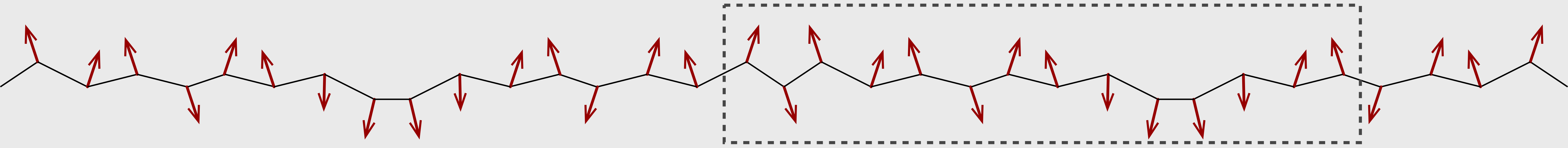
Complex quantum systems:

— many correlated degrees of freedom

— no integrability

— exponential complexity increase



SYK model

fermions or bosons

$$\hat{H} = \sum_{ijkl}^{N} J_{ijkl}\, c_i^\dagger c_j^\dagger c_k c_l$$

drawn from static random distribution

I: nuclear physics (1970s)

Volume 34B. number 4 PHYSICS LETTERS 1 March 1971

TWO-BODY RANDOM HAMILTONIAN AND LEVEL DENSITY

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Received 22 December 1970

Volume 33B, number 7 PHYSICS LETTERS 7 December 1970

VALIDITY OF RANDOM MATRIX THEORIES FOR MANY-PARTICLE SYSTEMS *

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Received 19 October 1970

II: cond-mat early 90s

VOLUME 69, NUMBER 16 PHYSICAL REVIEW LETTERS 19 OCTOBER 1992

Universal Quantum-Critical Dynamics of Two-Dimensional Antiferromagnets

Subir Sachdev and Jinwu Ye

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(Received 13 April 1992)

The universal dynamic and static properties of two-dimensional antiferromagnets in the vicinity of a zero-temperature phase transition from long-range magnetic order to a quantum-disordered phase are studied. Random antiferromagnets with both Néel and spin-glass long-range magnetic order are considered. Explicit quantum-critical dynamic scaling functions are computed in a $1/N$ expansion to two-loop level for certain nonrandom, frustrated square-lattice antiferromagnets. Implications for neutron scattering experiments on the doped cuprates are noted.

PACS numbers: 75.10.Jm, 05.30.Fk, 75.50.Ee

… discovery of conformal symmetries

III: Sachdev-Ye-Kitaev Model (15)

A model of N randomly interacting *Majorana* fermions

$$\hat{H} = \sum_{ijkl}^{N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l, \qquad \{\chi_i, \chi_j\} = 2\delta_{ij}$$

SYK model

where the interaction constants are static and random,

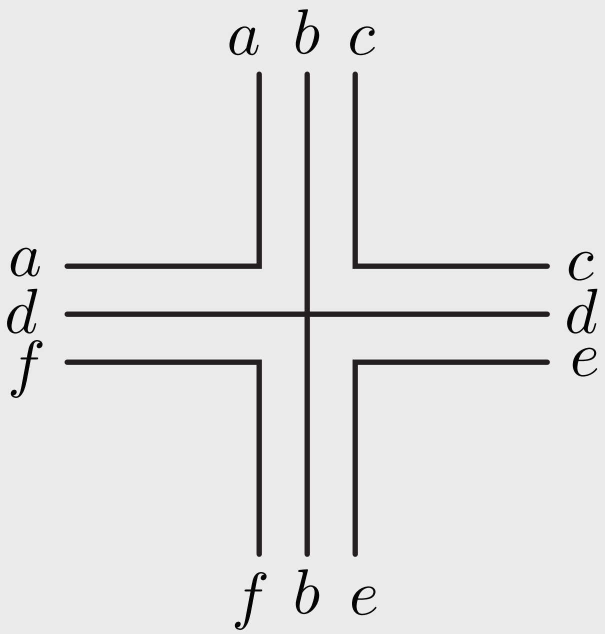
$$\langle |J_{ijkl}|^2 \rangle = \frac{6J^2}{N^3}$$

high energy scale

Three perspectives:

- quantum chaos
- strong correlation physics
- holography

IV: clean version (Witten 16, Klebanov & Tarnopolski 17)



$$\hat{H} = g \sum_{abcdef} c^{abc\dagger} c^{cde\dagger} c^{ebf} c^{fda}$$

No disorder required!

V: Russian potato (Wiegmann, unpublished)

Quantum ergodicity in the SYK model

Tbilisi, Jun. 3d, 2019

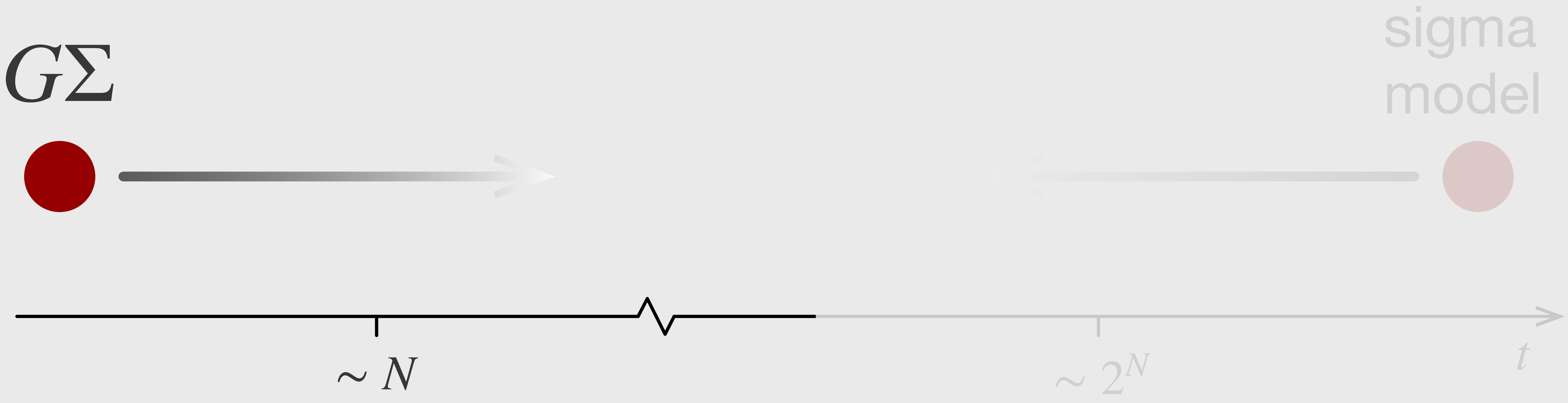
Alexander Altland, Dmitry Bagrets (Cologne),
Tobias Micklitz, Felipe Monteiro (Rio de Janeiro)

short times: exponential instabilities and collective quantum fluctuations

long times: spectral and wave function correlations

Nucl. Phys. B **930**, 45 (2018)
arXiv 1901.02389

time line



exponential instabilities

scrambling

conformal symmetry (breaking)

strong collective quantum fluctuations

quantum ergodicity

collective spectral fluctuations

causal symmetry (breaking)

short time dynamics

conformal symmetry and its breaking

$$Z = \left\langle \mathrm{tr}\left(e^{-\beta \hat{H}}\right)\right\rangle_{\mathrm{dis}} \qquad \beta = 1/T$$

$$= \int DX \exp\left(-S[X]\right) \qquad S[X] = N\int_0^\beta d\tau \left(\ldots\partial_\tau\ldots\right)$$

$G\Sigma$ functional, Sachdev and Ye, 92

$$Z = \int D[G,\Sigma]\, \exp(-S[G,\Sigma])$$

$$S[\Sigma, G] = -\frac{N}{2}\left[\mathrm{tr}\ln(\partial_\tau + \Sigma) + \frac{J}{4}\int_0^\beta d\tau d\tau' \left((G_{\tau,\tau'})^4 + \Sigma_{\tau,\tau'} G_{\tau',\tau}\right)\right]$$

mean field analysis

Universal properties of effective action #1: large factor N up front -> mean field approach.

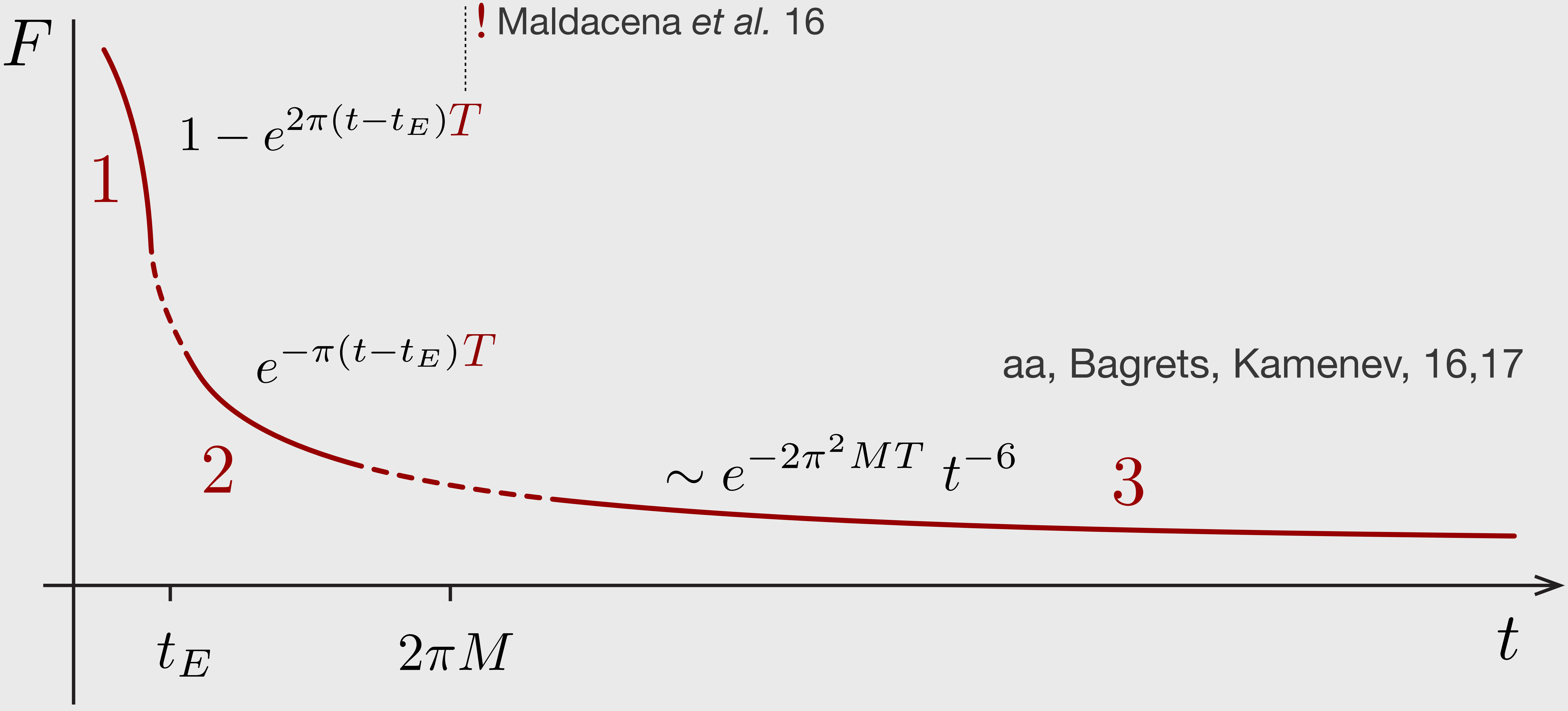
Mean field Majorana Green function:

