{e}_2) \ ,$$

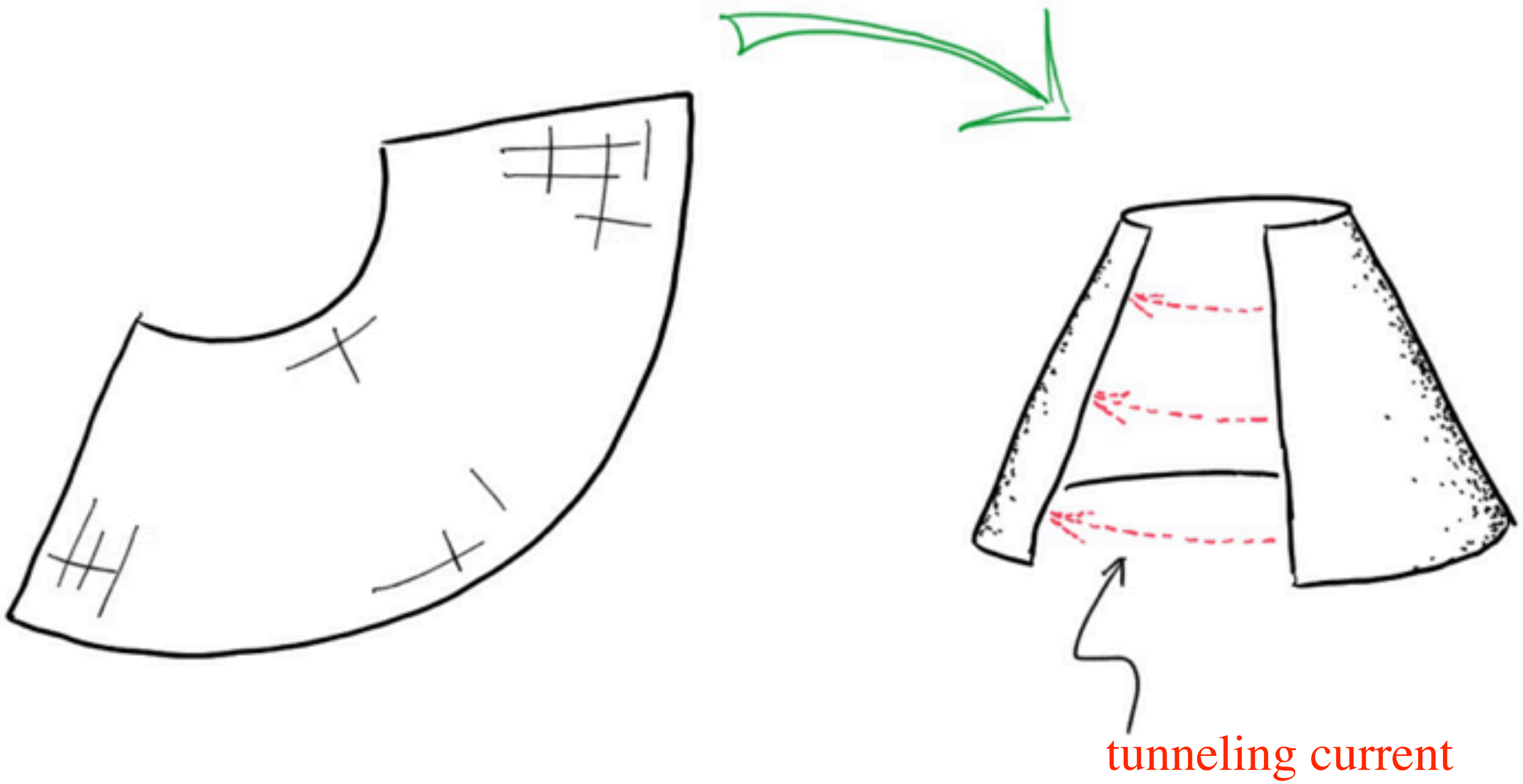
**assuming** $\bar{\rho} = \bar{\rho}_+ \neq 0 \ , \ \bar{\rho}_- = 0$ **gives the London free energy:**

$$F_L = \int \sqrt{g}\, d^2x \left( \frac{\hbar^2}{8\mu_0 e^2 \lambda_L^2} \left( \vec{\nabla}\theta_+ + \vec{\omega} - 2\frac{e}{\hbar}\vec{A} \right)^2 + \frac{B^2}{2\mu_0} \right)$$