$$G(\tau,\tau') \equiv \langle T_\tau\, \chi_i(\tau)\chi_i(\tau')\rangle = \frac{\text{const.}}{J^{1/2}} \frac{\text{sgn}(\tau-\tau')}{|\tau-\tau'|^{1/2}}$$

different from Fermi liquid Green function.

SYK OTO correlation function

$$F(t) = \mathrm{tr}\left(e^{-\beta \hat{H}} \chi_i \chi_j(t) \chi_i \chi_j(t) \right)$$



long time dynamics

exponential instabilities

scrambling

conformal symmetry (breaking)

strong collective quantum fluctuations

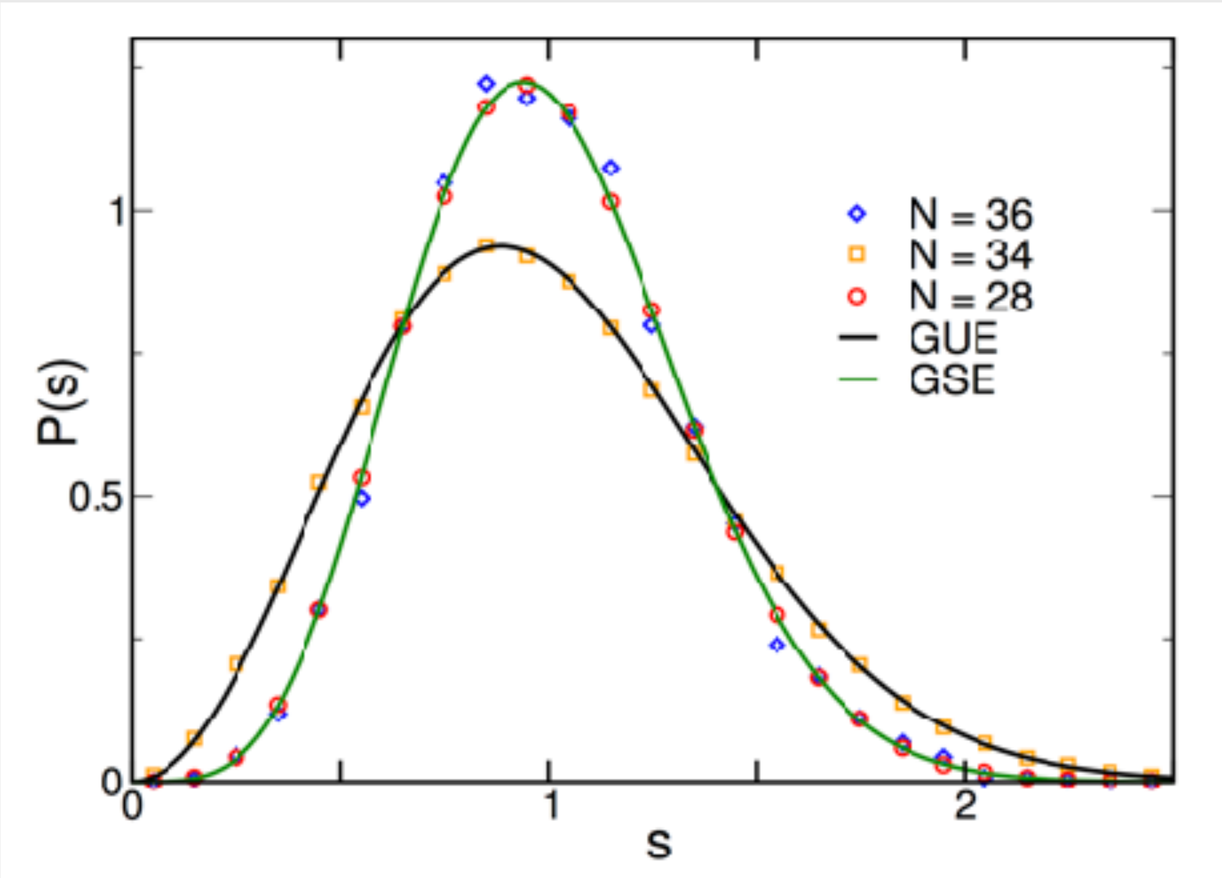
quantum ergodicity

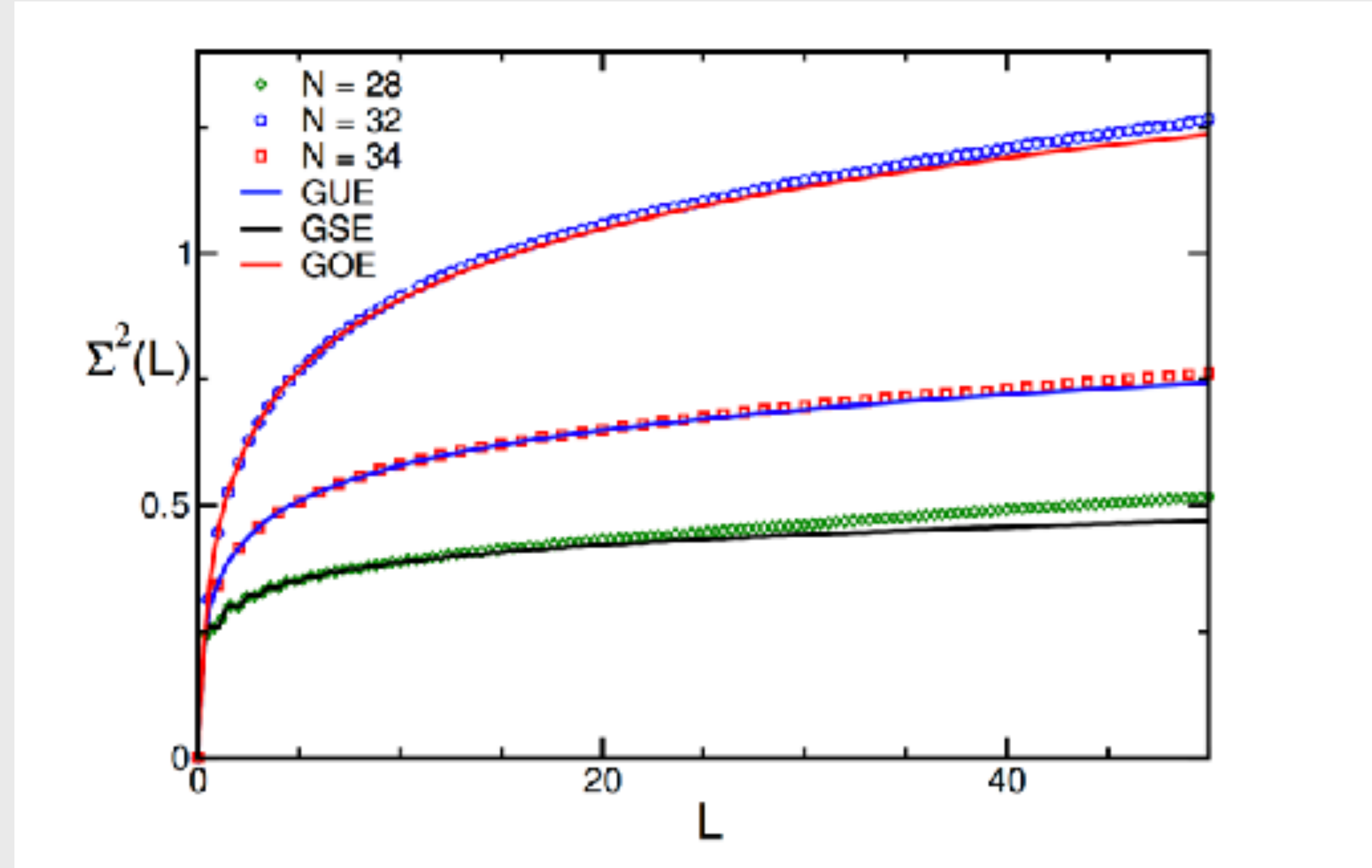
collective spectral fluctuations

causal symmetry (breaking)

random matrix correlations:

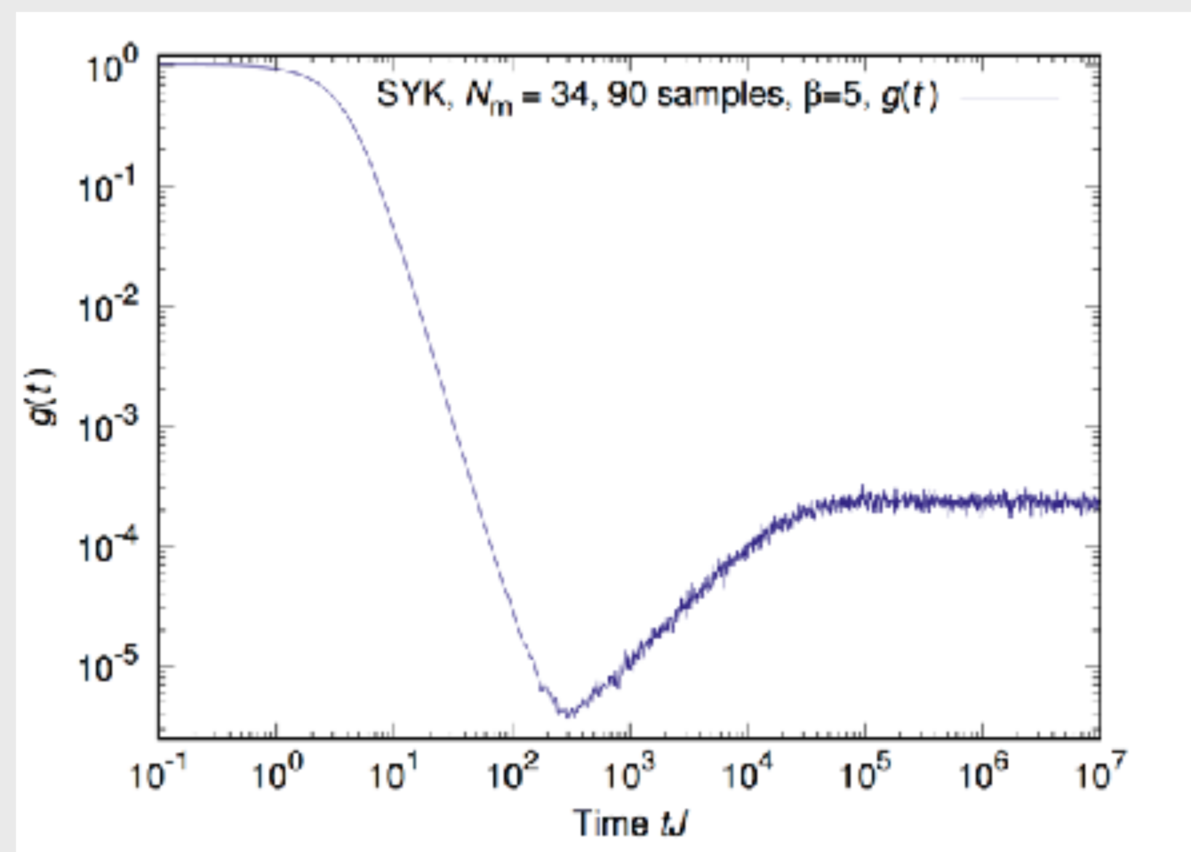
Verbaarschot, Garcia-Garcia, 16





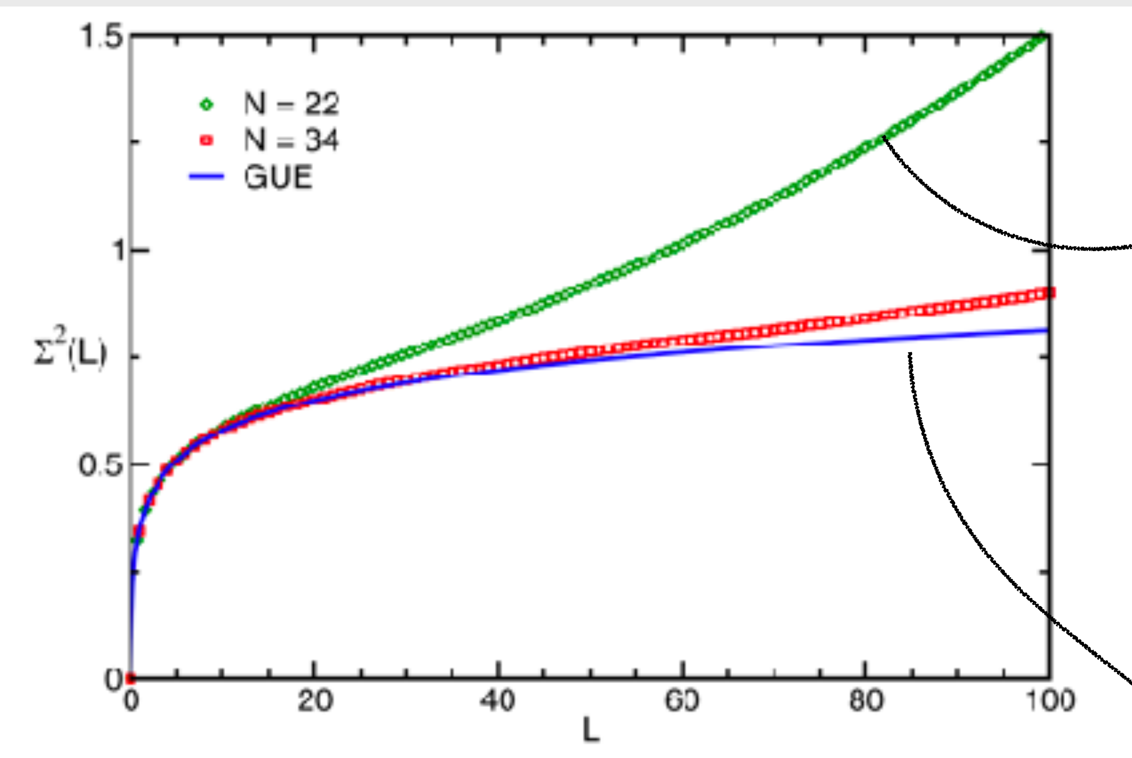
Note: depending on the value of *N* mod 8 the model realizes different symmetries

Cotler et al. 17



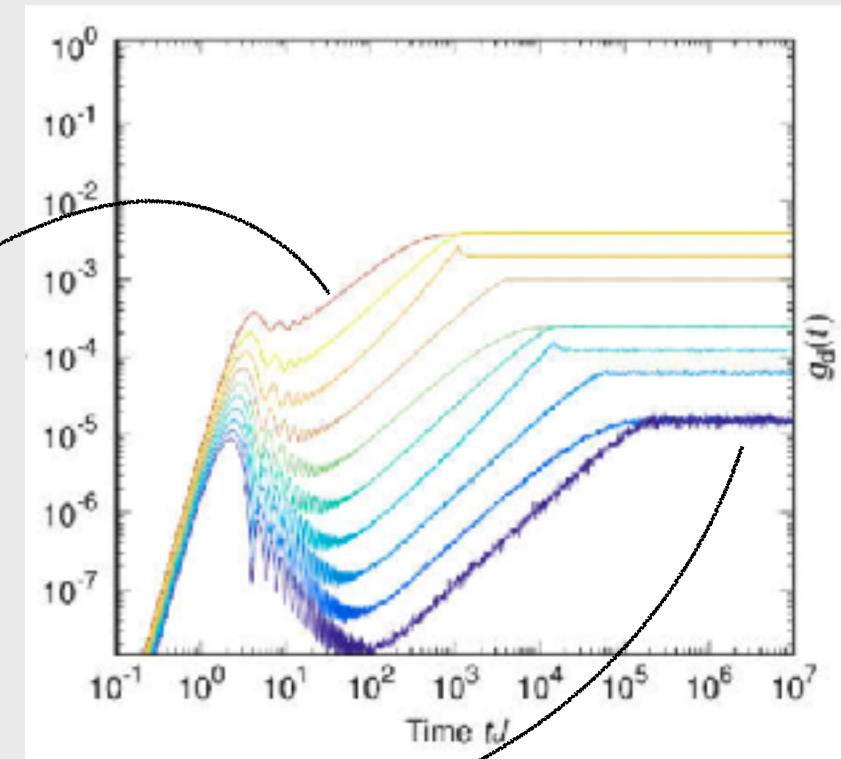
Deviations from RMT

For times shorter than an ergodic time $t_{\rm erg}$ universal deviations from RMT behavior are observed.



Verbaarschot, Garcia-Garcia, 16

not RMT
(still universal)



Cotler et al. 17

RMT

$$R_2(\omega) \equiv \Delta^2 \left\langle \rho(E+\frac{\omega}{2})\rho(E-\frac{\omega}{2}) \right\rangle_c$$

exploring quantum chaos in SYK — philosophy

- consider SYK Hamiltonian $\hat{H}$ from a first quantized perspective — a (complicated) matrix.

- physical information contained in resolvents $\hat{G}^{\pm} \equiv (E \pm i0 - \hat{H})^{-1}$

- apply methods of single particle quantum physics.

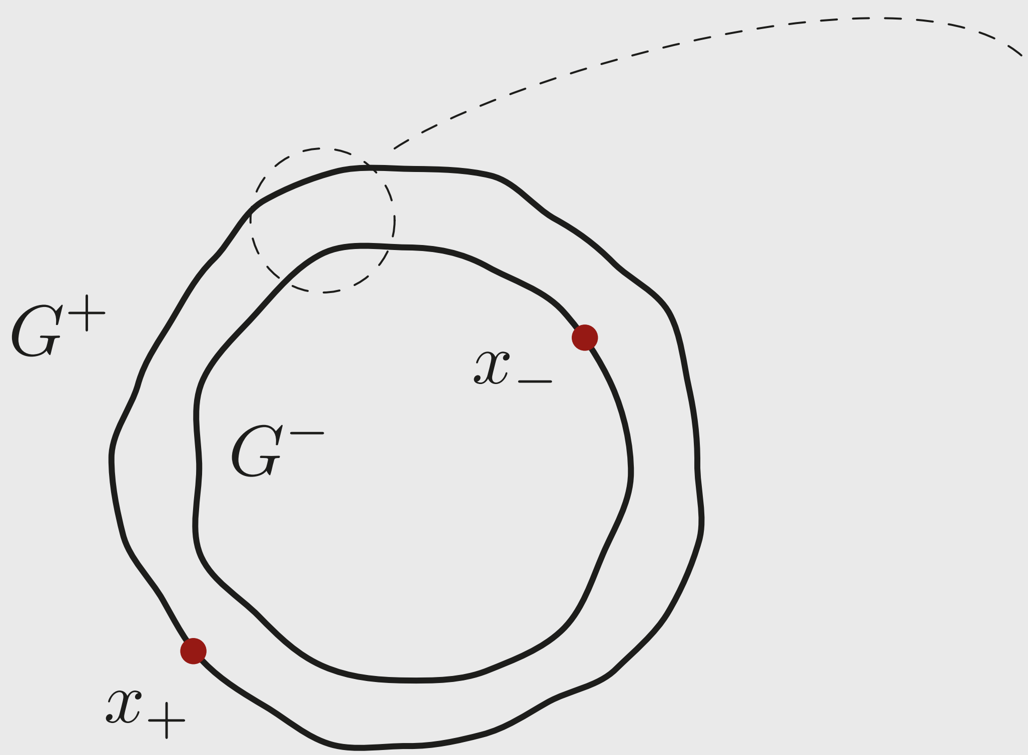
semiclassical interpretation of spectral correlations

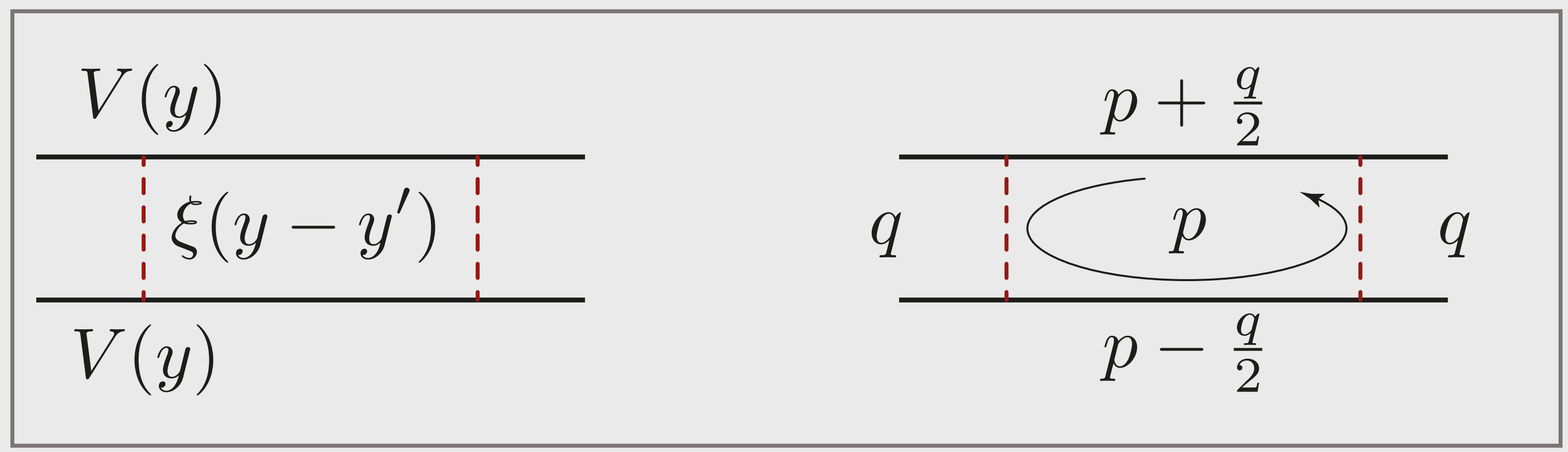
Compare to the physics of dirty metals

Diagnostic:

$$R_2(\omega) \equiv \Delta^2 \left\langle \rho(E + \frac{\omega}{2})\rho(E - \frac{\omega}{2}) \right\rangle_c$$

probe fluctuations of $\rho(E) = -\frac{1}{\pi}\int dx \,\mathrm{Im}(G^+(E,x,x))$



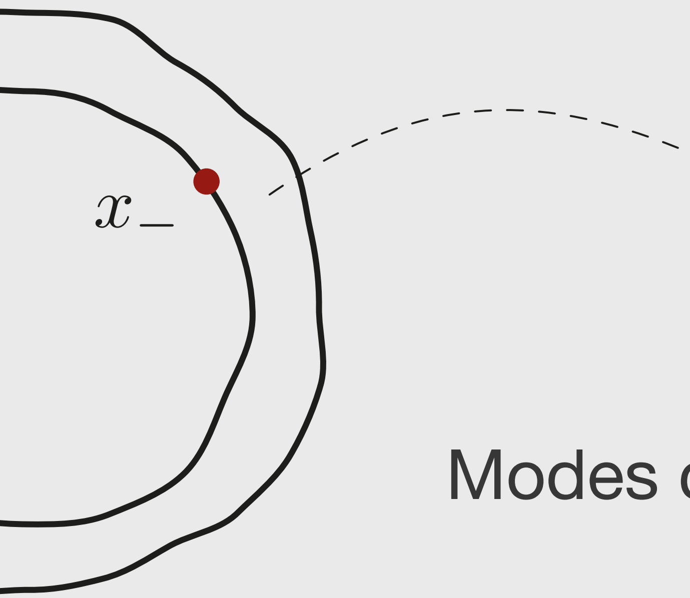


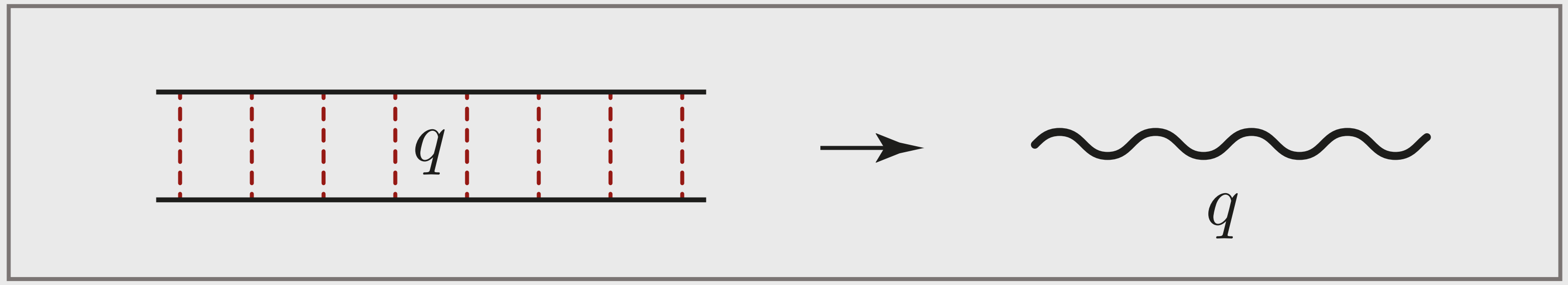
Momentum *q* difference conserved in scattering.

Idea: (i) basis change $|x\rangle\langle y| \longrightarrow |p+\frac{q}{2}\rangle\langle p-\frac{q}{2}|$

(ii) interpret this as a basis change in the space of Hilbert space *operators.*

dirty metals cont'd





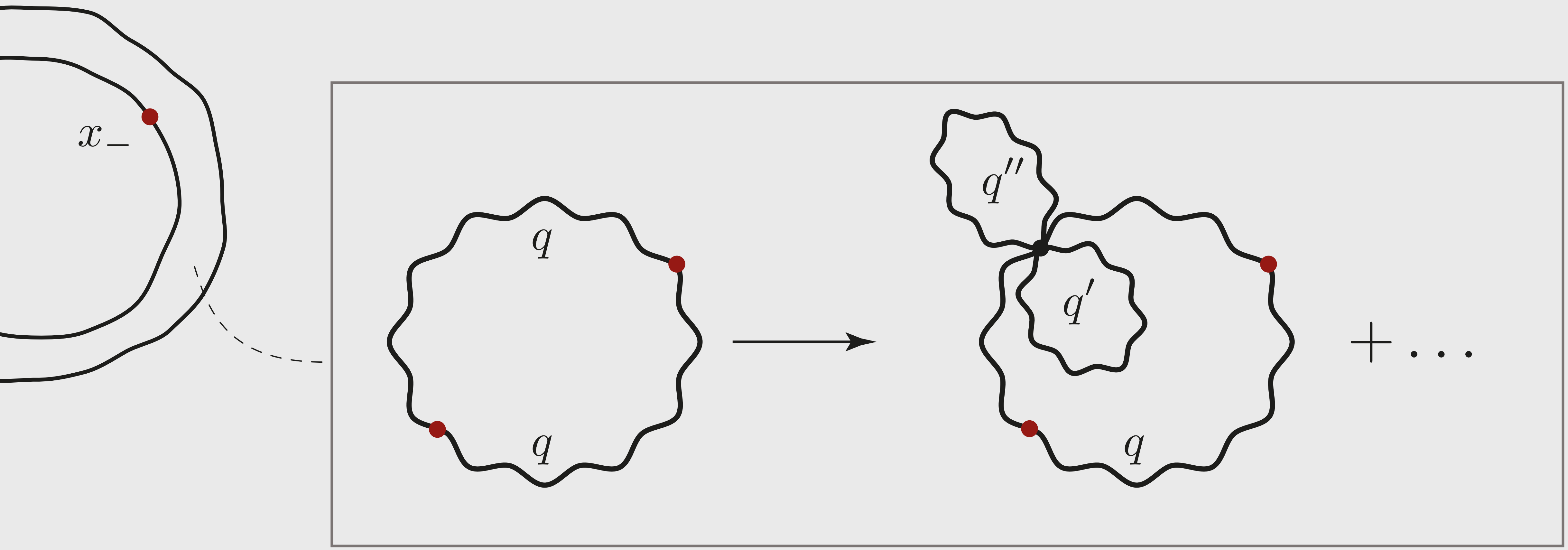
Modes characterized by

(i) 'discrete quantum numbers' $q \in \frac{2\pi}{L}\mathbb{Z}^3$

(ii) decay constants $Dq^2 + i\omega$

(iii) physical interpretation as irreversible relaxation modes

Dirty metals cont'd



$$R_2(\omega) = R_{2,\mathrm{RMT}}(\omega) + \frac{1}{2}\left(\frac{\Delta}{\pi}\right)^2 \mathrm{Re}\sum_{q\neq 0}\frac{1}{(i\omega - Dq^2)^2}$$