**and by variation w.r.t. A,**

$$\boxed{\left( \lambda_L^2 \triangle - 1 \right) B = \frac{\Phi_0}{4\pi} K}$$

# The geo-Josephson effect

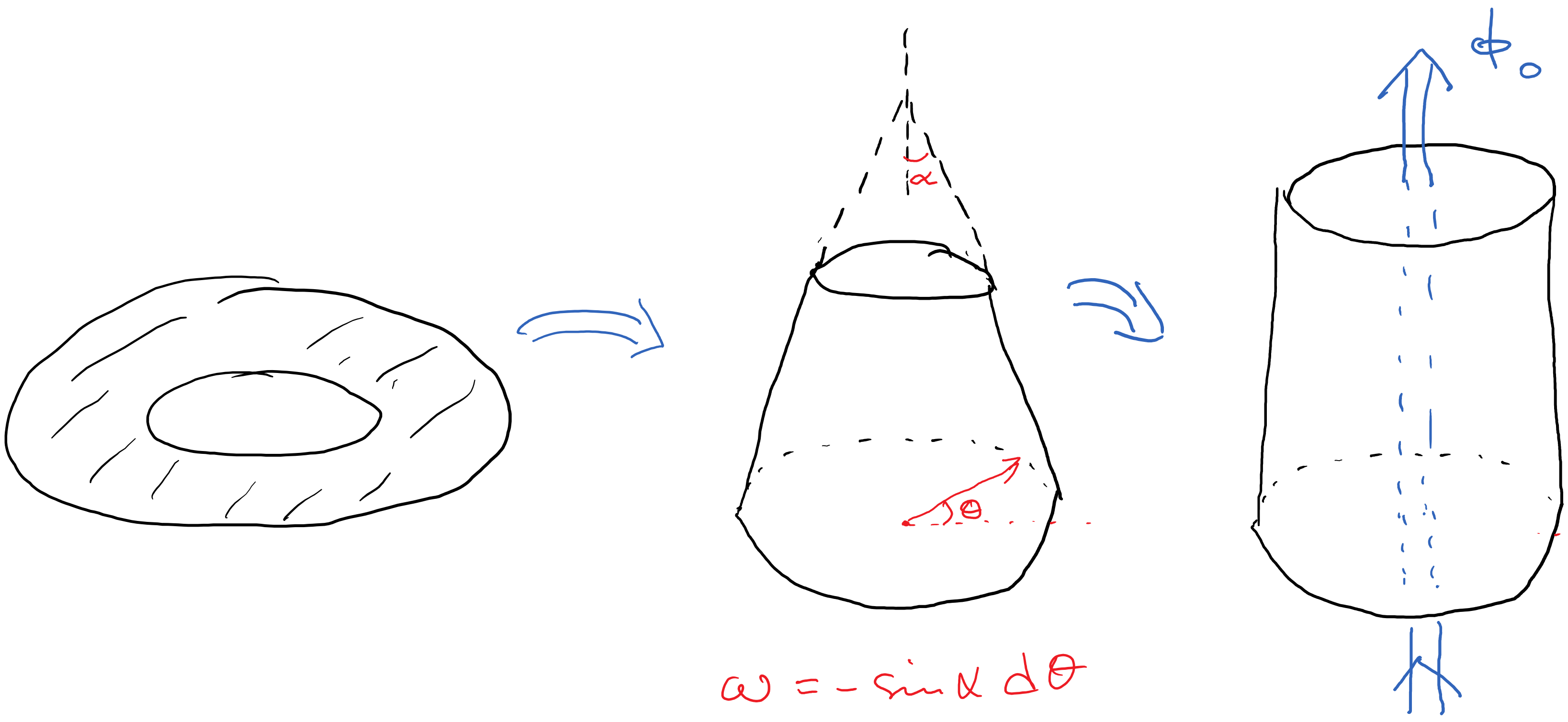



Figures by S. Holst

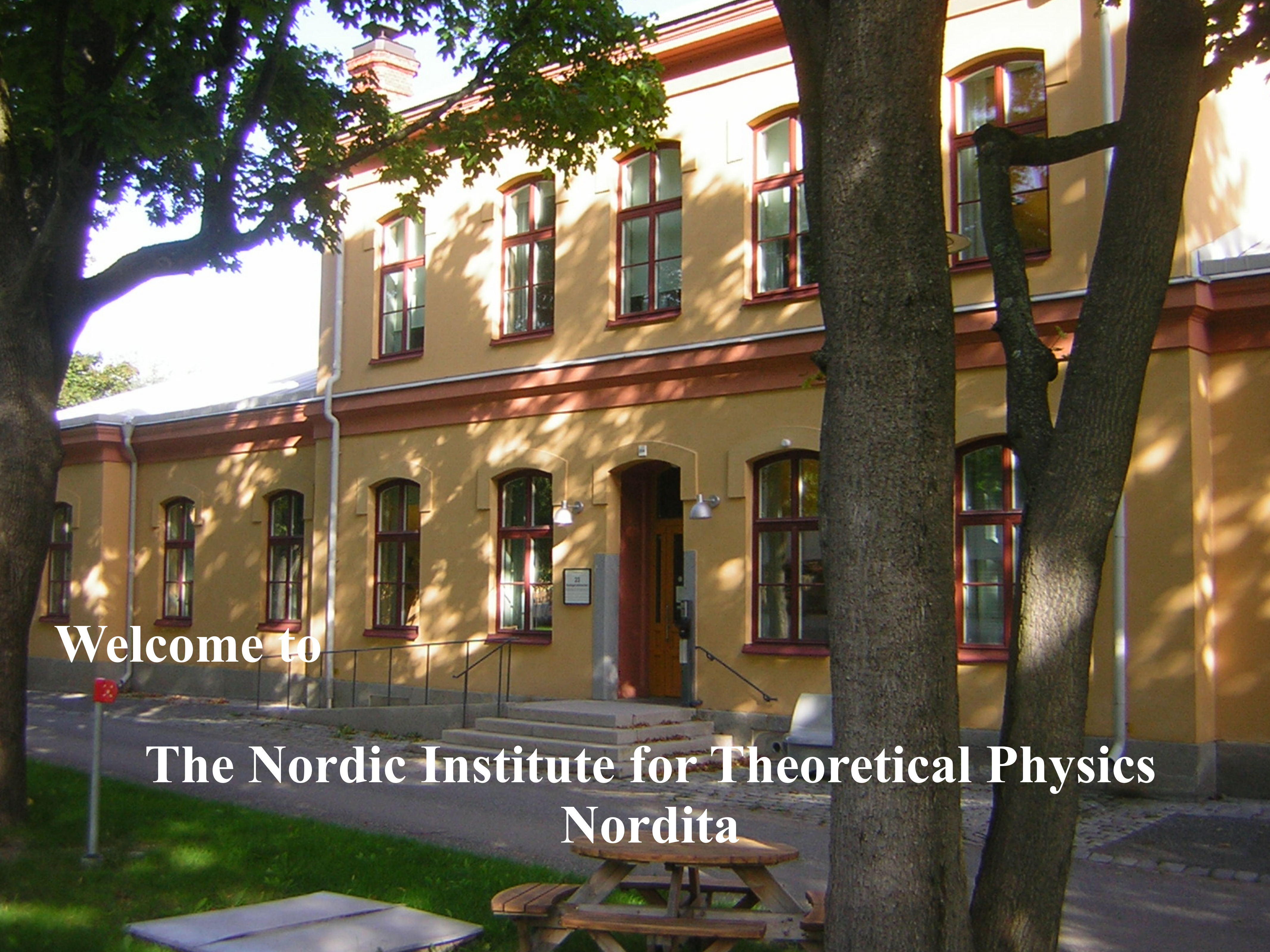
**The spin connection enters just as an electromagnetic vector potential giving a geometric version of the Josephson effect - the *geo-Josephson effect*!**

**As the edges come closer together the current increases, and the phase difference decreases, to completely vanish when the superconductor is healed an we get back a homogeneous state.**

# From Corbino disc to cylinder via the cone:



# Thank you for listening!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The Nordic Institute for Theoretical Physics
Nordita

# A very brief history

Nordita was founded in 1957 as the *Nordic Institute for Theoretical Atomic Physics*, located next to the Niels Bohr Institute in Copenhagen.

Over the years, the scope of Nordita activities has widened to include new and emerging areas while maintaining a strong focus on basic theoretical physics.

On Jan 1, 2007 Nordita moved to Stockholm, where it is hosted by Stockholm University, the Royal Institute of Technology (KTH) and Uppsala University

Nordita is now located at the *AlbaNova University Center*

More information is found on Nordita's homepage http://www.nordita.org

# Nordita activities

- In house research
    - Prof. + students
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    - Postdocs
- Schools 1-2 weeks
- Conferences 1week
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- Programs 2-4 weeks
    - Anyone can apply
    - Deadline early December
    -