Altshuler & Shklovskii, 86

relaxation dynamics in Fock space

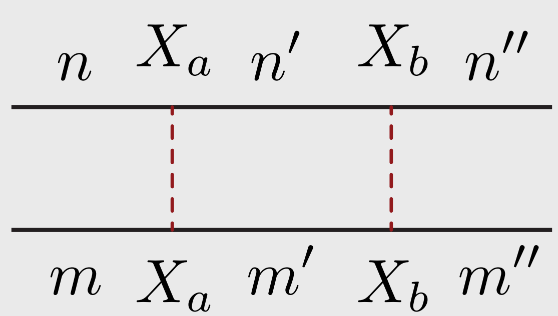
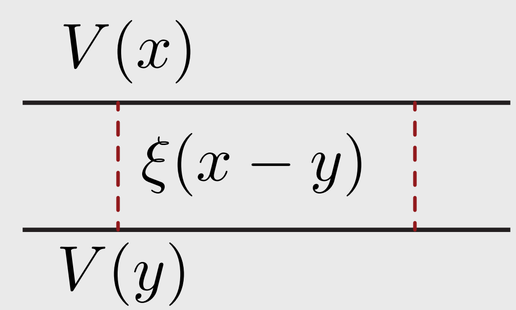
perspective of current approach:

1) consider many body Hamiltonian from first quantized persepective — a large, sparse, correlated matrix.

2) exploit simplicity of Clifford algebra to make analytic progress

The setting

$$\hat{H} = \sum_{ijkl}^{N} J_{ijkl}\chi_i\chi_j\chi_k\chi_l \equiv \sum_a J_a \hat{X}_a, \qquad a = (i,j,k,l)$$

	SYK	dirty metal
Hilbert space	Fock space of *N/2* fermions	function space
dimension	$2^{N/2}$	∞
basis states	$m = (1,0,0,1,\ldots)$	$\vert x\rangle$
scattering vertex		
scattering states	$\vert n\rangle \otimes \langle m\vert$	$\vert x\rangle \otimes \langle y\vert$
basis of conserved states	$\hat{X}_\mu \equiv \chi_{\mu_1}\chi_{\mu_2}\cdots\chi_{\mu_k},$ $\mu \equiv (\mu_1, \mu_2, \ldots, \mu_k)$	$\vert p+\frac{q}{2}\rangle \otimes \langle p-\frac{q}{2}\vert$

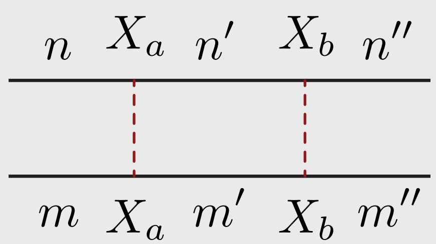
setting cont'd

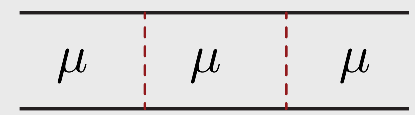
SYK

dirty metal

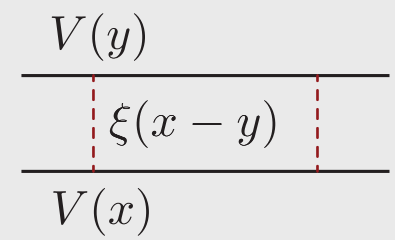
conserved modes

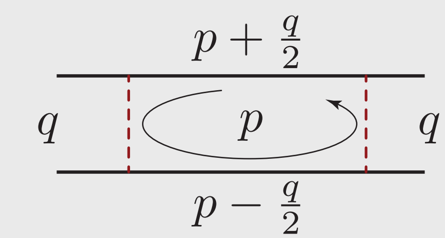
SYK:





dirty metal:





eigenvalues

SYK:

$$\epsilon(|\mu|) - i\omega$$

\# of Majoranas in state

$$\epsilon(k) \sim 2^{N/2}\Delta \times k$$

$$\Delta \sim \frac{JN^{1/2}}{2^{N/2}}$$

many body level spacing

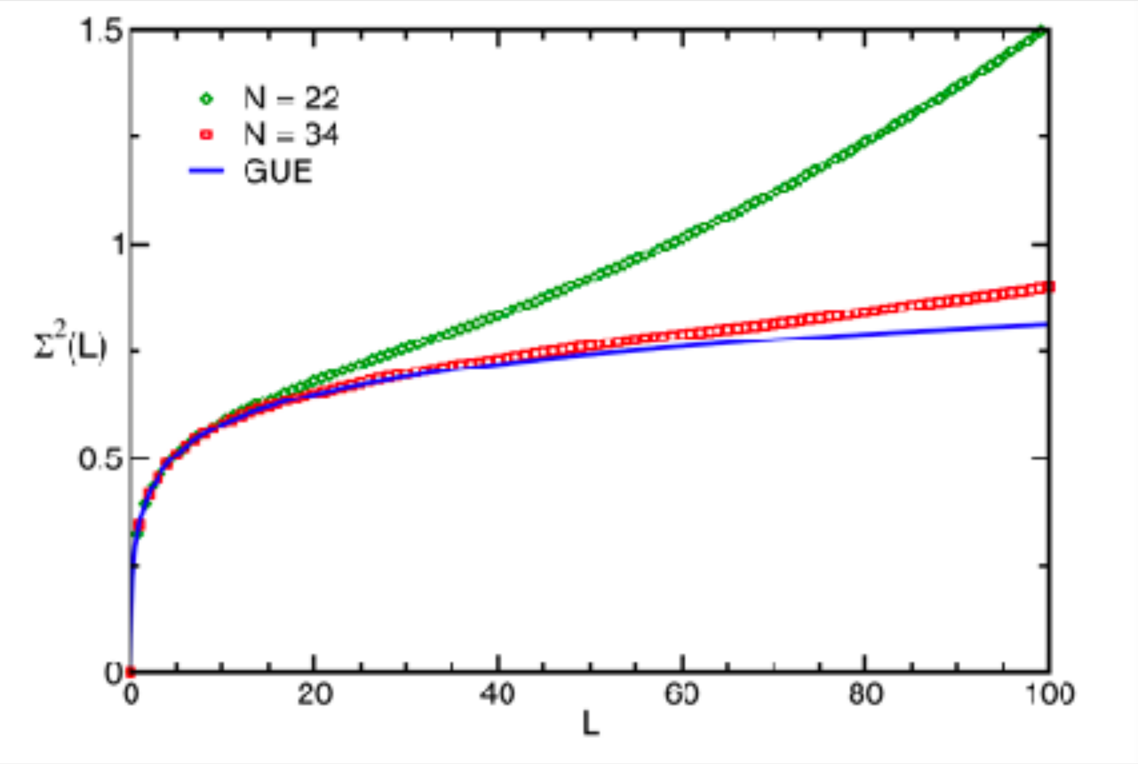
dirty metal:

$$Dq^2 - i\omega$$

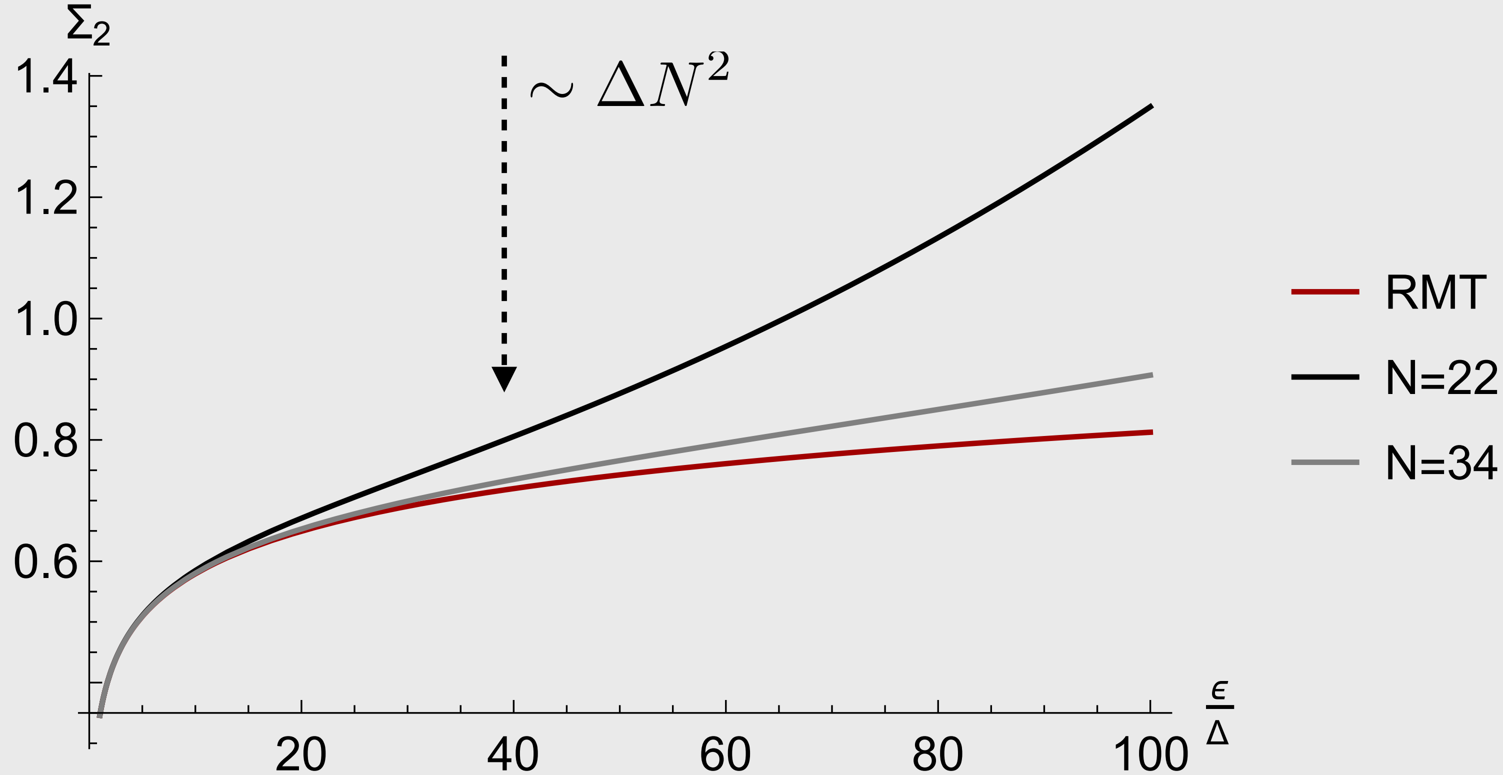
SYK spectral correlation function

$$R_2(\omega) = R_{2,\mathrm{RMT}}(\omega) + \frac{1}{2}\left(\frac{\Delta}{\pi}\right)^2 \mathrm{Re} \sum_{k\neq 0,\mathrm{even}} \binom{N}{k} \frac{1}{(i\omega - \epsilon(k))^2}$$

comparison to numerical data I: number variance



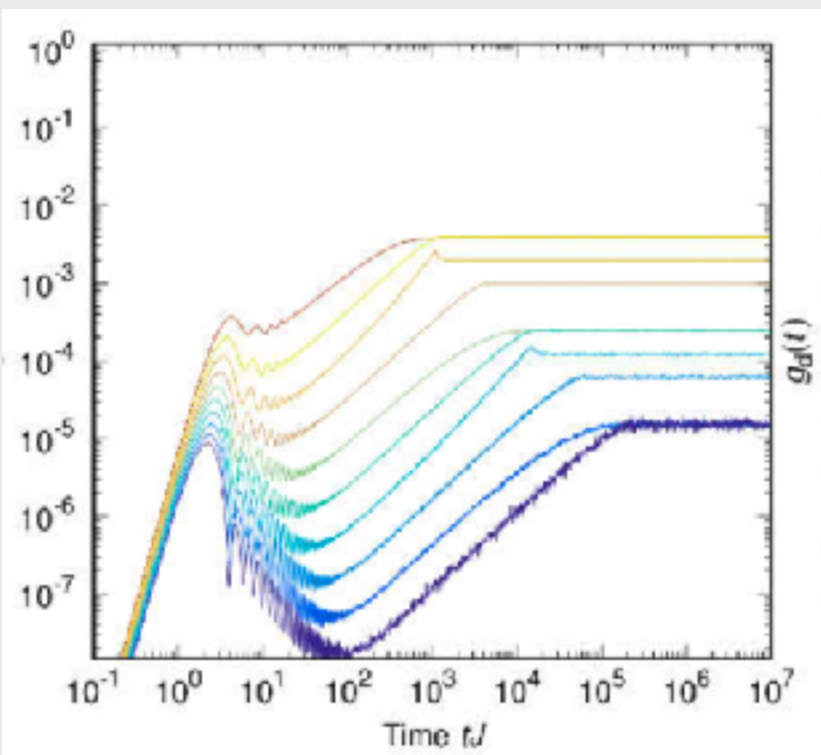
Verbaarschot, Garcia-Garcia, 16



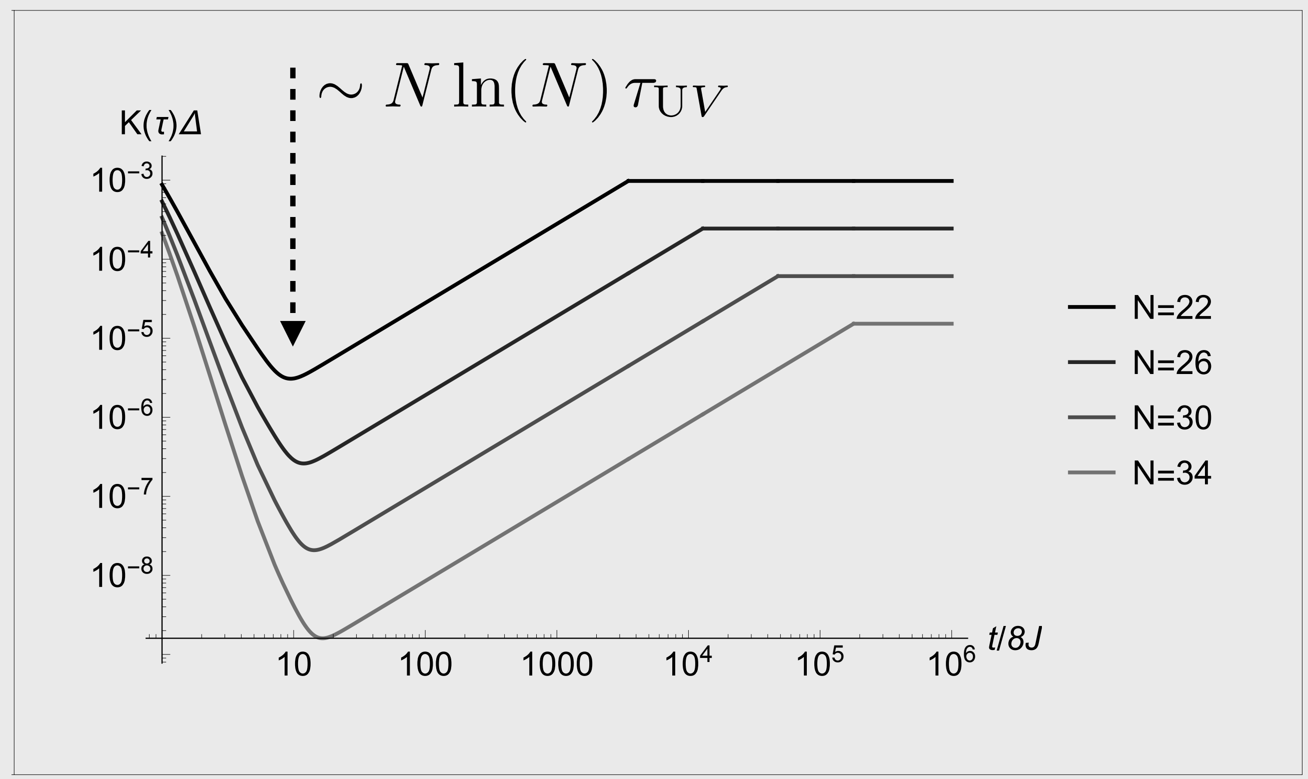
SYK spectral correlation function

$$K(\tau) = K_{\mathrm{RMT}}(\tau) + \tau \sum_{k\neq 0,\mathrm{even}} \binom{N}{k} e^{-\tau\frac{2\pi\epsilon(k)}{\Delta}}, \quad \tau > 0$$

comparison to numerical data II: spectral form factor



Cotler et al. 17



Qualitatively good agreement. However comparison problematic (1803.08050).

No universal ‘Thouless energy’ in this system.

matrix integral representation

4×2^N dim matrix

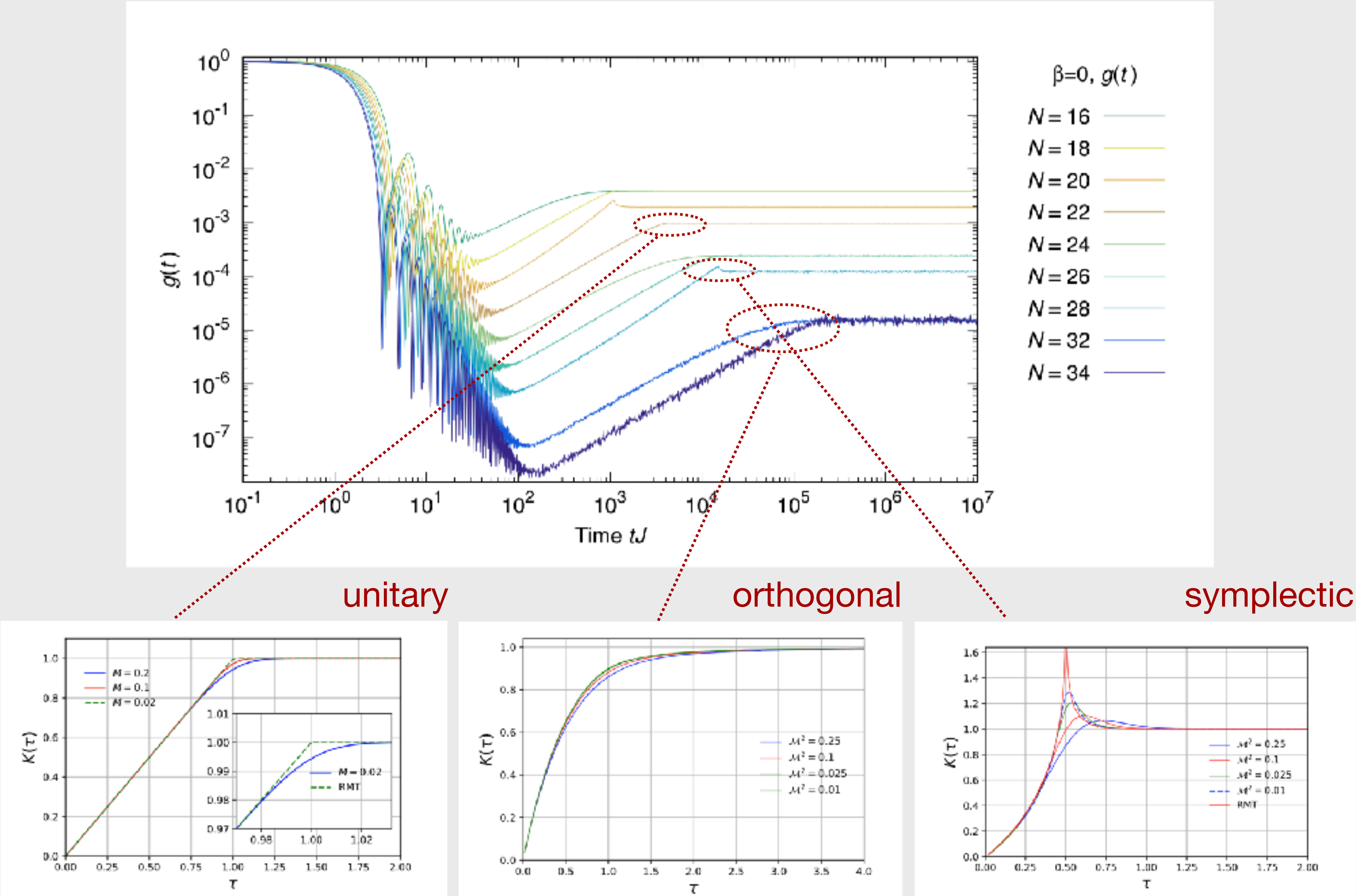
$$S(Y) = -\frac{1}{2}\mathrm{STr}(Y\mathscr{P}^{-1}Y) + \mathrm{STr}\ln\left(i\delta\sigma_3 + i\gamma Y\right),$$

$$\mathscr{P}Y \equiv \frac{1}{\mathscr{N}}\sum_a X_a Y X_a^\dagger$$

action has two saddle points: 'standard' $Y = \sigma_3^{\mathrm{ar}} \otimes 1_{\mathscr{F}}$

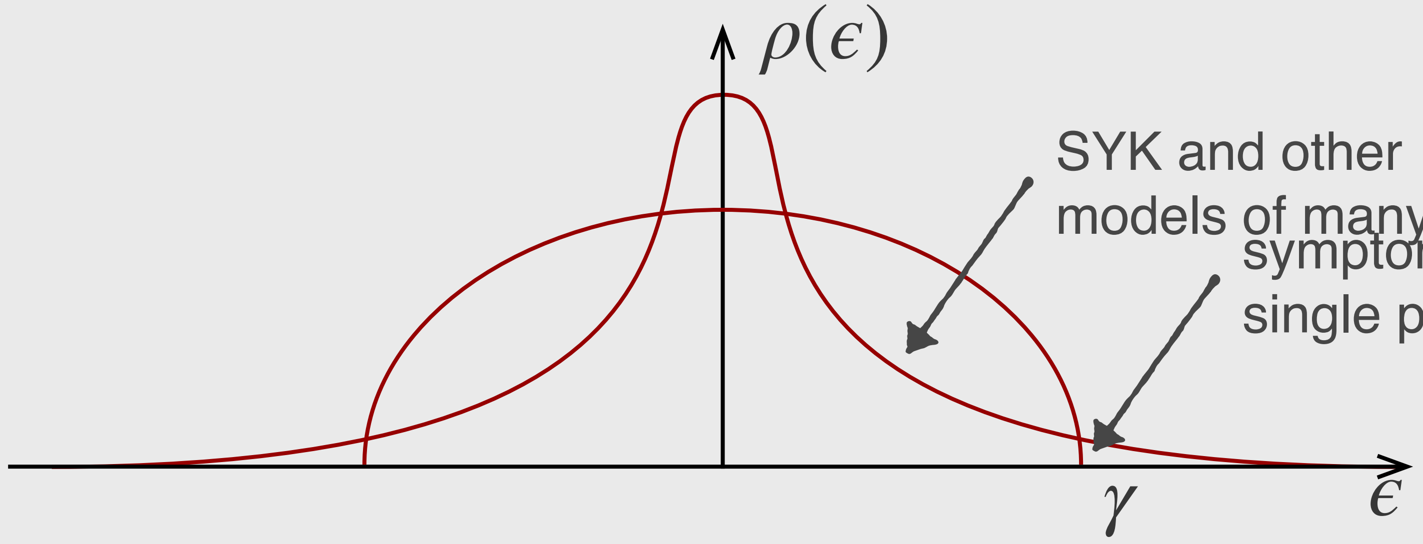
Altshuler-Andreev $Y = \sigma_3^{\mathrm{ar}} \otimes \sigma_3^{\mathrm{bf}} \otimes 1_{\mathscr{F}}$

Non perturbative physics near Heisenberg time



Physics of the second contribution (Seligman 00, Gharibyan *et al.* 18)

$$R_2(\omega) = -\frac{\sin\left(\frac{\pi\omega}{\delta}\right)^2}{\left(\frac{\pi\omega}{\delta}\right)^2} + \frac{1}{2}\sum_{\mu\neq 0}\frac{1}{\frac{D}{\pi}(S_\mu^{-1}-1)^2}$$



- Gaussian tails in reflect large collective fluctuations in coarse grained DoS
- a structureless (energy independent) contribution to the spectral correlation function

wave function statistics

background: non-ergodic extended (NEE) states

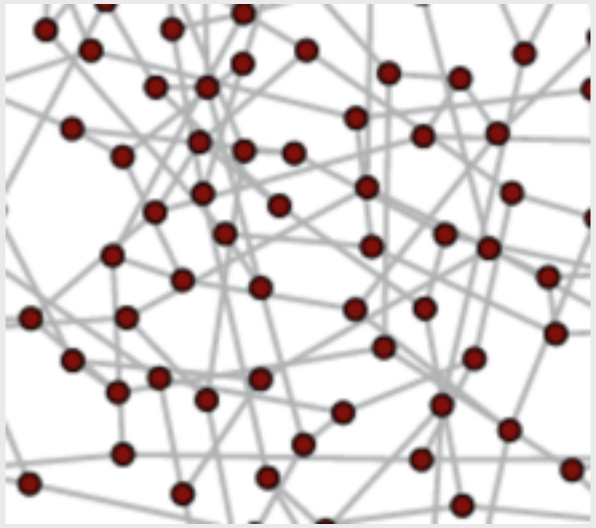
- 2005: many body localization

- ~2015: wave function multifractality

NEE states cont'd

indirect evidence from numerical/semi-analytic analysis of effective models.

- numerical and semi-analytical analysis of disordered random regular graphs (Kravtsov *et al.* 16)



- numerical (Kravtsov *et al.* 15) and analytical (Truong & Ossipov, 16) analysis of Rosenzweig-Porter model (1960)

$$\hat{H} = \begin{pmatrix} \ddots & & & \\ & \ddots & H_{ij} & \\ & & \ddots & \\ & & & \ddots \end{pmatrix} + \begin{pmatrix} \ddots & & & \\ & v_i & & \\ & & v_{i+1} & \\ & & & \ddots \end{pmatrix}$$

Wave function fractality in Rosenzweig–Porter model

Consider D–dimensional Hamiltonian $\hat{H} = \hat{H}_{\mathrm{RMT}} + \hat{H}_V,$ $\quad \hat{H}_V = \mathrm{diag}(d_1, \ldots, d_D)$

$$\mathrm{var}(d_i) \sim D^{\gamma}$$

Three phases:

	$\sum_i \langle \lvert \psi_i \rvert^{2q} \rangle$	spectral statistics
$\gamma < 0$	$\sim D^{1-q}$	WD
$0 < \gamma < 1$	$\sim D^{-(\gamma-1)(1-q)}$	WD
$\gamma > 1$	~ 1	Poisson

NEE phase: wave function occupy fractal subset of Hilbert space

Results obtained by sigma model representation (Truong & Ossipov, 16)

$$S[Q] = -\,\mathrm{tr}\ln(i\delta\sigma_3^{\mathrm{ra}} - iyQ - \hat{H}_V)$$

Definition of deformed SYK model

Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_V$

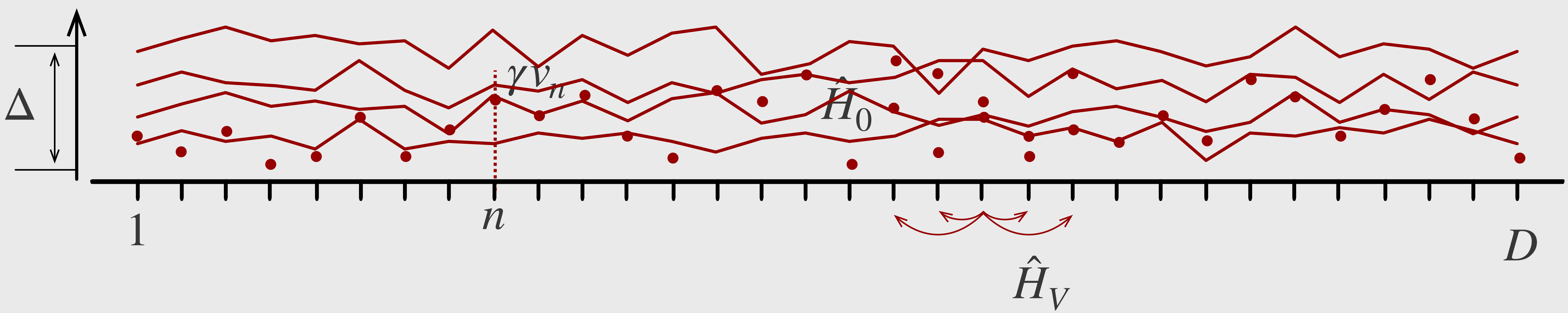
$$\hat{H}_0 = \frac{1}{4!} \sum_{i,j,k,l=1}^{2N} J_{ijkl} \hat{\chi}_i \hat{\chi}_j \hat{\chi}_k \hat{\chi}_l$$

$$\hat{H}_V = \gamma \sum_{n}^{D} v_n |n\rangle\langle n|$$

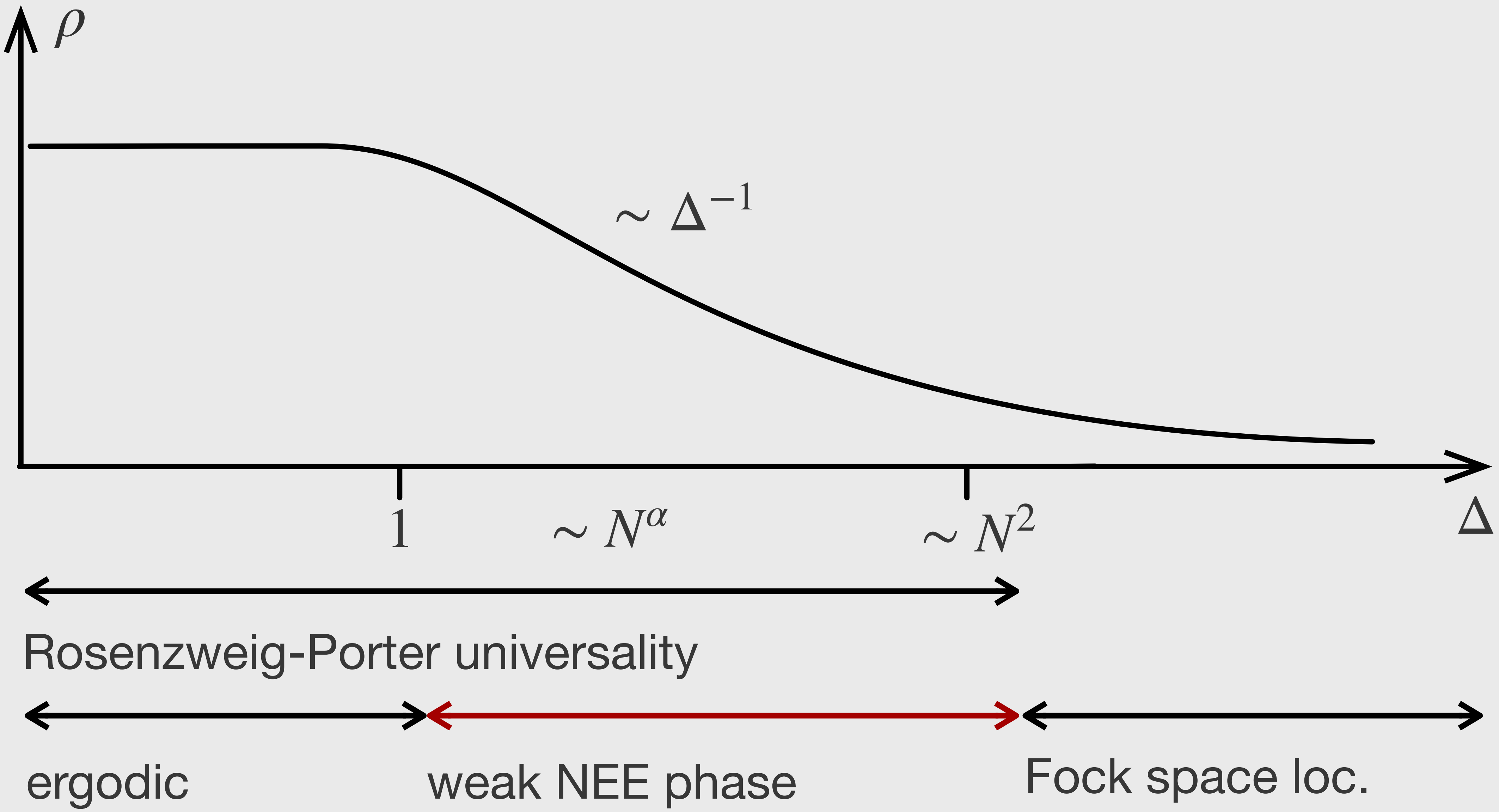
comments

$$\hat{H}_V = \frac{1}{4!}\sum_{i,j,k,l=1}^{2N} J_{ijkl}\,\hat{\chi}_i\hat{\chi}_j\hat{\chi}_k\hat{\chi}_l$$

$$\hat{H}_0 = \sum_{n}^{D} |n\rangle\langle n|$$



Results



Up to $\Delta \sim N^2$: equivalence SYK/RP

For larger H_V, Fock space localization

quantitative results

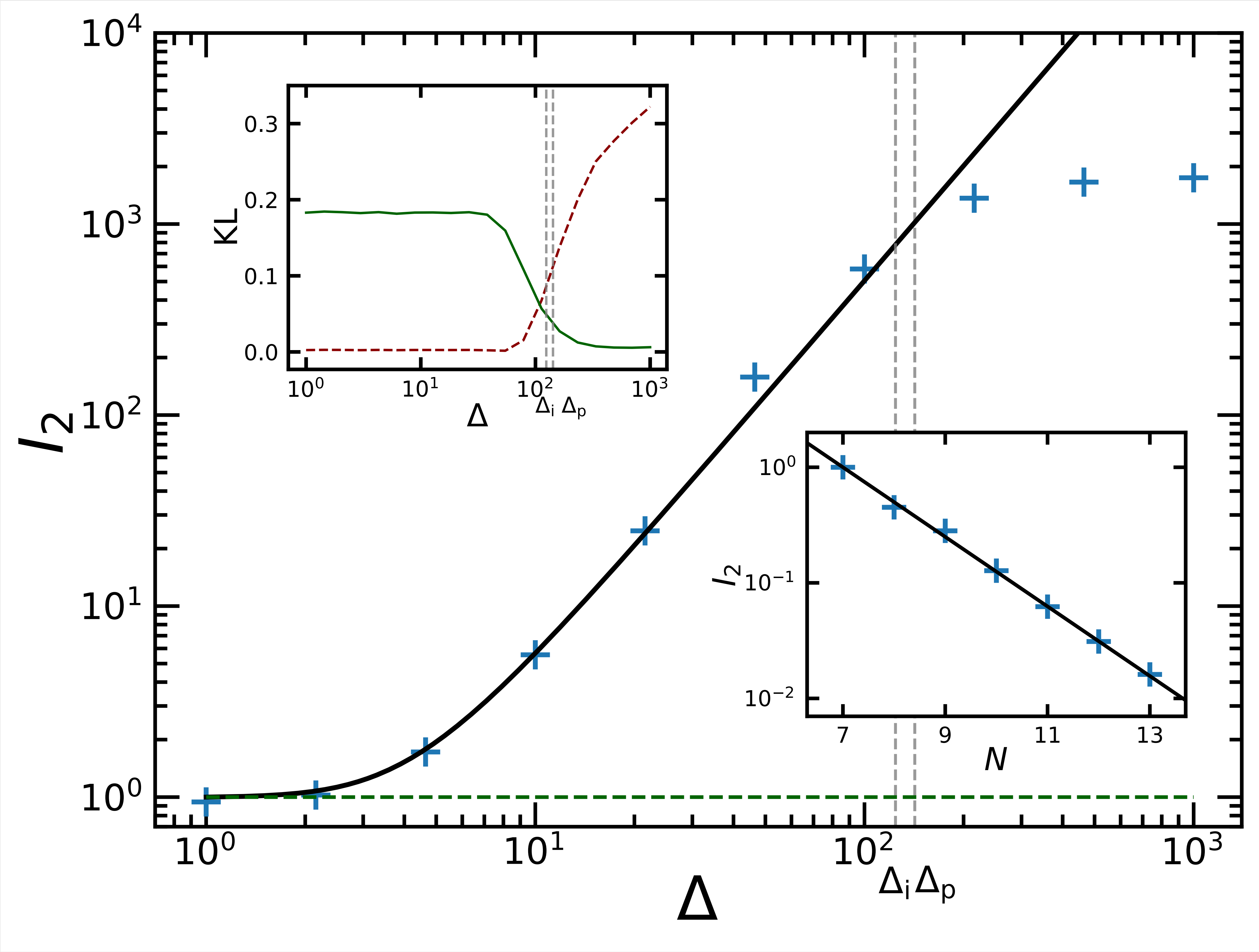
$$I_q \equiv \sum_n \langle |\psi_n|^{2q} \rangle = -(-2)^q q D^{1-q} \partial_{y_0^2}^{q-1} (1/y_0\Delta) \arctan\left(\Delta/2y_0\right)$$

$$y_0 = \frac{2}{\Delta} \arctan\left(\frac{\Delta}{2y_0}\right)$$

$\Delta \lesssim 1$	$I(q) = q!(D/2)^{1-q}$	RMT
$\Delta \gg 1$	$I_q = (2\pi^2)^{1-q} q(2q-3)!!\Delta^{2(q-1)} D^{1-q}$	parametric deviations from RMT
$\Delta \sim N^\alpha$	$I_q \sim [D/(\log D)^{2\alpha}]^{1-q}$	wave function nonergodicity

arXiv 1901.02389

Main message: for $1 < \Delta < N^\alpha, \alpha < 2$ wave functions are fragmented and populate only fraction $\sim N^{-\alpha}$ of Fock space. For $\alpha > 2$: Fock space localization


10^4
10^3
10^2
10^1
10^0
I_2
10^0
10^1
10^2
10^3
Δ
$\Delta_i \Delta_p$
KL
0.3
0.2
0.1
0.0
10^0
10^1
10^2
10^3
Δ
Δ_i Δ_p
I_2
10^0
10^{-1}
10^{-2}
7
9
11
13
N

1901.02389

summary

SYK model a paradigm of many body quantum chaos

shows exponential instabilities, strong quantum fluctuations, spectral and wave function correlations

a testbed for numerical and analytical theories

ramifications in holography